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SALEM - 636 011.

B.Sc. CHEMISTRY

FIRST YEAR

PAPER – II : PHYSICS ALLIED - I

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B.Sc. CHEMISTRY

FIRST YEAR

PAPER – II : PHYSICS ALLIED - I

BLOCK	**INTRODUCTION**
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UNIT II	**HEAT & OPTICS**
UNIT III	**ELECTRICITY, MAGNETISM & ATOMIC PHYSICS**
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MECHANICS AND SOUND

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ACOUSTICS & PROPERTIES OF MATTER : Production of ultrasonic waves – detection – properties – applications – bending of beams – expression for bending moment – Theory of uniform and non uniform bending – experiments – expression for couple per unit twist – rigidity modulus by torsion pendulum – static torsion method – excess pressure inside a curved surface – surface tension and interfacial surface tension by drop weight method.

UNIT – II

HEAT & OPTICS : Vander Waal's equation – expression for critical constants – Joule Thomson effect Theory and experiment – inversion temperature – Boyle temperature – liquefaction of hydrogen and helium – Interference of light – air wedge method – thickness of a wire – Jamin's interferometer – Rayleigh's interferometer – specific rotatory power and its determination.

UNIT – III

ELECTRICITY, MAGNETISM & ATOMIC PHYSICS : Carey Foster bridge – Theory – measurement of resistance – Potentiometer – low range voltmeter and ammeter calibration – Theory of moving coil ballistic galvanometer – determination of charge sensitivity – Comparison of capacities – Vector atom model – spatial quantization – spinning electron – quantum numbers associated with vector atom model – coupling schemes – L-S coupling – j-j coupling – Pauli's exclusion principle – periodic table – examples of electron configuration.

UNIT – IV

NUCLEAR PHYSICS & SOLID STATE PHYSICS : Nuclear models – liquid drop model – shell model – merits and demerits – Nuclear detectors – Ionisation chamber – GM counter – cloud chamber – Particle accelerator – synchrocyclotron – betatron – bonding in crystals – ionic bond – covalent bond – metallic bond – molecular bond – hydrogen bond.

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ELECTRONICS : Construction and characteristics of FET, SCR and UJT – Phase shift oscillator with theory – Multivibrator – astable – monostable and bistable – basic circuits – binary, octal and hexadecimal numbers and their inter conversions – laws of Boolean algebra – DeMorgan's theorems – NAND, NOR gates as universal building blocks – Karnaugh mapping – pairs, quads, octets – simplification of Boolean expressions – Half and full adders – Half and full subtractors.

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BLOCK INTRODUCTION

This book has been strictly in accordance with syllabus in Allied Physics for B.Sc students of PRIDE. The subject matter in this book has been astutely developed. A large number of questions and problems have been given at the end of each chapter. The student who attempts to tackle them successfully is bound to get a better insight and understanding of the subject.

The book illustrated with a large number of carefully drawn diagram which the students can easily reproduced.

The subject matter is divided into five units. Each unit is self contained and is treated in a comprehensive way.

In first unit we will discuss about the Acoustics and Properties of matter.

In the second unit we are going to learn about the Heat and Optics

In the third unit we are going to study about the Electricity, Magnetism and Atomic physics.

In the fourth unit we are going to deal about the Nuclear Physics & Solid state physics.

In the fifth unit we are going to know and discuss the Electronics. Suggestions and Opinions for improvement of the book are cordially invited.

UNIT - I

ACOUSTICS & PROPERTIES OF MATTER

ACOUSTICS

1.1 Ultrasonics

The human ear is unable to hear sounds of frequency less than 20 and more than 20000. Sounds of frequency less than 20 are called infrasonics. Sound waves of frequency more than 20000 Hz are called ultrasonics. These waves also travel with the speed of sound ($330ms^{-1}$). Hence they have a wavelength smaller than 330/20000 or 0.0165m. So the ultrasonic waves will have wavelengths of 1.65cm and less.

1.1.1 Production of ultrasonic waves:

(i) Magnetostriction oscillator

Principle: when a rod of ferromagnetic material like nickel is magnetized longitudinally, it undergoes a very small change in length. This is known as magnetostriction effect. A nickel rod placed in a rapidly varying magnetic field alternately expands and contracts with twice the frequency of the applied magnetic field to be equal to the natural frequency of longitudinal vibration of the rod, resonance can be produced. Due to resonance, vibrations of large amplitude are produced in the rod which generates ultrasonic waves.

Experimental arrangement : The ferromagnetic rod (nickel) is clamped at its mid-point O. Two magnetizing coils L_1 and L_2 are wound round the two halves of the rod respectively (Fig. 1.1.1(i)) L_1 is connected in the grid circuit and L_2 is connected in the plate circuit of a triode valve. The valve acts as an oscillator and supplies high frequency alternating current to the coils. Then the rod is subjected to periodic changes in length. The frequency of the valve oscillator is adjusted by means of the variable condenser C so that it is equal to the natural frequency of the rod. Then the rod vibrates with a large amplitude. Producing ultrasonic waves in the surrounding medium. Magnetostrction oscillators are mechanically rugged. They are capable of producing large acoustical power with fairly good efficiency (eg. 60%). Their deficiencies are : (1) low upper frequency limit and (2) conversion losses due to hysterisis and eddy currents.

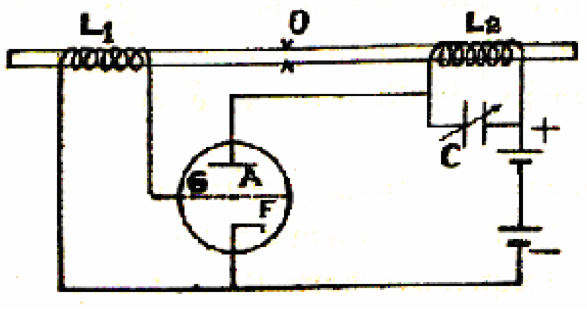


Fig.1.1.1 (i)

(ii) Piezo-eletric generator : Principle : If one pair of opposite faces of certain doubly refracting crystals, cut in a specified manner, is subjected to pressure, the other pair or opposite faces develop equal and opposite electric charges on them. The sign of the charges is reversed, when the faces are subjected to tension instead of pressure. The electric charge developed is proportional to the amount of pressure or tension. This phenomenon is known as piezo-eletric effect. The effect is reversible i.e. if an electric field is applied across one pair of faces of the crystal, contraction or expansion occurs across the other pair.

The crystal commonly used for this purpose is quartz. The natural quartz crystal is as shown in Fig. 1.1.1 (ii). Its cross-section is hexagonal. The axis along the longest dimension it the optic axis called Z axis. The lines joining the opposite corners are called the electric axes (or X-axes). We have three such axis. An axis perpendicular to both these axes is the mechanical axis (or Y axes). The type of crystal used for ultrasonic vibrations is generally the X-cut crystal, is obtained by cutting the hexagonal slice along Y axis such that the largest face are silvered to ensure proper contact and alternating field is applied by means of metallic plates pressing against the faces.

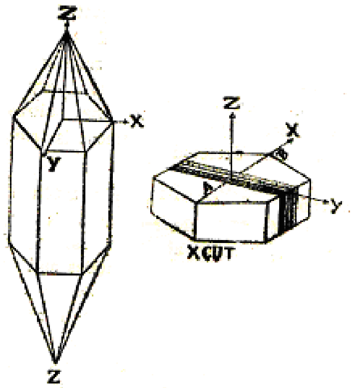


Fig. 1.1.1 (ii).

Experimental arrangement: The circuit diagram is shown in Fig 1.1.1(iii). The quartz crystal Q is placed between two metal plates A and B. The metal plates are connected to the primary (L_3) of a transformer which is inductively coupled to the oscillatory circuit of a triode valve. Coil L_2 is connected in the plate circuit. The tank circuit L_2 and C_1 is connected between the gird and the cathode. When the valve oscillates, high frequency alternating voltages are impressed on the plates A and B.

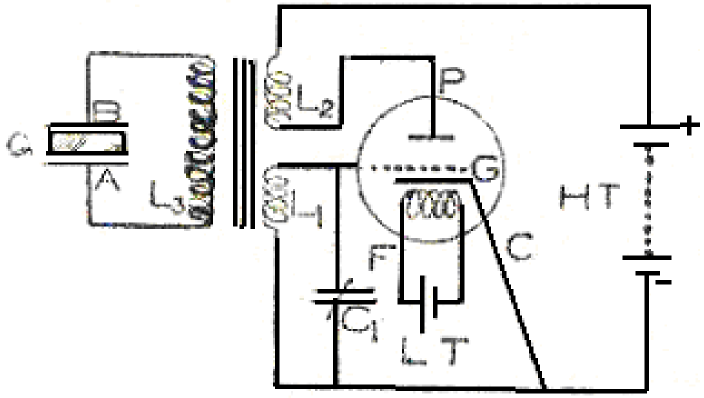


Fig. 1.1.1 (iii)

Inverse piezo-electric effect takes place and the crystal contracts and expands periodically. The variable condenser C_2 is adjusted so that the frequency of the oscillatory circuit is equal to one of the natural frequencies of the crystal. Due to resonance, the amplitude of the vibrations of the crystal becomes very large. The crystal can be immersed in a liquid so that ultrasonic waves are produced in the liquid. This arrangement is suitable for experiments in liquids.

Detection of ultrasonic waves: (1) Kundt's tube method:

If the wavelength of the ultrasonic waves is greater than a few millimeters, a Kundt's tube can be used to form a stationary wave pattern with well defined nodes and antinodes. In the air or gas medium, lycopodium power collects in the form of heaps at the nodes. In a liquid medium, powdered coke is used to detect the position of nodes.

(2) Piezo-electric detectors : This method is based on piezo-electric effect. When one pair of opposite faces of a quartz crystal is exposed to ultrasonic waves opposite charges are developed on the other pair of opposite faces, perpendicular to the first. These charges are amplified and detected using an electronic circuit.

(3) Thermal detectors: Modern methods of detection use thermal detectors. A platinum wire is placed in the region to be tested for ultrasonic waves. At nodes, due to alternate compressions and rarefactions. Alternate heating and cooling is produced, change in temperature at the node brings about changes in the electrical resistance of the platinum wire. This is detected by means of a Wheastone's bridge. No change in temperature occurs at the antinode.

Properties of ultrasonic waves:

(1) They have a high energy content.

(2) Just like ordinary sound waves, ultrasonic waves get reflected, refracted and absorbed.

(3) **Thermal detectors**: Modern methods of detection use thermal detectors. A platinum wire is placed in the region to be tested for ultrasonic waves. At nodes, due to alternate compressions and rarefactions, alternate heating and cooling is produced, change in temperature at the node brings about changes in the electrical resistance of the platinum wire. This is detected by means of a Wheatstone's bridge. No change in temperature occurs at the antinode.

(4) They can be transmitted over large distances with no appreciable loss of energy.

(5) If an arrangement is made to form stationary waves of ultrasounds in a liquid, it serves as a diffraction grating. It is called an acoustic grating.

(6) They produce intense heating effect when passed through a substance.

Applications of ultrasonic waves:

(1) etermination of the depth of sea: The ultrasonic waves are directed to the bottom of the sea. The time (t) that elapses between the emission of ultrasonic waves and the reception of the echo is recorded. Then the depth of the sea $= h = vt/2.\ where\ v =$ the velocity of ultrasonic waves through sea-water.

(2) Detection of air craft, submarines etc :- Bacause of their short wavelength, ultrasonic waves are highly directional. A piezo-electric quartz crystal oscillator is used for sending out a beam of ultrasonic waves. This is reflected back from an aircraft, if one comes in its way. The reflected beam is detected by a quartz receiver. The entire system is called SONAR from sound navigation and ranging.

(3) Sound signaling: The highly directional ultrasonic sound beam is used for purposes of signaling to a distant ship.

(4) Chemical Applications: (i) Ultrasonic waves are used to form stable emulsions of even immiscible liquids like water and oil or water and mercury. This finds an application in the preparation of photographic films, face cream etc, (ii) They are used to liquefy gels like aluminum hydroxide in the same way as they are liquefied like by shaking (iii) They are used to coagulate fine solid or liquid particles in a gas for example, dust, smoke. Mist etc. Ultrasonics thus find use in collecting factory dust and purifying the air (iv) Ultrasonics act like a catalytic agent and accelerate chemical reactions. Ultrasonic waves accelerate crystalisation

(5) Science

i. Investigation of Structure of Matter

We can determine the velocity of ultrasonics in liquids and gases and its variation with frequency and temperature. This study gives information about a number of properties of the medium such as its compressibility, absorption, concentration, specific heat capacity, chemical structure, arrangement of atoms in them etc.

ii. Study of Molecular Energies

The frequencies of molecular vibrations are of the same order as the ultrasonic vibrations. So ultrasonic waves are used in the study of molecular energies. They are used in molecular acoustics for investigating structure and properties of substances.

iii. Elastic Symmetries of Crystals

When ultrasonic waves are applied to certain crystals, they give rise to diffraction images. The diffraction images reveal the elastic symmetries of crystals.

(6) Industrial Application

a. Non-Destructive Testing (NDT)

Principle: Whenever there is a change in medium, the ultrasonic waves will be reflected. Since the flaws can be detected without destroying the materials, it is called non-destructive testing.

Working: The pulse echo system used to determine the various flaws like cracks, holes, air bubbles, laminations, etc., in the specimen

a - Incident pulse

b - Pulse from flaw

c - Reflected pulse from the boundary of the specimen

TR - Transducer

CRT – Cathode Ray tube.

Here short pulse of ultrasonic waves are transmitted into the material being tested. These pulses get reflected from discontinuities on their path or from any boundary of the material on which they strike. The received echoes are then displayed on a cathode ray tube screen. The CRT screen furnishes specific data as to the relative size of a discontinuity in terms of signal amplitude. The location of the discontinuity with respect to the scanning surface can be obtained by proper calibration of the CRT time base scale.

b. Ultrasonic Soldering

Ultrasonic solders are used for soldering aluminium coil capacitors, aluminium wires and plates without using any fluxes.

An ultrasonic soldering iron consists of an ultrasonic generator having a tip fixed at its end. The tip is heated by an electrical heating element. The tip of the soldering iron melts solder on the aluminium. The ultra vibrator removes the aluminium oxide layer. The solder thus gets fastened to the clear metal without any difficulty.

c. Ultrasonic Welding

The properties of some metals changes on heating. Therefore, they cannot be welded by electric or gas welding. In such cases, the metal sheets are welded together at room temperature using ultrasonic waves.

d. Ultrasonic Drilling and cutting

Ultrasonics are used for making holes in very hard materials such as glass, diamond, gems and ceramics.

e. Ultrasonics in Metallurgy

To irradiate molten metals which are in the process of cooling, so as to refine the grain size and to prevent the formation of cores and to release trapped gases, the ultrasonic waves are used.

f. Formation of Alloys

The constituents of alloys, having widely different densities, can be mixed uniformly by a beam of ultrasonics. Thus it is easy to get alloys of uniform composition.

g. Acoustic Halograms

Surface structures of various engineering materials used for space applications can be studied by using acoustic halograms.

h. Sound Navigation and Ranging (SONAR)

Ultrasonic waves sent form a point A travel through sea of water and get reflected back from the bottom of the sea. The reflected waves are received at the point B. Using a CRO the time taken t, for the ultrasonic wave to travel to the bottom of the sea and reflected back to the surface is calculated.

Let v = velocity of ultrasonic wave in sea water.

$$\text{Depth of the sea} = \frac{vXt}{2}.$$

The same method is used for finding the distance and direction of a submarine. The change in frequency of the echo signal due to Doppler Effect helps to determine the velocity of the submarine and its direction. The whole system is called SONAR.

(7) Biological Applications: small animals like frog, fish, rat etc, are maimed or killed when exposed to ultrasonic waves. Ultrasonic waves can kill bacteria and therefore they are used for sterilizing milk.

(8) Medical Applications: They are used for (i) treatment of neuralgic and rheumatic pains (ii) relieving pain in Arthritis (iii) restoring contracted fingers (iv) extraction of broken teeth etc. When ultrasonic waves are focused on a sharp instrument, it becomes capable of destroying tissues without loss of blood. Such instruments are used in performing bloodless brain surgery.

(9) Industrial uses: (i) A glass rod oscillating with ultrasonic frequency can be used to bore holes in steel and other hard metals. (ii) Ultrasonic waves are used to detect creaks or flaws in metal structures. In this respect they are much better then X-rays (iii) They are used for producing alloys of uniform composition. (iv) They are used for cleansing clothes and parts of watches etc.

(10) Determination of velocity of ultrasonic waves in a liquid

Ultrasonic diffraction : The apparatus consists of a quartz crystal forming ultrasonic waves in a liquid contained in a glass vessel (fig.1.1.(10)). These waves are reflected by the reflector. Due to superposition of direct and reflected waves, stationary waves are formed between the crystal and reflector. The density of the liquid is maximum at the nodal planes of the stationary wave pattern. The density is minimum at the antiondal planes. So alternate regions of high and low density are formed in the liquid behaves as the acoustic grating the distance between two adjacent nodal planes constitutes the grating element d.

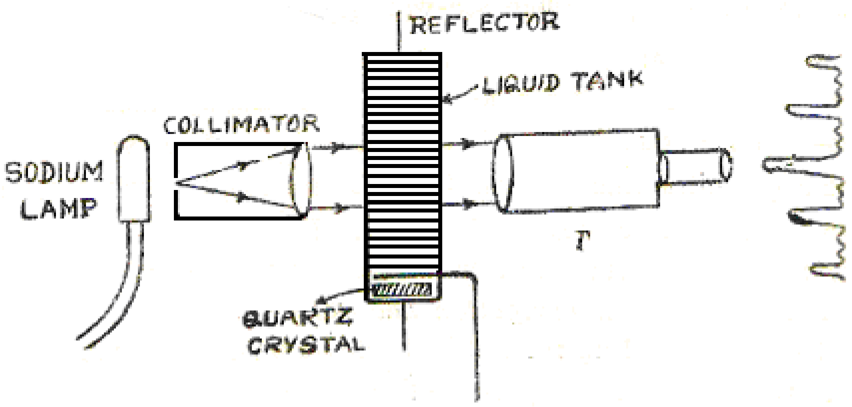


Fig 1.1.(10)

The acoustic grating is mounted on the prism table of a spectrometer. A parallel beam incident on the acoustic grating, the diffracted beam of light is viewed through the telescope T. The diffraction pattern consists of a central maximum and principal maxima on either side. If θ is the angle of diffraction for the nth order principal maximum $2d\sin\theta, n\lambda$ Hence, λ wavelength of the light used; knowing $\theta, n and \lambda$ the value of d can be calculated. If λu is the wavelength of ultrasonic waves in the liquid, $d = \lambda u / 2 or \lambda u = 2d$. The frequency N of ultrasonic waves produced by the quartz oscillator is found by means of a wave meter. The velocity of ultrasonic waves $= N\lambda^{u}$ can then be determined.

PROPERTIES OF MATTER

ELASTICITY (Bending of Beams)

1.2 Introduction

A body can be deformed (i.e., changed in shape or size) by the suitable application of external forces on it. A body is said to be perfectly elastic, if it regains its original shape or size, when the applied forces are removed. This property of a body to regain its original state or condition on removal of the applied forces is called elasticity. A body which does not tend to regain its original shape or size, even when the applied forces are removed, is called a perfectly plastic body. No body, in nature, is either perfectly elastic or perfectly plastic. Quartz fibre is the nearest approach to a perfectly elastic body.

When an external force is applied on body, there will be relative displacement of the particles and due to the property of elasticity, the particles tend to regain their original positions. Stress is defined as the restoring force per unit area. If a force F is applied normally to the area of cross-section A of a wire, then stress = F/A. Its dimensions are $ML^{-1}\ T^{-2}$.

Definitions

Beam: A beam is defined as a rod or bar of uniform cross-section (circular or rectangular) whose length is very much greater than its thickness.

Bending Couple: If a beam is fixed at one end and loaded at the other end, it bends. The load acting vertically downwards at its free end and the reaction at the support acting vertically upwards, constitute the bending couple.

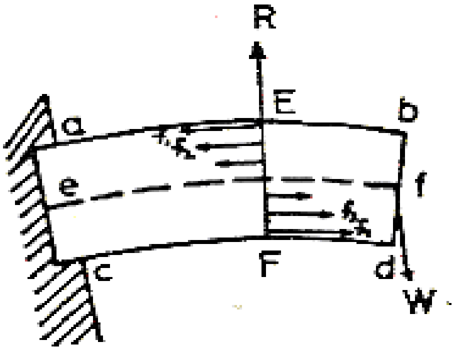


Fig. 1.2

This couple tends to bend the beam clockwise. Since there is no rotation of the beam, the external bending couple must be balanced by another equal and opposite couple which comes into play inside the body due to elastic nature of the body. The moment of this elastic couple is called the internal bending moment. When the beam is in equilibrium, the external banding moment = the internal bending moment.

Plane of Bending : The plane of bending is the plane is which the bending takes place and the bending couple acts in this plane. In Fig. 1.2, the plane of paper is the plane of bending.

Neutral Axis: When a beam is bent as in Fig. 1.2, filaments like ab in the upper part of the beam are elongated and filament like cd in the lower part are compressed. Therefore, there must be filament like ef in between, which is neither elongated nor compressed. Such a filament is known as the neutral filament and the axis of the beam lying on the neutral filament is the neutral axis. The change in length of any filament is proportional to the distance of the filament from the neutral axis.

1.3 Expression for the bending moment

Consider a portion of the beam to be bent into a circular arc, as shown in Fig. 1.3. ef is the neutral axis. Let R be the radius of curvature of the neutral axis and θ the angle subtended by it at its centre of curvature C.

Filaments above ef are elongated while filaments below ef are compressed. The filament ef remains unchanged in length.

Let ab be a filament at a distance z from the neutral axis. The length of this filament a′b′ before bending is equal to that of the corresponding filament on the neutral axis ab.

We have, original length = ab = Rθ

Its extended length = a′b′ = (R+z) θ

Increase in its length = a′b′ -ab = (R+z) θ - Rθ = z. θ.

$$\therefore \quad \text{Linear strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{z.\theta}{R.\theta} = \frac{z}{R}$$

If E is the Young’s modulus of the material,

E = Stress / Linear strain

ie., Stress = E X Linear strain = E(z/R)

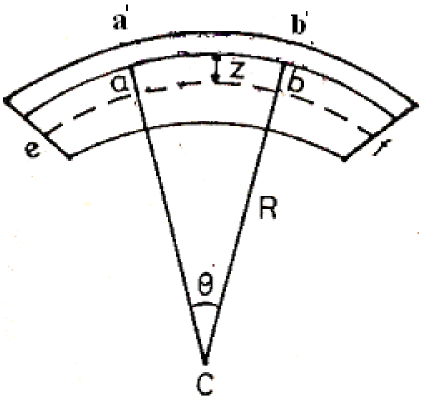


Fig 1.3.

If δA is the area of cross-section of the filament,

the tensile force on the area δA = stress X area = $\frac{E.z}{R}\delta A.$

Moment of this force about the neutral axis ef

$$= \frac{E.z}{R}\delta A.z = \frac{E}{R}\delta A.z^2.$$

$$\left.\begin{matrix}\text{The sum of the moments of}\\ \text{forces acting on all the filaments}\end{matrix}\right\} = \sum \frac{E}{R}\delta A.z^2.$$

$$= \frac{E}{R}\sum \delta A.z^2$$

$\sum \delta A.z^2$ is called the geometrical moment of inertia of the cross-section of the beam about an axis through its centre perpendicular to the plane of bending. It is written as equal to Ak^2. i.e., $\sum \delta A.z^2 = Ak^2$. (A = Area of cross-section and k = radius of gyration).

But the sum of moments of forces acting on all the filaments is the internal bending moment which comes into play due to elasticity.

Thus, bending moment of a beam = $E\ Ak^2 / R$.

Notes : (i) For a rectangular beam of breadth b, and depth (thickness) d, $A = bd$ and $k^2 = d^2 / 12$.

∴ $Ak^2 = bd^3 / 12$.

(ii) For a beam of circular cross-section of radius r, $A = \pi r^2$ and $k^2 = r^2 / 4$.

∴ $Ak^2 = \pi r^4 / 4$.

(iii) $E\ Ak^2$ is called the flexural rigidity of the beam.

1.3.1 Depression of the loaded end of a cantilever

Cantilever: A cantilever is a beam fixed horizontally at one end and loaded at the other end.

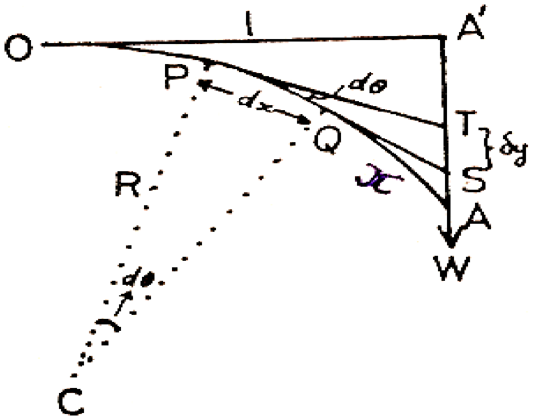


Fig. 1.3.1

Let OA be a cantilever of length l fixed at O and loaded with a weight W at the other end. OA′ is the unstrained position of the beam. Let the depression A′A of the free end be y (Fig. 1.3.1). Let us consider an element PQ of the beam of length dx at a distance (QA = x) from the loaded end. C is the centre of curvature of the element PQ and R its radius of curvature. The load W at A and the force of reaction W at Q constitute the external couple, so that, the external bending moment = W.x.

The internal bending moment = $\frac{E.Ak^2}{R}$

For equilibrium, Wx = $\frac{E.Ak^2}{R}$ or R = $\frac{E.Ak^2}{Wx}$

…(1)

Draw tangent at P and Q meeting the vertical line at T and S respectively. Let TS = dy and dθ = Angle between the tangents. Then, ∠PCQ also = dθ.

Now, PQ = dx = R dθ or dθ = $\frac{dx}{R} = dx.\frac{Wx}{EAk^2}$ (From Eq. 1)

We have, dy = x dθ = x.$\frac{Wxdx}{EAk^2}$ = $\frac{Wx^2dx}{EAk^2}$ …(2)

$$\therefore \quad \left.\begin{array}{l}\text{the total depression of}\\ \text{the end of the cantilever}\end{array}\right\} = y = \int_0^1 \frac{Wx^2}{EAk^2}dx = \frac{Wl^3}{3EAk^2}$$

Angle between the tangents at the ends of a cantilever:

Since the beam is fixed horizontally at O, the tangent at O is horizontal. If a tangent is drawn at A (the free end of the bent bar), it makes an angle θ with the horizontal.

$$\left.\begin{array}{l}\text{Angle between the}\\ \text{tangents at P and Q}\end{array}\right\} = d\theta = \frac{Wx}{EAk^2}dx.$$

$$\left.\begin{array}{l}\text{Angle between the}\\ \text{tangents at O and A}\end{array}\right\} = \theta = \int_0^1 \frac{Wx}{EAk^2}dx.$$

$$\therefore \quad \theta = \frac{Wl^2}{2EAk^2}.$$

Work done in uniform bending. Consider a beam bent uniformly by an external couple. Let A be the area of cross-section of the beam. Consider a filament of area of cross-section δA at a distance z from the neutral axis . Then,

The tensile force on the area δA = $\frac{Ez}{R}\delta A.$.

The linear strain of this filament = z / R. If l is the length of the filament, then,

The extension of the filament = zl / R.

$$\left.\begin{array}{l}\text{The work done in}\\ \text{bending the filament}\end{array}\right\}=\frac{1}{2}\,forceXextension$$

$$=\frac{1}{2}\frac{Ez}{R}\delta AX\frac{zl}{R}=\frac{1}{2}\frac{El}{R^2}Xz^2.\delta A.$$

For uniform bending R is constant. Hence, the work done in bending the whole beam is

$$W=\frac{1}{2}\frac{El}{R^2}\sum z^2\delta A=\frac{1}{2}\frac{El}{R^2}XAk^2=\frac{1}{2}\frac{EAk^2}{R}X\frac{1}{R}$$

Here, $\frac{EAk^2}{R}$ = the bending moment and l / R = the angle subtended by the bent beam at its centre of curvature.

∴ The work done in uniform bending = $\frac{1}{2}$(bending moment) X (Angle subtended by the bent beam at its centre of curvature).

Example 1: Obtain an expression for the depression at the free end of a heavy beam clamped horizontally at end and loaded at the other end.

Consider an element PQ of the beam of length dx at a distance x from the fixed end O. Now, in addition to the load W acting at A, a weight equal to that of the portion (*l* – x) of the beam also acts at is mid – point. Let W_1 be the weight of the beam. Then, the weight per unit length of the beam = W_1/l. Now, we have an additional weight $W_1(l-x)/l$ acting at a distance (*l* – x)/2 from Q. Therefore,

$$\left.\begin{array}{l}\text{total moment of the}\\ \text{external couple applied}\end{array}\right\}=W(l-x)+\frac{W_1}{l}(l-x)\frac{(l-x)}{2}$$

$$=W(l-x)+\frac{W_1}{2l}(l-x)^2$$

The beam being in equilibrium, this must be balanced by the bending moment $\frac{EAk^2}{R}$. Therefore,

$$W(l-x)+\frac{W_1}{2l}(l^2-2l.x+x^2)=\frac{EAk^2}{R}=EAk^2\left(\frac{d^2y}{dx^2}\right)$$

Integrating,

$$W\left(lx-\frac{x^2}{2}\right)+\frac{W_1}{2l}\left(l^2.x-l.x^2+\frac{x^3}{3}\right)=EAk^2\frac{dy}{dx}+C$$

where C is constant of integration.

Since at x = 0, dy/dx =0, we have C = 0.

Integrating once again,

$$EAk^2\int_0^y dy = W\int_0^1 (lx - x^2/2)dx + \frac{W_1}{2l}\int_0^1 (l^2x - lx^2 + x^3/3)dx$$

or $$EAk^2 y = W\left(\frac{l^3}{3}\right) + \frac{W_1}{2l}\left(\frac{l^4}{4}\right)$$

or $$EAk^2 y = \frac{Wl^3}{3} + \frac{W_1 l^3}{8}.$$

or $$y = (W + \frac{3}{8}W_1)\frac{l^3}{3EAk^2}$$

1.3.2 Measurement of E

(1) Cantilever depression :

The given beam is clamped rigidly at on end (Fig. 1.3.2(a)). A weight – hanger (H) is suspended at the free end of the beam. A pin (P) is fixed vertically by some wax at the free end of the beam. A traveling microscope (M) is focused on the pin. The microscope is adjusted so that the horizontal cross-wire coincides with the tip of the pin and the reading on the vertical scale is noted. Then weights m, 2m, 3 m, 4 m, etc., are added to the weight-hanger. The microscope is adjusted each time to make the horizontal cross-wire coincide with the tip of the pin and the reading on the vertical scale of the microscope is noted in each case. Observations are made for decreasing loads also. The results are tabulated as follows:

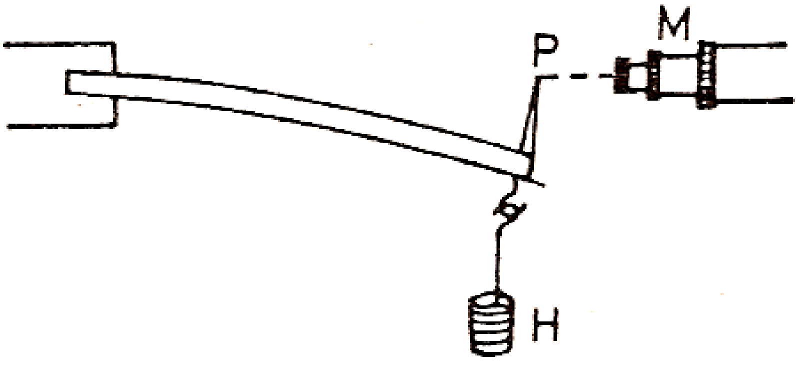


Fig. 1.3.2(a)

The length of the beam (l0 between the clamped end and the loaded end is measured. The mean breadth (b) of the beam and its mean thickness (d) are determined.

If y is the depression produced for a load of Mg, then,

$$y = \frac{Mgl^3}{3EAk^2} \; or E = \frac{Mgl^3}{3Ak^2 y}.$$

Load in kg	Microscope Reading			Depression For M(= 4 m)	Mean depression for a load of M kg
	Load Increasing	Load Decreasing	Mean		
W W + m W + 2m W + 3m			x x_1 x_2 x_3		
W + 4m W + 5m W + 6m W + 7m			x_4 x_5 x_6 x_7	$x_4 - x$ $x_5 - x_1$ $x_6 - x_2$ $x_7 - x_3$	

The mean depression(y) for a load M kg is found.

∴ Now, $Ak^2 = bd^3 / 12$ for a rectangular beam.

$$\text{Hence, } E = \frac{Mgl^3}{3(bd^3/12)y} = \frac{4Mgl^3}{bd^3 y.}$$

The young’s modulus of the material of the beam is calculated using this relation.

(2) E - by measuring the tilt in a loaded cantilever.

The given rectangular beam is rigidly clamped at one end and a small plane mirror M is fixed at the free end [Fig. 1.3.2(b)]. A weight hanger (H) is attached at the free end of the beam. A vertical scale (S) and telescope (T) are arranged in front of the mirror. The telescope is focused so that the image of the vertical scale due to reflection in the mirror is obtained in the telescope. The reading on the scale which coincides with the horizontal cross-wire is noted. Then weights m, 2m, 4m etc., are added to the weight-hanger and the readings of the scale as observed in the telescope, are noted in each case.

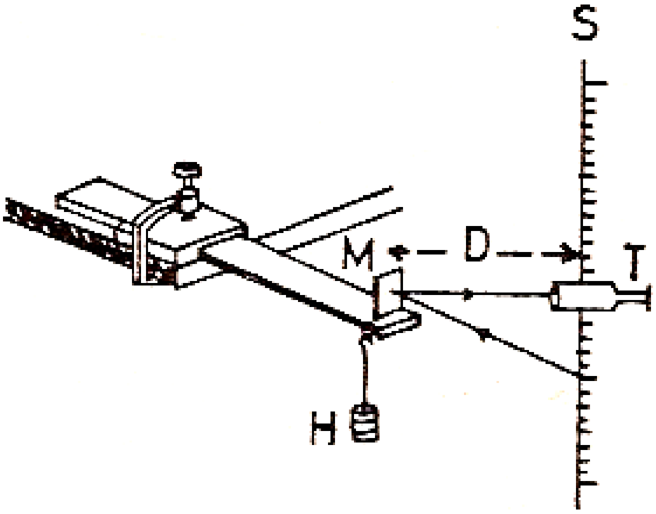


Fig 1.3.2(b)

Observations are made for decreasing loads also. **The results are tabulated as follows:**

Load in kg	Readings on the scale			Change in scale reading for M kg
	Load increasing	Load decreasing	Mean	

The average of the readings in the last column gives the mean change in scale reading s for a load of M kg. The distance D between the mirror and the scale is found. The breadth (b) and the thickness (d) of the beam are accurately measured.

$$\left.\begin{array}{l}\text{The angle between the two ends}\\\text{of the cantilever for a load of M kg}\end{array}\right\} = \theta = \frac{s}{2D}$$

…(1)

But, $$\theta = \frac{\mathrm{Mg.l}^2}{2\mathrm{EAk}^2} = \frac{\mathrm{Mg.l}^2}{2\mathrm{E.bd}^3/12}$$

$$\left(Since Ak^2 = \frac{bd^3}{12} \right)$$

$$\theta = \frac{6Mgl^2}{bd^3E}. \qquad \text{…(2)}$$

From (1) and (2), $\dfrac{s}{2D} = \dfrac{6Mg.l^2}{bd^3E} or E = \dfrac{12Mg.l^2D}{bd^3s}$.

1.3.3 Oscillations of a cantilever

Let OA be a cantilever of length l, of negligible mass fixed at O. Let a mass M be attached at the other end A (Fig.1.3.3). If the mass is slightly depressed and then released, the cantilever will execute simple harmonic motion about its original depressed position.

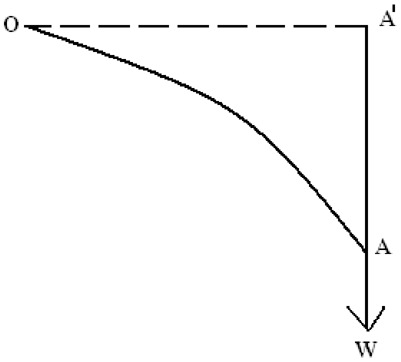


Fig.1.3.3

The depression of the loaded end of the cantilever is

$$y = \frac{Wl^3}{3EAk^2}$$

or $$W = \frac{3EAk^2}{l^3} y.$$

This must be equal to the elastic reaction of the cantilever balancing it and hence directed oppositely to it.

If M is the mass of the weight W and d^2y/dt^2, the acceleration (upwards), we have,

$$\text{elastic reaction} = M\frac{d^2y}{dt^2}$$

$$\therefore \quad -M\frac{d^2y}{dt^2} = \frac{3EAk^2}{l^3} y$$

or $$\frac{d^2y}{dt^2} = \frac{-3EAk^2}{Ml^3} y.$$

But, $$\frac{3EAk^2}{Ml^3} = \text{A constant}$$

The acceleration of mass M or the free end of the cantilever is thus proportional to its displacement and is directed opposite to it.

It, therefore, executes a S.H.M. of time period T, given by

$$T = 2\pi\sqrt{\frac{Displacement}{Acceleration}} = 2\pi\sqrt{\frac{y}{\left(\frac{3EAk^2 y}{M.l^3}\right)}} = 2\pi\sqrt{\frac{Ml^3}{3EAk^2}}$$

If the mass of the cantilever is not negligible, it can be shown that,

$$T = 2\pi\sqrt{\frac{(M + \frac{1}{3}m)l^3}{3EAk^2}}$$ where m = mass of the cantilever.

The mass of the cantilever can be eliminated by finding the periods T_1 and T_2 for two different masses M_1 and M_2 attached to the cantilever at the same length. Then,

$$= T_1^2 = 4\pi^2 \frac{(M_1 + \frac{1}{3}m)l^3}{3EAk^2} \text{ and } T_2^2 = 4\pi^2 \frac{(M_2 + \frac{1}{3}m)l^3}{3EAk^2}$$

or $$T_2^2 - T_1^2 = \frac{4\pi^2 (M_2 - M_1)l^3}{3EAk^2}.$$

∴ $$E = \frac{4\pi^2 (M_2 - M_1)l^3}{3Ak^2 (T_2^2 - T_1^2)}.$$

Experiment: The given beam is rigidly clamped at O. A certain load of M_1 kg is suspended from the other end A. The beam is set in transverse oscillations and the time for 25 oscillations is found. From this the period of oscillation T_1 is calculated. Similarly, the period T_2 with at load M_2 is found.

We have, $$E = \frac{4\pi^2 (M_2 - M_1)l^3}{3Ak^2 (T_2^2 - T_1^2)}.$$

For a rectangular bar, $Ak^2 = bd^3 / 12$.

Hence, $$E = \frac{16\pi^2 l^3}{bd^3} \frac{(M_2 - M_1)}{(T_2^2 - T_1^2)}$$

The length of the cantilever l, the breadth b and depth d are measured. E is calculated using the above formula.

Example 1: A steel bar, 0.3 m long, 2 X 10^{-2} m broad and 2 X 10^{-3} m thick is clamped at one end and loaded at the other with a mass of 0.01 kg. Calculate the period of vibration of the bar, neglecting the effect of weight of the bar. (E for steel = 20 X 10^{10} Nm^{-2}).

$$\left.\begin{array}{l}\text{For a cantilever of length l, loaded}\\ \text{with a mass M, period of vibraiton}\end{array}\right\} = T = 2\pi\sqrt{\frac{Ml^3}{3EAk^2}}$$

Here, l = 0.3m, M = 0.01 kg; E = 20 X 10^{10} Nm^{-2}

$Ak^2 = bd^3 / 12 = (2 \text{ X } 10^{-2})(2 \text{ X } 10^{-3})^3/12 = \frac{4}{3} X 10^{-11}$

$$\therefore \qquad T = 2\pi \sqrt{\frac{(0.01) X (0.3)^3}{3X(20X10^{10})X\left(\frac{4}{3}X10^{-11}\right)}} = 0.0365 \sec onds.$$

1.3.4 Depression at the mid-point of a beam loaded at the middle

Let AB represent a beam of length l, supported on two knife-edges at A and B and loaded with a weight W at the centre C. The reaction at each knife-edge is W/2 acting vertically upwards. The beam bends as shown in Fig. 1.3.4, the depression being maximum at the centre. The bending is non-uniform. Let CD = y.

The portion DA of the beam may be considered as a cantilever of length *l*/2, fixed at D and bending upwards under a load W/2. Hence the elevation of A above D or,

the depression of D below A = y = $\frac{(W/2)(l/2)^3}{3EAk^2} = \frac{Wl^3}{48EAk^2}$

Note: The inclination of the tangent at the points A and B is given by

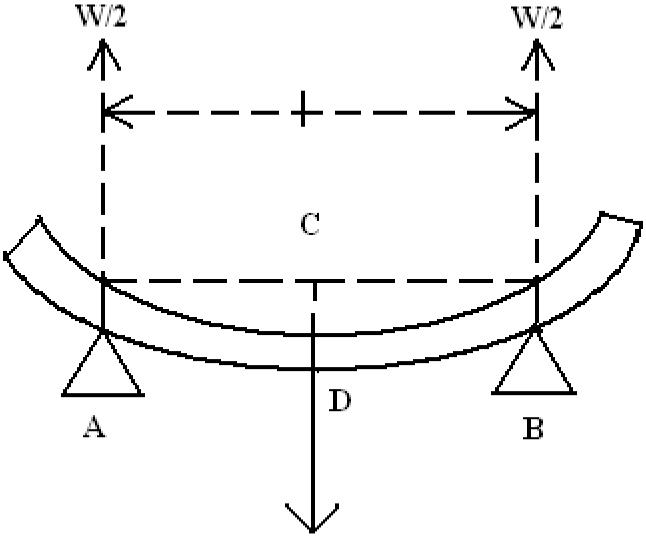


Fig. 1.3.4

$$\tan\theta = \frac{dy}{dx} = \frac{Wl^2}{16EAK^2}$$

Since θ is small, $\tan\theta = \theta.$

$$\therefore \qquad \theta = \frac{Wl^2}{16EAK^2}$$

1.4 Uniform bending of a beam

Consider a beam of negligible mass supported symmetrically on two knife-edges A and B in a horizontal level (Fig. 1.4 (a)). Let AB = 1.

Let equal weights W, W be added to the beam at its ends C and D. Let AC = BD = a. Then the beam is bent into an arc of a circle. The reactions on the knife-edges will then be W and W, acting vertically upwards. Consider the cross-section of the beam at any point P. The only forces acting on the part PC of the beam are the forces W at C and the reaction W at A.

The external bending moment with respect to P

$$= W.Cp - W.AP = W(CP - AP) = W.AC = Wa.$$

This must be balanced by the internal bending moment EAk^2/R.

Hence, $Wa = EAk^2/R$ …(1)

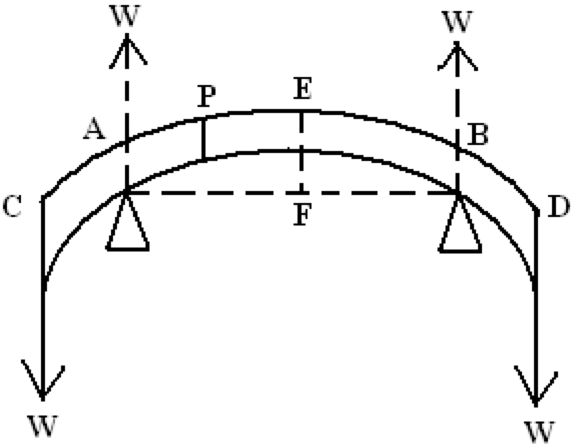


Fig. 1.4(a)

Since for a given load W, E, a and Ak^2 are constant, R is a constant. The bending is then said to be uniform. If y is the elevation of the mid-point of AB above its normal position (Fig. 1.4(b)),

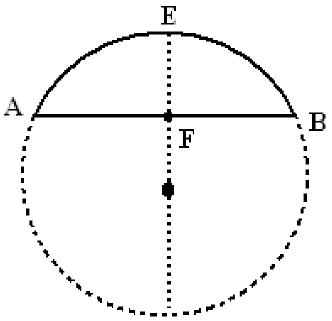


Fig. 1.4(b)

$$EF(2R - EF) = AF^2$$

$$y(2R - y) = (l / 2)^2$$

$$y.2R = l^2 / 4$$

$$(\because y^2 \text{ is negligible})$$

$$y = l^2 / 8R$$

From (1), $$\frac{1}{R} = \frac{Wa}{EAk^2}$$

$$\therefore \quad y = \frac{Wal^2}{8EAk^2}$$

1.4.1 Measurement of Young's modulus – By bending of a beam

(1) Non-uniform Bending:

The given beam is symmetrically supported on two knife-edges (Fig. 1.4.1 (1)). A weight-hanger is suspended by means of a loop of thread from the point C exactly midway between the knife-edges. A pin is fixed vertically at C by some wax. A traveling microscope is focused on the tip of the pin such that the horizontal cross-wire coincides with the tip of the pin. The reading in the vertical traverse scale of microscope is noted. Weights are added in equal steps of m kg and the corresponding readings are noted. Similarly, readings are noted while unloading. The results are tabulated as follows: -

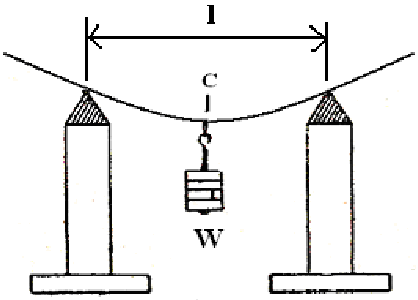


Fig. 1.4.1 (1)

Load in kg	Readings on the microscope			y for M kg
	Load increasing	Load decreasing	Mean	

The mean depression y is found for a load of M kg. The length of the beam (l) between the knife-edges is measured. The breadth b and the thickness d of the beam are measured with a vernier calipers and screw gauge respectively.

Then, $$y = \frac{Wl^3}{48EAk^2} or E = \frac{Wl^3}{48Ak^2 y}$$

or $$E = \frac{Mgl^3}{48X(bd^3/12)Xy} \qquad (\because W = Mg \text{ and } A\,k^2 = bd^3/12)$$

$$\therefore \qquad E = \frac{Mgl^3}{4bd^3 y}$$

Example 1: In an experiment a rod of diameter 0.0126 m was supported on two knife-edges, placed 0.7 metre apart. On applying a load of 0.9 kg exactly midway between the knife-edges, the depression on the middle point was observed to be 0.00025 m. Calculate the Young's modulus of the substance.

$$E = \frac{Mgl^3}{12y\pi r^4} = \frac{(0.9)(9.8)(0.7)^3}{12(0.00025)\pi(0.0063)^4}$$

$$\therefore \qquad E = 2.039X10^{11}Nm^{-2}$$

(2) Uniform bending:

The given beam is supported symmetrically on two knife-edges A and B (Fig. 1.4.1 (2)). Two equal weight-hangers are suspended, so that their distances from the knife-edges are equal. The elevations of the centre of the beam may be measured accurately by using a single optic level (L). The front leg of the single optic lever rests on the centre of the loaded beam and the hind legs are supported on a separate stand. A vertical scale (S) and telescope (T) are arranged in front of the mirror. The telescope is focused on the mirror and adjusted so that the reflected image of the scale in the mirror is seen through the telescope. The load on each hanger is increased in equal steps of m kg and the corresponding readings on the scale are noted. Similarly, readings are noted while unloading. The results are tabulated as follows:

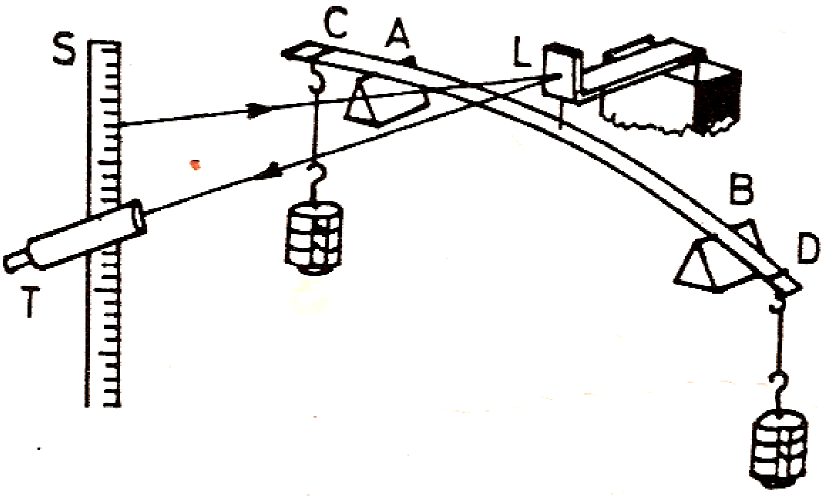


Fig. 1.4.1 (2)

Load in kg	Readings of the scale as seen in the telescope			Shift in reading for M kg
	Load increasing	Load decreasing	Mean	

The shift in scale reading for M kg is found from the table. Let it be S. If

D = The distance between the scale and the mirror,

x = the distance between the front leg and the plane containing the two hind legs of the optic lever,

then $y = Sx / 2D$.

The length of the beam l between the knife-edges, and a, the distance between the point of suspension of the load and the nearer knife-edge (AC = BD = a) are measured. The breadth b and the thickness d of the beam are also measured.

Then, $$y = \frac{Wal^2}{8EAk^2} \text{ or } \frac{Sx}{2D} = \frac{Mgal^2}{8E(bd^3/12)}$$

[Since $W = Mg$ and $Ak^2 = bd^3 / 12$]

$$\therefore \quad E = \frac{3Mgal^2D}{Sxbd^3}$$

Pin and Microscope Method: The given beam is supported symmetrically on two knife-edges A and B. Two equal weight-hangers are suspended so that their distances from the knife-edges are equal. A pin is placed vertically at the centre of the beam. The tip of the pin is viewed by a microscope. The load on each hanger is increased in equal steps of m kg and the corresponding microscope readings are noted. Similarly, readings are noted while unloading. The results are tabulated as follows:

Load in kg	Readings of the microscope			y for M kg
	Load increasing	Load decreasing	Mean	

The mean elevation (y) of the centre for M kg is found. The length of the beam l between the knife-edges and a, the distance between the point of suspension of the load and the nearer knife-edge (AC = BD = a) are measured. The breadth b and the thickness d of the beam are also measured.

$$y = \frac{Wal^2}{8EAk^2} = \frac{Mgal^2}{8E(bd^3/12)} \qquad \left(\because W = Mg \text{ and } \mathrm{Ak}^2 = \frac{\mathrm{bd}^3}{12} \right)$$

$$\therefore \qquad E = \frac{Mgal^2}{2bd^3y}$$

Using the above formula we can calculate the Young's modulus of the material of the beam.

Example 1: **Distinguish between uniform and non-uniform bending.**

In uniform bending every element of the beam is bent with the same radius of curvature (R). In non-uniform bending, R is not the same for all the elements in the beam.

1.5 Theory of Non-Uniform Bending

(Beam supported at its Ends and Loaded in the Middle)

Consider a beam supported on two knife-edges A and B (Fig. 1.5). The length of the beam between the two points A and B is *l*. It is loaded with weight W at the centre C. The reaction at each knife-edge is W/2 acting vertically upwards.

Consider a cross section of the beam at P, at a distance x from A.

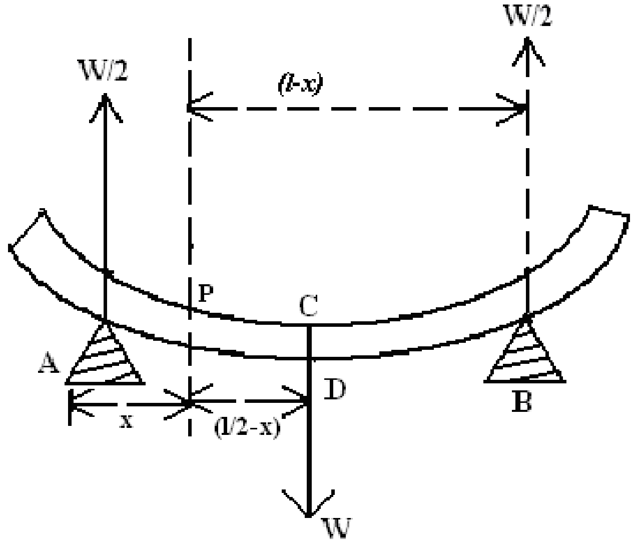


Fig. 1.5

For equilibrium,

$$\frac{EI_g}{R} = W\left[\frac{l}{2} - x\right] - \frac{W}{2}(l = x)$$

$$\frac{EI_g}{R} = -\frac{W}{2} \times x$$

$$\frac{1}{R} = -\frac{Wx}{2EI_g} \qquad \ldots(1)$$

But $$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$\therefore$ $$\frac{d^2y}{dx^2} = -\frac{Wx}{2EI_g} \qquad \ldots(2)$$

Integrating Eq. (2),

$$\frac{dy}{dx} = -\frac{Wx^2}{4EI_g} + C_1 \qquad \ldots(3)$$

At $$x = \frac{l}{2}, \frac{dy}{dx} = 0$$

$\therefore$ $$0 = -\frac{W(l/2)^2}{4EI_g} + C_1$$

$$C_1 = \frac{Wl^2}{16EI_g}$$

$\therefore$ $$\frac{dy}{dx} = -\frac{Wx^2}{4EI_g} + \frac{Wl^2}{16EI_g} \qquad \ldots(4)$$

Integrating Eq. (4),

$$y = -\frac{Wx^3}{12EI_g} + \frac{Wl^2x}{16EI_g} + C_2$$

At $x = 0, y = 0;$ $\therefore C_2 = 0$

$$y = -\frac{Wx^3}{12EI_g} + \frac{Wl^2x}{16EI_g} \qquad \ldots(5)$$

At the mid point C, $x = l/2.$

$$y = -\frac{W(l/2)^3}{12EI_g} + \frac{Wl^2(l/2)}{16EI_g}$$

$\therefore$ $$y = \frac{Wl^3}{48EI_g} \qquad \ldots(6)$$

Special cases

1. For a beam of rectangular cross-section,

$$I_g = bd^3/12$$

$$\therefore \qquad y = \frac{Wl^3}{4Ebd^3} \qquad \ldots(7)$$

2. For a beam of circular cross-section.

$$I_g = \pi r^4/4.$$

$$\therefore \qquad y = \frac{Wl^3}{12\pi Er^4} \qquad \ldots(8)$$

Torsion

1.6 Torsion of a body

When a body fixed at one end and twisted about its axis by means of a torque at the other end, the body is said to be under torsion. Torsion involves shearing strain and so the modulus involved is the rigidity modulus.

Torsion of a cylinder-Expression for torque per unit twist

Consider a cylindrical wire of length L and radius a fixed at its upper end and twisted through an angle θ by applying a torque at the lower end. Consider the cylinder to consist of an infinite number of hollow co-axial cylinders. Consider one such cylinder of radius x and thickness dx. [Fig. 1.6(1)].

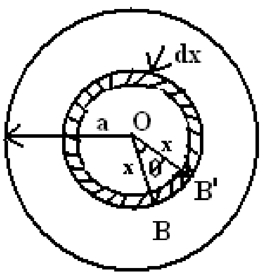


Fig. 1.6 (1)

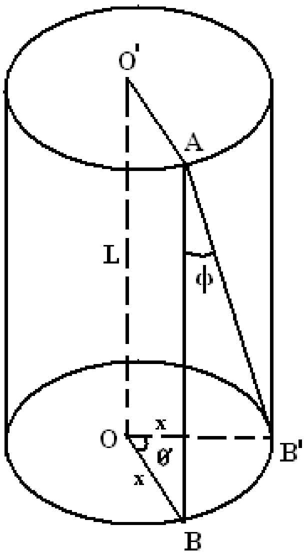


Fig. 1.6 (2)

A line such as AB initially parallel to the axis OO' of the cylinder is displaced to the position AB′ through an angle ϕ due to the twisting torque [Fig. 1.6(2)]. The result of twisting the cylinder is a shear strain. The angle of shear = $\angle$BAB' = ϕ.

Now BB′ = $x.\theta = L\,\phi$ or $\phi = x.\theta/L$

We have, rigidity modulus = G = $\dfrac{\text{Shearing stress}}{\text{angle of shear } (\phi)}$

$\therefore$ Shearing stress = $G.\phi = Gx\theta/L$

But, Shearing stress = $\dfrac{\text{Shearing force}}{\text{Area on which the force acts}}$

$\therefore$ Shearing force = Shearing stress X Area on which the force acts.

The area over which the shearing force acts = $2\pi x\,dx$

Hence, the shearing force = F = $\dfrac{Gx\theta}{L} X\, 2\pi x dx$

$$\left.\begin{array}{l}\text{The moment of this}\\ \text{force about the axis}\\ \text{OO' of the cylinder}\end{array}\right\} = \frac{Gx\theta}{L}2\pi x dx.x = \frac{2\pi G\theta}{L}x^3dx$$

$$\therefore \quad \left.\begin{array}{l}\text{Twisting torque on the}\\ \text{whole cylinder}\end{array}\right\} = C = \int_0^a \frac{2\pi G\theta}{L}x^3dx$$

or

$$C = \frac{\pi G a^4\theta}{2L}$$

$$\left.\begin{array}{l}\text{The torque per unit twist (i.e.,}\\ \text{the torque when } \theta = 1 \text{ radian)}\end{array}\right\} = c = \frac{\pi G a^4}{2L}$$

Note 1: When an external torque is applied on the cylinder to twist it, at once an internal torque, due to elastic forces comes into play. In the equilibrium position, these two torques will be equal and opposite.

Note 2: If the material is in the form of a hollow cylinder of internal radius a and external b, then,

$$\left.\begin{array}{l}\text{The torque acting}\\ \text{on the cylinder}\end{array}\right\} = C = \int_a^b \frac{2\pi G\theta}{L}x^3dx = \frac{\pi G\theta}{2L}(b^4 - a^4)$$

Torque per unit twist = c = $\pi G(b^4\text{-}a^4)/(2L)$

Example 1: What torque must be applied to a wire one metre long, 10^{-3} metre in diameter in order to twist one end of it through 90^{o}, the other end remaining fixed? The rigidity of the material of the wire is 2.8 X 10^{10} Nm^{-2}.

Here, L = 1m; G = 2.8 X 10^{10} Nm^{-2}; a = $\frac{10^{-3}}{2}m = 0.5X10^{-3}m$;

θ = 90° = π/2 radian;

$$\therefore \qquad C = \frac{\pi G a^4}{2L}\theta = \frac{\pi(2.8X10^{10})(0.5X10^{-3})^4}{2X1} X \frac{\pi}{2}$$

= 4.318 X 10^{-3} Nm.

Example 2: A circular bar one metre long and 8 X 10^{-3} metre diameter is rigidly clamped at one end in a vertical position. A torque of magnitude 2.5 Nm is applied at the other end. As a result, a mirror fixed at this end deflects a spot of light by 0.15 metres on a scale one metre away. Calculate the modulus of rigidity of the bar.

For a twist θ, the mirror turns through θ, and the reflected beam through 2θ. If the deflection is d on a scale D away, 2θD = d or $\theta = \frac{d}{2D} = \frac{0.15}{2X1} = 0.075 radians$;

Here, C= 2.5 Nm; a = 4 X 10^{-3} m; θ = 0.075 radians; L = 1m; G = ?

Hence, $C = \frac{\pi G a^4 \theta}{2L}$ or $G = \frac{C.2L}{\pi a^4 \theta}$

i.e., $$G = \frac{2.5X2X1}{\pi(4X10^{-3})^4(0.075)} = 8.290X10^{10}Nm^{-2}$$

Example 3: A steel wire of diameter 3.6 X 10^{-4} m and length 4 m extends by 1.8 X 10^{-3} m under a load of 1 kg and twists by 1.2 radians when subjected to a total torsional torque of 4 X 10^{-5} Nm at one end. Find the values of E, G and v for steel.

We have, $$E = \frac{F/A}{l/L} = \frac{FL}{A.l}$$

Here, F = mg = 1 X 9.8 = 9.8N: L = 4m;

A = πa^2 = π(1.8 X 10^{-4})^{2}m^2 and l = 1.8 X 10^{-3}m.

$$\therefore \qquad E = \frac{9.8X4}{\pi(1.8X10^{-4})^2X1.8X10^{-3}} = 2.139X10^{11}Nm^{-2}$$

$$\left.\begin{array}{l}\text{Torque which must be applied}\\ \text{to twist one end of the wire}\\ \text{through an angle } \theta \text{ radians}\end{array}\right\} = C = \frac{\pi G a^4 \theta}{2L}$$

or $$G = \frac{C.2L}{\pi a^4 \theta}$$

Here, C = 4 X 10^{-5} Nm; L = 4m; a = 1.8 X 10^{-4}m;

θ = 1.2 radians

$$\therefore \qquad G = \frac{C.2L}{\pi a^4 \theta} = \frac{(4X10^{-5})X\,2X\,4}{\pi(1.8X10^{-4})^4\,X1.2} = 0.8083X10^{11}\,Nm^{-2}$$

$$v = \frac{E}{2G} - 1 = \frac{2.139X10^{11}}{2X\,0.8083X10^{11}} - 1 = 1.323 - 1 = 0.323.$$

Example 4: Explain why a hollow rod is a better shaft than a solid one of the same mass, length and material.

Consider a solid cylinder of length L, radius r and shear modulus G.

$$\left.\begin{array}{l}\text{The torque required to twist the}\\ \text{solid cylinder through an angle } \theta\end{array}\right\} = C_1 = \frac{\pi G r^4 \theta}{2L} \qquad \ldots(1)$$

Let r_1 and r_2 be the inner and outer radii of the hollow cylinder of the same length, mass and material.

Then the torque required to twist it through the same angle θ is

$$C_2 = \frac{\pi G(r_2{}^4 - r_1{}^4)\theta}{2L} \qquad \ldots(2)$$

Hence, $$\frac{C_2}{C_1} = \frac{r_2{}^4 - r_1{}^4}{r^4} = \frac{(r_2{}^2 + r_1{}^2)(r_2{}^2 - r_1{}^2)}{r^4}$$

Since the two cylinders have the same mass,

$$\pi(r_2{}^2 - r_1^2)l\rho = \pi r^2 l.\rho$$

(where ρ is the density of the material of the cylinders).

or $$r_2{}^2 - r_1{}^2 = r^2 \qquad \ldots(3)$$

Adding $2r_1{}^2$ to both sides, $r_2{}^2 + r_1{}^2 = r^2 + 2r_1^2$ …(4)

Hence, $$\frac{C_2}{C_1} = \frac{(r^2 - 2r_1^2)r^2}{r^4} = \frac{r^2 + 2r_1^2}{r^2}$$

or $$\frac{C_2}{C_1} = 1 + \frac{2r_1^2}{r^2}$$

∴ $C_2 > C_1$ i.e., the twisting torque for a hollow cylinder is greater than that for a solid cylinder of the same mass, length and material. Hence a hollow cylinder is stronger and a better shaft than a solid one of the same mass, length and material.

Torsional oscillations of a body

Suppose a wire is clamped vertically at one end and the other end carries a body (i.e., a disc, bar or a cylinder) of moment of inertia I about the wire as the axis. Let the length, radius and rigidity modulus of the wire be respectively l, a and G. When the body is given a slight rotation by applying a torque, say by the hand, the wire is twisted. If the body is released, the body oscillates in the horizontal plane about the wire as axis. These oscillations are called Torsional oscillations and the arrangement is known as a Torsion pendulum.

Let us consider the energy of the vibrating system when the angle of twist is θ. Let ω be the angular velocity of the body.

The potential energy of the wire due to the twist = $\frac{1}{2}c.\theta^2$.

$$\left.\begin{array}{l}\text{The kinetic energy of the}\\ \text{body due to its rotation}\end{array}\right\} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2$$

$$\left.\begin{array}{l}\text{The total energy}\\ \text{of the system}\end{array}\right\} = \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}c\theta^2 = cons\tan t$$

Differentiating this with respect to t,

$$\frac{1}{2}I.2\frac{d\theta}{dt}.\frac{d^2\theta}{dt^2} + \frac{1}{2}c2\theta\frac{d\theta}{dt} = 0.$$

or $$I\frac{d^2\theta}{dt^2} + c\theta or \frac{d^2\theta}{dt^2} + \frac{c}{I}\theta = 0$$

The body has simple harmonic motion and its period is given by

$$T = 2\pi\sqrt{\frac{I}{c}}$$

1.7 Rigidity modulus by Torsion pendulum (Dynamic torsion method):

The torsion pendulum consists of a wire with one end fixed in a split chuck and the other end to the centre of a circular disc as in Fig. 1.7.

Two equal symmetrical masses (each equal to m) are placed along a diameter of the disc at equal distances d_1 on either side of the centre of the disc. The disc is rotated through an angle and is then released. The system executes torsional oscillations about the axis of the wire. The period of oscillations T_1 is determined.

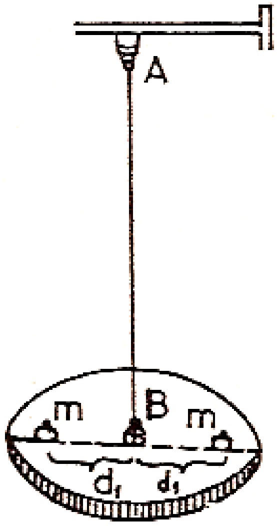


Fig. 1.7.

Then $T_1 = 2\pi\sqrt{\frac{I_1}{c}}$

or $T_1^2 = \frac{4\pi^2}{c} I_1.$

Here, I_1 = Moment of inertia of the whole system about the axis of the wire and c= torque per unit twist.

Let I_0 = M.I. of the disc alone about the axis of the wire.

i = M.I. of each mass about a parallel axis passing through its centre of gravity.

Then by the parallel axes theorem,

$$I_1 = I_0 + 2i + 2md_1^2$$

$$\therefore \quad T_1^2 = \frac{4\pi^2}{c}\left[I_0 + 2i + 2m.d_1^2\right]. \qquad \ldots(1)$$

The two masses are now kept at equal distances d_2 from the centre of the disc and the corresponding period T_2 is determined. Then,

$$T_2^2 = \frac{4\pi^2}{c}\left[I_0 + 2i + 2m.d_2^2\right]. \qquad \ldots(2)$$

$$\therefore \quad T_2^2 - T_1^2 = \frac{4\pi^2}{c}.2m.\left(d_2^2 - d_1^2\right) \qquad \ldots(3)$$

But c = $\frac{\pi G a^4}{2L}$

Hence $$T_2^2 - T_1^2 = \frac{4\pi^2.2m.\left(d_2^2 - d_1^2\right)2L}{\pi G a^4}$$

or $$G = \frac{16\pi Lm(d_2^2 - d_1^2)}{a^4(T_2^2 - T_1^2)}$$

Using this relation, G is determined.

M.I. of the disc by torsional oscillations. The two equal masses are removed and the period T_0 is found when the disc alone is vibrating. Then,

$$T_0^2 = \frac{4\pi^2}{c} I_0 or I_0 = \frac{cT_0^2}{4\pi^2} \qquad \ldots(4)$$

From (3), $$c = \frac{4\pi^2.2m.\left(d_2^2 - d_1^2\right)}{T_2^2 - T_1^2}$$

Hence $$I_0 = \frac{4\pi^2.2m.\left(d_2^2 - d_1^2\right)}{T_2^2 - T_1^2}.\frac{T_0^2}{4\pi^2} = \frac{2m.\left(d_2^2 - d_1^2\right)T_0^2}{T_2^2 - T_1^2}$$

From this relation, the moment of inertia of the disc about the axis of the wire is calculated.

1.7.1 Determination of rigidity modulus – Static torsion method

Searle's apparatus: The experimental rod is rigidly fixed at one end A and fitted into the axle of a wheel W at the other end B (Fig. 1.7.1.). The wheel is provided with a grooved edge over which passes a tape. The tape carries a weight hanger at its free end. The rod can be twisted by adding weights to the hanger. The angle of twist can be measured by means of two pointers fixed at Q and R which move over circular scales S_1 and S_2. The scales are marked in degrees with centre zero.

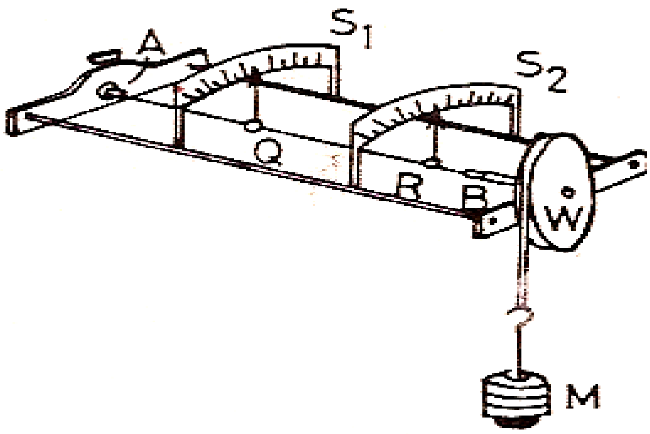


Fig. 1.7.1

With no weights on the hanger, the initial readings of the pointers on the scales are adjusted to be zero. Loads are added in steps of m kg (conveniently

0.2 kg). The readings on the two scales are noted for every load, both while loading and unloading. The experiment is repeated after reversing the twisting torque by winding the tape over the wheel in the opposite way. The observations are tabulated.

The readings in the last column give the twist for a load of M kg for the length QR (=L) of the rod.

Load	Reading on S$_1$				
	Torque Clockwise		Torque Anticlockwise		Mean θ_1
	Load increasing	Load decreasing	Load increasing	Load decreasing	

Reading on S$_2$					$\theta = \theta_2 - \theta_1$	θ for M kg
Torque Clockwise		Torque Anticlockwise		Mean θ_2		
Load increasing	Load decreasing	Load increasing	Load decreasing			

The radius a of the rod and the radius R of the wheel are measured. If a load of M kg is suspended from the free end of the tape, the twisting torque = MgR.

The angle of twist = θ degrees = $\theta.\pi/180$ radians.

$\therefore$ The restoring torque = $\frac{\pi G a^4}{2L} \cdot \frac{\theta\pi}{180}$.

For equilibrium, $MgR = \frac{\pi G a^4}{2L} \frac{\theta.\pi}{180} or G = \frac{360 MgRL}{\pi^2 a^4 \theta}$

Since a occurs in the fourth power in the relation used, it should be measured very accurately.

Notes: (1) We eliminate the error due to the eccentricity of the wheel by applying the torque in both clockwise and anticlockwise directions.

(3) We eliminate errors due to any slipping at the clamped end by observing readings at two points on the rod.

1.7.2 Determination of rigidity modulus – Static torsion method. (Searle's apparatus – Scale and Telescope)

A plane mirror strip is fixed to the rod at a distance L from the fixed end of the rod [Fig. 1.7.2]. A vertical scale (S) and telescope (T) are arranged in front of the mirror. The telescope is focused on the mirror and adjusted so that the reflected image of the scale in the mirror is seen through the telescope. With some dead load W on the weight-hanger, the reading of the scale division coinciding with the horizontal cross-wire is taken. Weights are added in steps of m kg and the corresponding scale readings are taken. Weights are then decreased continuously in steps of m kg and the readings taken again. The torque is reversed now, by passing the tape anticlockwise on the wheel. The readings are taken as before. From these readings, the shift in scale reading (s) for a load m kg is found.

The length L of the rod from the fixed end to the mirror is measured. The mean radius a of the rod is accurately measured with a screw gauge. The radius (R) of the wheel is found by measuring its circumference with a thread.

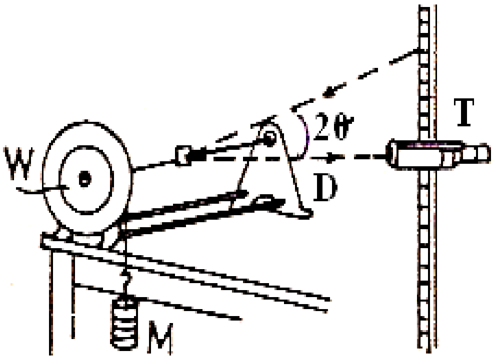


Fig 1.7.2

The distance (D) between the scale and the mirror is measured with a metre scale.

G is calculated using the formula $G = \dfrac{4mgRLD}{\pi a^4 s}$

Load in kg	Telescope Reading						$\frac{X \sim Y}{2}$	Shift in scale reading for 4m kg
	Torque clockwise			Torque anticlockwise				
	Loading	Unloading	Mean (X)	Loading	Unloading	Mean (Y)		
W								
W+m								
W+2m								
W+3m								
W+4m								
W+5m								
W+6m								
W+7m								

Work done in twisting a wire

Consider a cylindrical wire of length L and radius a fixed at its upper end and twisted through an angle θ by applying a torque at the lower end.

If c is the torque per unit angular twist of the wire, then the torque required to produce a twist θ in the wire is

$$C = c\theta.$$

The work done in twisting the wire through a small angle dθ is

$$Cd\theta = c\theta\, d\theta.$$

$$\therefore \quad \left.\begin{array}{l}\text{The total work done in twisting}\\ \text{the wire through an angle } \theta\end{array}\right\} = W = \int_0^\theta c\theta d\theta$$

Or $$W = \frac{1}{2}c.\theta^2$$

The work done in twisting the wire is stored up in the wire as potential energy.

Example 1: Find the amount of work done in twisting a steel wire of radius 10^{-3} m and of length 0.25 m through an angle of 45^o. Given G for steel = 8 X 10^{10} Nm^{-2}.

We have, $$W = \frac{1}{2}c.\theta^2 = \frac{1}{2}\frac{\pi G a^4}{2L}\theta^2 \left(\because c = \frac{\pi G a^4}{2L} \right)$$

Here, G = 8 X 10^{10} Nm^{-2} ; a = 10^{-3} m; θ = 45^o = π/4 rad;

L = 0.25 m

$$\therefore \qquad W = \frac{1}{2}\frac{\pi(8X10^{10})(10^{-3})^4\left(\pi/4\right)^2}{2X0.25} = 0.1550J.$$

Surface Tension

1.8 Introduction

Any liquid in small quantity, so that gravity influence is negligibly small, will always assume the form of a spherical drop – e.g., rain drops, small quantities of mercury placed on a clean glass plate etc. So a liquid must experience some kind of force, so as to occupy a minimum surface area. This contracting tendency of a liquid surface is known as surface tension of liquid. This is a fundamental property of every liquid.

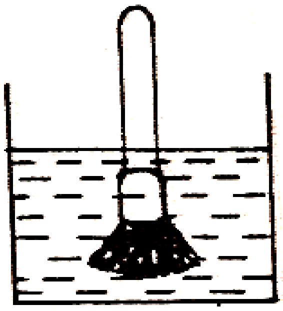


Fig. 1.8(a)

Fig. 1.8(b)

The following experiment illustrates the tendency of a liquid to decrease its surface area.

When a camel hair brush is dipped into water, the bristles spread out [Fig. 1.8 (a)]. When the brush is taken out, the bristles cling together on account of the films of water between them contracting [Fig. 1.8 (b)]. This experiment clearly shows that the surface of a liquid behaves like an elastic membrane under tension with a tendency to contract. This tension or pull in the surface of a liquid is called its surface tension.

Definition: It may be defined as the force per unit length of a line drawn in the liquid surface, acting perpendicular to it at every point and tending to pull the surface apart along the line.

Unit of surface tension. Surface tension being force per unit length, its SI unit is Newton per meter (Nm^{-1}).

Dimensions of surface Tension: Since it is the ratio of a force to a length, its dimensions are $MLT^{-2}/L = MT^{-2}$.

1.8.1 Pressure difference across a liquid surface

(a) If the free surface of the liquid is plane, as in Fig. 1.8.1 (a), the resultant force due to S.T. on a molecule on the surface is zero.

(b) IF the free surface of the liquid is concave, as if Fig. 1.8.1 (b), the resultant force due to S.T. on a molecule on the surface acts vertically upwards.

(c) If the free surface of the liquid is convex, as in Fig. 1.8.1 (c), the resultant force due to S.T. on a molecule on the surface acts vertically downwards (into the liquid).

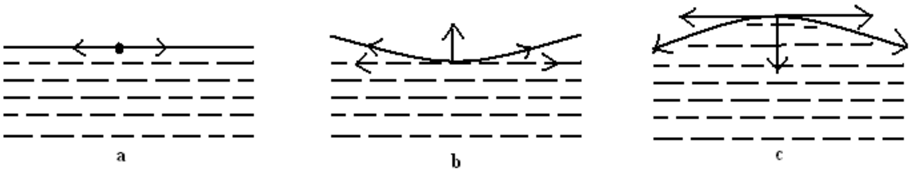
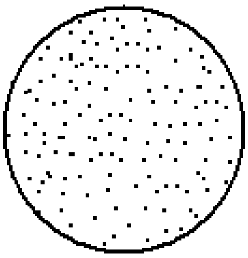


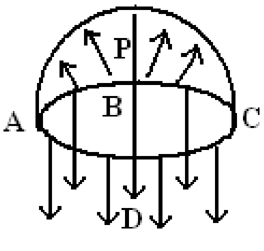
Fig. 1.8.1

Excess pressure inside a liquid drop: A spherical liquid drop has a convex surface, as in Fig. (i). The molecules near the surface of the drop experience a resultant force, acting inwards due to surface tension. Therefore, the pressure inside the drop must be greater than the pressure outside it. Let this excess pressure inside the liquid drop over the pressure outside it be p.

Imagine the drop to be divided into two exactly equal halves. Consider the equilibrium of the upper-half (or the upper hemisphere) of the drop as shown in Fig. (ii). IF r is the radius of the drop, and σ its S.T.,



(i)



(ii)

$$\left.\begin{array}{l}\text{The upward force on the plane face}\\ \text{ABCD due to the excess pressure p}\end{array}\right\} = p\pi r^2$$

$$\left.\begin{array}{l}\text{The downward force due to surface tension acting}\\ \text{along the circumference of the circle ABCD}\end{array}\right\} = \sigma 2\pi r$$

Since the hemisphere is in equilibrium, the two forces are equal.

$$\therefore \qquad p\pi r^2 = \sigma 2\pi r \text{ or } p = 2\sigma / r$$

Excess pressure inside a soap bubble: A soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble.

The force due to S.T. in this case = $2X\sigma 2\pi r = 4\pi r\sigma$.

Therefore, for equilibrium of the hemisphere,

$$p\pi r^2 = 4\pi r\sigma \text{ or } p = 4\sigma / r.$$

Thus the excess pressure inside a drop or a bubble is inversely proportional to its radius (i.e., $p\alpha 1/r$). Since $p\alpha 1/r$, the pressure needed to form a very small bubble is high. This explains why one needs to blow hard to start a balloon growing. Once the balloon has grown, less air pressure is needed to make it expand more.

1.8.2 Excess pressure inside a curved liquid surface

When the pressure on both sides of a liquid surface is same, then the surface is flat, without any curvature. But when it is curved convex upwards, then the pressure inside must be greater than the pressure outside and the excess pressure inside is balanced by the force of S.T.

To find the excess pressure, consider a small curvilinear rectangular element $A_1B_1C_1D_1$ of a liquid surface [Fig. 1.8.2]. A_1B_1 has a radius of curvature R_1 with centre at O_1. B_1C_1 has a radius of curvature R_2 with centre at O_2. Let p be the excess of pressure inside the surface over that outside. Then the outward thrust on the surface $A_1B_1C_1D_1$ = p X Area of the element $A_1B_1C_1D_1$ = p X A_1B_1 X B_1C_1.

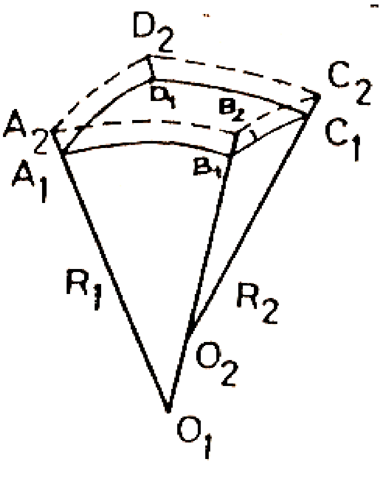


Fig. 1.8.2

Now, let the surface be moves outward through a very small distance δx.

Let the new position of the surface be $A_2B_2C_2D_2$.

Work done in the displacement = $p.A_1B_1.B_1C_1.\ \delta x$ …(1)

$$\left.\begin{array}{r}\text{Now, increase}\\ \text{in surface area}\end{array}\right\} = \text{Area of } A_2B_2C_2D_2 - \text{Area of } A_1B_1C_1D_1$$

$$= A_2B_2.B_2C_2 - A_1B_1.B_1C_1$$

From similar triangles, $A_1B_1O_1$ and $A_2B_2O_1$,

$$\frac{A_1B_1}{A_2B_2} = \frac{A_1O_1}{A_2O_1} \text{ or } \frac{A_1B_1}{A_2B_2} = \frac{R_1}{(R_1 + \delta x)}$$

$\therefore$ $$A_2B_2 = \frac{A_1B_1(R_1 + \delta x)}{R_1} = A_1B_1\left[1 + \frac{\delta x}{R_1}\right]$$

Similarly, $$B_2C_2 = B_1C_1\left(1 + \frac{\delta x}{R_2}\right)$$

Hence $$A_2B_2.B_2C_2 = A_1B_1\left(1+\frac{\delta x}{R_1}\right)B_1C_1\left(1+\frac{\delta x}{R_2}\right)$$

$\therefore$ $$\left.\begin{array}{r}\text{Increase in}\\ \text{surface area}\end{array}\right\} = A_1B_1.B_1C_1\left(1+\frac{\delta x}{R_1}+\frac{\delta x}{R_2}\right) - A_1B_1.B_1C_1$$

$$= A_1B_1.B_1C_1\,\delta x\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\left.\begin{array}{l}\text{Work done in increasing}\\ \text{the area of a surface}\end{array}\right\} = \left\{\begin{array}{l}\text{Surface tension X increase}\\ \text{in surface area}\end{array}\right\}$$

$$= \sigma.\, A_1B_1.B_1C_1.\delta x\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \qquad \ldots(2)$$

Equating (1) and (2) we have,

$$p.A_1B_1.B_1C_1.\delta x = \sigma.\, A_1B_1.B_1C_1.\delta x\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

0or $$p = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

The excess of pressure inside any particular surface can be deduced from the above expression.

(1) **Spherical liquid drop** (an air bubble in a liquid).

It has only one surface and the radius of curvature is the same everywhere, i.e., $R_1 = R_2 = r$. $\therefore p = 2\sigma/r$.

(2) **Spherical soap bubble.** Here there are two surfaces having the same radius of curvature $\therefore p = 4\sigma/r$.

(3) **Cylindrical drop.** $R_1 = r$ = Radius of the cylinder and $R_2 = \propto$, $\therefore p = \sigma/r$

(4) **Cylindrical bubble.** p = 2σ/r since it has two surfaces.

Note: We have, $p = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$. The expression holds good for surfaces such as spherical or ellipsoidal, for which the principal radii of curvature are on the same side. Such surfaces are called synclastic surfaces [Fig. 1.8.2 (a)]. But in cases where the two radii are in opposite directions, the surface is called anticlastic [Fig. 1.8.2 (b)]. The expression for such surfaces is

$$p = \sigma\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

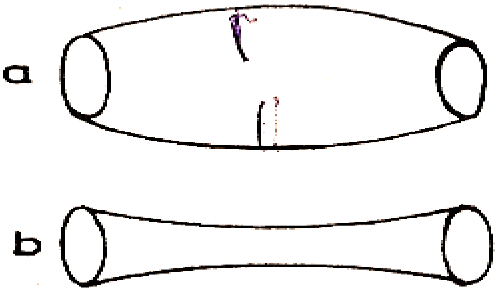


Fig. 1.8.2(a) & (b)

Combining the two cases, the general relation is

$$p = \sigma\left(\frac{1}{R_1}\pm\frac{1}{R_2}\right).$$

Example 1: Calculate the work done in spraying a spherical drop of water of 10^{-3} m radius into million droplets, all of the same size, the surface tension of water being 72 X10^{-3} Nm^{-1}.

Breaking the liquid drop means an increase in surface area. Therefore work has to be done for this purpose.

Work done = Surface tension X Increase in surface area.

Let us calculate the increase in surface area.

Let R be the radius of the larger drop and r the radius of the smaller droplets.

Volume of the original drop = Volume of 10^6 droplets.

$$\frac{4}{3}\pi R^3 = 10^6 X \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi(10^{-3})^3 = 10^6 X \frac{4}{3}\pi r^3 \qquad [\because R=10^{-3}m]$$

or $r^3 = 10^{-15}$m or $r=10^{-5}$m

Area of original large drop $= 4\pi R^2 = 4\pi(10^{-3})^2 = 4\pi X 10^{-6} m^2$

Area of 10^6 droplets $= 10^6 X 4\pi r^2 = 10^6 X 4\pi(10^{-5})^2$

$= 4\pi X 10^{-4}$ m^2

Hence increase in surface area is

$$= 4\pi X 10^{-4} - 4\pi X 10^{-6}$$
$$= 1.244 X 10^{-3} m^2$$

∴ Work done = Surface tension X Increase in surface area.

$$= (72 X 10^{-3}) X (1.244 X 10^{-3})$$
$$= 8.956 X 10^{-5} \text{ joules.}$$

Example 2: Calculate the amount of work done if a soap bubble is slowly enlarged from a radius of 0.1 m to a radius of 0.2m. $\sigma = 30 X 10^{-3} Nm^{-1}$.

Increase in surface area = $2[4\pi(r_2^2-r_1^2)]$

$= 8\pi[(0.2)^2-(0.1)^2] = 0.7536$ m^2

∴ Work done = Increase in surface area X σ

$= 0.7536 X (30 X 10^{-3}) = 0.02261$J.

Example 3: What is the work done in blowing a soap bubble of radius 0.1 m? What additional work will be performed in further blowing it, so that its radius becomes 0.15 m? $\sigma = 30 X 10^{-3} Nm^{-1}$.

$$\left.\begin{array}{l}\text{Work done in blowing a soap}\\ \text{bubble of radius 0.1 m}\end{array}\right\} = \textit{surface} \text{ tension X increase in surface area}$$

$$W = \sigma X 8\pi r_1^2 = (30 X 10^{-3})\, 8\pi\, (0.1)^2 = 7.536 X 10^{-3} J.$$

$$\left.\begin{array}{l}\text{Work done in increasing the radius of}\\ \text{the soap bubble from 0.1m to 0.15m}\end{array}\right\} = \sigma X 8\pi(r_2^2 - r_1^2)$$

i.e., $W = (30X10^{-3})\, 8\pi[(0.15)^2-(0.1)^2] = 9.42X10^{-3}$J.

Example 4: There is a minute circular hole at the bottom of a small hollow vessel. The vessel has to be immersed in water to a depth of 0.4m, before any water penetrates inside. Find the radius of the hole, if the surface tension and density of water be $73 X 10^{-3} Nm^{-1}$ and 1000 kg m^{-3} respectively.

Water cannot penetrate till the hydrostatic pressure is greater that the excess of pressure that comes into play due to surface tension. At the limiting condition, when water just penetrates, the two pressures should be equal; i.e., $h\rho g = 2\sigma/r$ where r is the radius of the hole.

$$\therefore \qquad r = 2.\ \sigma/(h\rho g)$$

Here, $\sigma = 73 \text{ X } 10^{-3}\ Nm^{-1}$; $h = 0.4m$; $\rho=1000kgm^{-3}$; $r = ?$

$$r = \frac{2X(73X10^{-3})}{0.4X1000X9.8} = 3.724X10^{-5}m$$

Example 5: What would be the pressure inside a small air bubble of 10^{-4} m radius, situated just below the surface of water? S.T. of water may be taken to be 70 X 10^{-3} Nm^{-1} and the atmospheric pressure to be 1.012 X 10^5 Nm^{-2}.

$$\left.\begin{array}{l}\text{Excess of pressure inside the spherical}\\ \text{air bubble over that of the atmosphere}\end{array}\right\} = p = \frac{2\sigma}{r}$$

Here, $\sigma = 70 \text{ X } 10^{-3}\ Nm^{-1}$; $r = 10^{-4}m$.

$$\left.\begin{array}{l}\text{Excess}\\ \text{pressure}\end{array}\right\} = p = \frac{2\sigma}{r} = \frac{2X(70X10^{-3})}{10^{-4}} = 1400Nm^{-2}$$

$$\therefore \qquad \left.\begin{array}{r}\text{Total pressure inside the}\\ \text{air bubble}\end{array}\right\} = \left\{\begin{array}{l}\text{Atmospheric pressure}\\ \text{+Excess pressure}\end{array}\right\}$$

$$= 1.012 \text{ X } 10^5 + 1400 = 1.026 \text{ X } 10^5\ Nm^{-2}.$$

Example 6: The pressure of air in a soap bubble of 7 X 10^{-3} m diameter is 8 X 10^{-3} m of water above the atmospheric pressure. Calculate the S.T. of the soap solution.

$$\left.\begin{array}{l}\text{Excess of pressure inside a soap}\\ \text{bubble over that outside it}\end{array}\right\} = p = \frac{4\sigma}{r}.$$

Here, $p = 8 \text{ X } 10^{-3}$ m of water = $(8 \text{ X } 10^{-3}) \text{ X } 1000 \text{ X } 9.81\ Nm^{-2}$.

$= 78.48\ Nm^{-2}$

$r = (7 \text{ X } 10^{-3})/2 = 3.5 \text{ X } 10^{-3}$ m.

$$\therefore \qquad \sigma = \frac{pr}{4} = \frac{78.48X(3.5X10^{-3})}{4} = 68.67X10^{-3}Nm^{-1}$$

Example 7: A spherical bubble of radius 0.001m is blown in an atmosphere whose pressure is 10^5 Nm^{-2}. If the S.T. of the liquid comprising the film is 0.05 Nm^{-1}, to what pressure must the surrounding atmosphere be brought in order that the radius of the bubble may be doubled?

The pressure inside the bubble initially is

$$p_i = \frac{4\sigma}{r} + P = \frac{4X0.05}{0.001} + 10^5 = 1.002X10^5Nm^{-2}$$

$$\left.\begin{array}{l}\text{The volume of}\\ \text{the bubble}\end{array}\right\} = \mathrm{V_i} = \frac{4}{3}\pi \mathrm{r}^3 = \frac{4}{3}\pi X10^{-9}m^3$$

In the second case, the radius of the bubble is 0.002m. Let the required pressure p_f. Then the total pressure inside the bubble,

$$P_f = \frac{4\sigma}{0.002} + P_f = \frac{4X0.05}{0.002} + P_f = (100 + P_f)Nm^{-2}$$

In the second case, volume of the bubble = $V_f = \frac{4}{3}\pi(0.002)^3$

Applying Boyle's law, $p_iV_i = p_fV_f$,

$$(1.002X10^5)\left(\frac{4}{3}\pi X10^{-9}\right) = (100 + P_f)\frac{4}{3}\pi(0.002)^3$$

$$Simplifying, P_f = 1.252X10^4 Nm^{-2}$$

Example 8: Two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. If the external pressure is P, prove that the S.T. of the solution from which the soap bubble is formed is given by $\sigma = \frac{1}{4}P(r^3 - r_1^3 - r_2^3)/(r_1^2 + r_2^2 - r^2)$.

Pressure inside the first bubble = $p_1 = (4\sigma/r_1)+P$

Pressure inside the second bubble = $p_2 = (4\sigma/r_2)+P$

Let the bubbles coalesce into a larger bubble of radius r.

Then, pressure inside this larger bubble = $p_3 = (4\sigma/r)+P$.

Let V_1, V_2 be the volumes of the two bubbles before they coalesce and V_3 the volume of the large bubble formed. Then by Boyle's law,

$$p_1V_1 + p_2V_2 = p_3V_3$$

$$\left(\frac{4\sigma}{r_1} + P\right)\frac{4}{3}\pi r_1^3 + \left(\frac{4\sigma}{r_2} + P\right)\frac{4}{3}\pi r_2^3 = \left(\frac{4\sigma}{r} + P\right)\frac{4}{3}\pi r^3.$$

$$or \qquad 4\sigma(r_1^2 + r_2^2 - r^2) = P(r^3 - r_1^3 - r_2^3)$$

$$\sigma = \frac{1}{4}P(r^3 - r_1^3 - r_2^3)/(r_1^2 + r_2^2 - r^2).$$

1.9 Drop-weight method of determining the surface tension of a liquid

Experiment: A short glass tube is connected to the lower end of a burette (or funnel) clamped vertically, by means of a rubber tube [Fig. 1.9.]. The funnel is filled with the liquid whose S.T. is to be determined. A beaker is arranged under the glass tube to collect the liquid dropping from the funnel. The stopcock is adjusted so that the liquid drops are formed slowly. In a previously weighed beaker a known number of drops, (say 50) are collected.

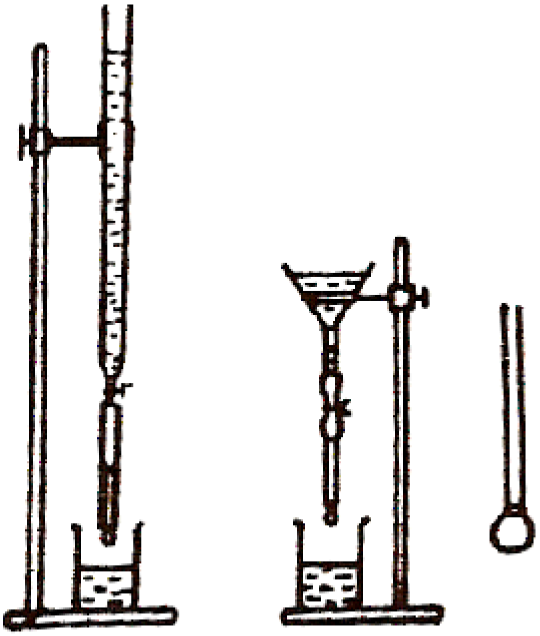


Fig. 1.9

The beaker is again weighed. The difference between this weight and the weight of the empty beaker gives the weight of 50 drops of the liquid. From this the mass m of each drop is calculated. The inner radius r of the tube is determined using a vernier calipers. The S.T. of the liquid at the room temperature is calculated using the formula,

$$\sigma = \frac{m.g}{3.8r}$$

Theory: Here, we consider the vertical forces that keep a small drop of liquid in equilibrium, just before it gets detached from the end of a vertical glass tube of circular aperture. At the instant the drop gets detached, it assumes a cylindrical shape at the orifice of the tube (Fig. 1.9). Let σ = S.T. of the liquid and r = radius of the orifice.

$$\left.\begin{array}{l}\text{Excess pressure (p) inside the drop over}\\ \text{the outside atmospheric pressure}\end{array}\right\} = \frac{\sigma}{r}$$

The area of the section is πr^2. Therefore,

$$\left.\begin{array}{l}\text{Downward forces on the drop}\\ \text{due to this excess of pressure}\end{array}\right\} = \pi r^2 \frac{\sigma}{r}$$

The weight mg of the drop also acts vertically downwards.

∴ Total force on the drop = $(\pi r^2 \sigma / r) + mg$

This downward force is balanced by the upward pull due to surface tension $2\pi r\sigma$ acting along a circle of radius r. Therefore

$$2\pi r\sigma = \frac{\pi r^2\sigma}{r} + mg \text{ or } 2\pi r\sigma = \pi r\sigma + mg$$

$$\therefore \qquad \sigma = \frac{m.g}{\pi.r}$$

But the equilibrium of the drop at the instant of its detachment is dynamic and not static. Lord Rayleigh, taking dynamical aspect into account, showed that

$$\sigma = \frac{m.g}{3.8r}$$

Example 1: In a drop weight method for the determination of S.T. between water and air, a glass tube of external diameter 2mm is used, and 100 drops of water are collected. The mass of these drops is 2.8 gms. Find the S.T. of water in air.

Here, $r = 10^{-3}$m.m $= 2.8 \text{ X } 10^{-3}/100 = 2.8 \text{ X } 10^{-5}$kg

$$\therefore \qquad \sigma = \frac{m.g}{3.8r} = \frac{(2.8 \text{ X } 10^{-5}) X 9.8}{3.8 X 10^{-3}} = 0.07221 Nm^{-1}$$

1.9.1 Interfacial Tension: At the surface of separation between two immiscible liquids there is a tension similar to surface tension. It is called the interfacial tension.

Definition: When one liquid rests on another without mixing with it, the interface between the two liquids possesses energy just like the surface of a liquid. The interfacial tension is the value of the force acting per metre normal to a line drawn on the interface.

Experiment to determine the interfacial tension between water and kerosene

Sufficient amount of the lighter liquid (kerosene) is taken in a beaker. The weight w_1 of the beaker with kerosene is determined. The heavier liquid (water) is taken in the burette [Fig 1.9.1]. The glass tube is fixed vertically with its end under the surface of kerosene. The flow of water is regulated so that drops of water detach themselves into kerosene one by one. After collecting 50 drops, the beaker is again weighed. Let this weight be w_2. Then w_2-w_1 gives the mass of 50 drops.

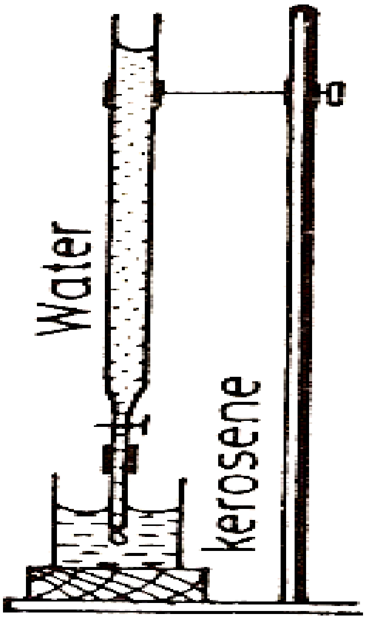


Fig 1.9.1

From this the average mass m of each drop is calculated. The interfacial tension σ between water and kerosene is calculated using the formula

$$\sigma = \frac{m.g}{3.8r}\left(1 - \frac{\rho_2}{\rho_1}\right)$$

No	*Mass of beaker + lighter liquid*		*Mean w_1 kg*	*Mass of beaker + lighter liquid +50 drops*		*Mean w_2 kg*	*Mass of 50 drops =(w_2-w_1) kg*
	Trial I	*Trial II*		*Trial I*	*Trial II*		

Theory: Let ρ_1 and ρ_2 be the densities of water and kerosene respectively. Let m be the mass of water drop in air.

∴ Volume of water drop = m/ρ_1

$$\left.\begin{array}{l}\text{Volume of kerosene displaced}\\ \text{by the water drop}\end{array}\right\} = \frac{m}{\rho_1}$$

$$\left.\begin{array}{l}\text{mass of kerosene displaced}\\ \text{by the water drop}\end{array}\right\} \frac{m\rho_2}{\rho_1}$$

$$\therefore \quad \left.\begin{array}{l}\text{Apparent weight of the}\\ \text{water drop in kerosene}\end{array}\right\} = mg - \frac{m\rho_2 g}{\rho_1}$$

Let σ be the S.T. at the interface between the two liquids.

Then, $$2\pi r\sigma = \frac{\pi r^2\sigma}{r} + m.g - \frac{m\rho_2 .g}{\rho_1}$$

$$\therefore \quad \sigma = \frac{m.g}{\pi r}\left(1 - \frac{\rho_2}{\rho_1}\right)$$

Again the more accurate equation will be

$$\sigma = \frac{m.g}{3.8r}\left(1 - \frac{\rho_2}{\rho_1}\right)$$

Questions:

1) What are ultrasonic waves?
2) What is piezoelectric effect? How is inverse piezoelectric effect used to produce ultrasonic wave?
3) Explain the principle of magnetostriction method of producing ultrasonics.
4) What are the usual methods employed to detect ultrasonic.
5) Mention the properties of ultrasonics.
6) Give important applications of ultrasonic.
7) Describe the piezoelectric method of producing ultrasonic waves.
8) Explain how ultra sonic are produced in a magetrostriction oscillator.
9) Write any four applications of ultrasonics
10) What are ultrasonic waves? Describe in detail one method of their production and

    Detection. Give some of their important applications.
11) What are ultrasonic waves? How are they produced and detected?
12) Calculate the work done in twisting a wire.
13) Derive an expression for the period of oscillation of a torsion pendulum.
14) What is meant by a beam? Explain the terms neutral surface, neutral axis, plane of bending, and bending moment of a beam.
15) Describe, with necessary theory, how the rigidity modulus of the material of a rod is determined by the static torsion method.
16) Describe, with necessary theory, how you would determine the rigidity modulus of a wire experimentally by using the torsion pendulum.

17) What is a cantilever? Obtain an expression for the depression at the free end of a thin light beam clamped horizontally at one end and loaded at the other.

18) Derive an expression for the bending of a bar supported at the two ends and loaded in the middle. Describe an experiment to determine E by bending.

19) Describe, with necessary theory, the oscillation method to determine E for the material of a cantilever.

20) A wire of length lm and diameter 10^{-3} m is fixed at one end and twisted at the other end through an angle of 70^{o} by applying a couple of value 0.01 Nm. Evaluate the rigidity modulus of the wire.

21) Explain the theory of uniform bending?

22) Explain the theory of non-uniform bending?

NOTES

UNIT – II

HEAT & OPTICS

HEAT

2. Postulates of the Kinetic Theory of Gases

1. A gas is composed of a large number of tiny, perfectly elastic particles, called molecules.
2. The molecules of a gas are identical in all respects.
3. The molecules are in a state of continuous motion. They move in all directions with all possible velocities. In their motion, they collide with each other and with the walls of the containing vessel. Collisions between molecules are elastic.
4. The molecules move in straight lines between any two consecutive collisions.
5. The average distance traveled by a molecule between two successive collisions is called the mean free path.
6. The time of impact is negligible in comparison to the time taken to traverse the free path.
7. The molecules do not exert any force of attraction or repulsion on one another.
8. The volume of the molecule is negligible, compared with the volume of the gas.
9. The molecules are perfectly hard elastic spheres and the whole of their energy is kinetic.

2.1 Van der Waals Equation of State

The perfect gas equation PV = RT was derived on the following assumptions:

(i) the size of the molecule of the gas is negligible.

(ii) the forces of inter-molecular attraction are absent.

But in actual practice, at high pressure, the size of the molecules of the gas becomes significant. So the size of the molecule cannot be neglected in comparison with the volume of the gas.

At high pressure, the molecules come closer and the forces of intermolecular attraction are appreciable.

Van der Waals modified the perfect gas equation PV =RT by applying corrections for (i) finite size of the molecules and (ii) intermolecular attractions.

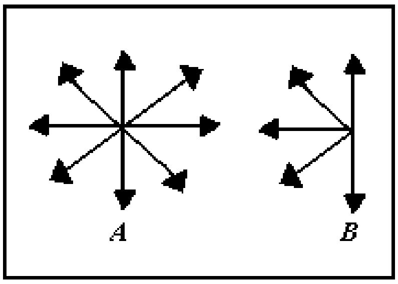


Fig. 2.1

Let us now derive Van der Waals equation of state.

(i) **Correction for Pressure:**

A molecule at a point A well within the interior of the gas is attracted by other molecules equally in all directions. Hence the net cohesive force on it is zero (Fig. 2.1). But when a molecule B strikes the walls of the container, it is pulled back by other molecules. Such molecules collide on the walls of the vessel with a diminished momentum. Due to this reason, the observed pressure of the gas is less than the actual pressure.

The correction for pressure p depends upon:

(i) the number of molecules striking unit area of the walls of the container per second and

(ii) the number of molecules present in a given volume.

Both these factors depend on the density ρ of the gas.

$\therefore$ Correction for pressure $p \propto \rho^2 \propto 1/V^2$

$$p = \frac{a}{V^2}$$

Here, a is a constant and V is the volume of the gas

Corrected pressure $(P + p) = \left(P + \frac{a}{V^2}\right)$ …(1)

Here, P is the observed pressure.

(iii) Correction for Volume:

In the case of a real gas, molecules have a definite size and therefore occupy some volume. Hence the actual space for the movement of the molecules is less than the volume of the vessel.

Corrected volume of the gas = (V-b) …(2)

Here, b is a constant for the given mass of the gas. It is called the co-volume of the gas. It is not equal to the actual volume occupied by the gas molecules. Van der Waals showed that b is equal to four times the actual volume of the molecules.

Thus the Van der Waals equation of state for a gas is

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT \qquad \ldots(3)$$

Here, a and b are known as van der Waals constants.

2.2 Derivation of Critical Constants

Consider the isothermal ACB, at the critical temperature T_c (Fig. 2.2). The values of volume, pressure and temperature of a gas corresponding to its critical point C are called the critical constants of the gas. They are called critical volume, critical pressure and critical temperature, denoted by V_c, P_c and T_c respectively. The critical point C is a point of inflexion.

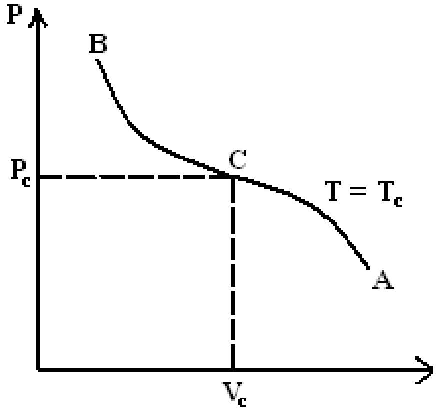


Fig. 2.2

Hence, at $T=T_c, \frac{dP}{dV}=0$ and $\frac{d^2P}{dV^2}=0.$

From Van der Waals equation, we get

$$P=\frac{RT}{(V-b)}-\frac{a}{V^2} \qquad \ldots(1)$$

At the critical point C,

$$\frac{dP}{dV}=-\frac{RT}{(V-b)^2}+\frac{2a}{V^3}=0 \qquad \ldots(2)$$

and $$\frac{d^2P}{dV^2}=\frac{2RT}{(V-b)^3}-\frac{6a}{V^4}=0 \qquad \ldots(3)$$

At the critical point,

$$P=P_c, V=V_c \text{ and } T=T_c$$

From (1), $$P_c=\frac{RT_c}{(V_c-b)}-\frac{a}{V_c^2} \qquad \ldots(4)$$

From (2), $$\frac{RT_c}{(V_c-b)^2}=\frac{2a}{V_c^3} \qquad \ldots(5)$$

From (3) $$\frac{2RT_c}{(V_c-b)^3}=\frac{6a}{V_c^4} \qquad \ldots(6)$$

Dividing Eq. (5) by (6),

$$\frac{V_c-b}{2}=\frac{V_c}{3}$$

$$\therefore \quad V_c=3b$$

Substituting $V_c=3b$ in Eqn. (5),

$$\frac{RT_c}{4b^2}=\frac{2a}{27b^3} \qquad \ldots(7)$$

$$\therefore \quad T_c=\frac{8a}{27Rb} \qquad \ldots(8)$$

Substituting $V_c=3b, T_c=\frac{8a}{\left(27Rb\right)}$ in Eq. (4),

$$P_c=\frac{R\times\left(8a/27Rb\right)}{\left(3b-b\right)}-\frac{a}{9b^2}$$

$$\therefore \quad P_c=\frac{a}{27b^2} \qquad \ldots(9)$$

Critical constants of a gas:

(i) *Critical temperature (T_c)* of a gas is that temperature above which it is not possible to liquefy the gas by the application of pressure alone. To liquefy a gas, it must be cooled below its critical temperature before compressed.

(ii) *Critical pressure (P_c)* is that pressure which just liquefies a gas at its critical temperature.

(iii) *Critical volume (V_c)* is the volume of unit mass of gas at its critical temperature and pressure. It is the reciprocal of the critical density. It is often taken as the volume of one mole of a gas at its critical temperature and pressure.

Demerits of Van der Waals equation

(i) Van der Waals assumed a and b to be constants but experiments show that a and b vary with temperature.

(ii) The ratio $RT_c/(P_cV_c)$ is called the critical coefficient.

$$\frac{RT_c}{P_cV_c} = R \times \frac{8a}{27Rb} \times \frac{27b^2}{a} \times \frac{1}{3b} = \frac{8}{3} = 2.67$$

Thus, the critical coefficient should be constant for all gases. But, actually it caries from 3.28 for hydrogen to 4.99 for acetic acid.

(iii) According to Van der Waals, V_c = 3 b for all gases. But, actually it varies form 1.4 b for argon to 2.8b for hydrogen.

LOW TEMPERATURE PHYSICS

Production of low temperature. The general principle involved in the production of low temperature is to device means for removing from a body its heat content. The following methods have been employed, with what results we shall presently see.

(i) Freezing mixtures of salts in ice.

(ii) Cooling due to evaporation of liquids under reduced pressure.

(iii) Cooling due to Joule-Thomson effect.

(iv) Cooking due to adiabatic expansion.

(v) Cooling due to desorption.

(vi) *Adiabatic demagnetization,*

i.e., by demagnetizing certain crystals adiabatically, magnetic energy is made to leave the system and thereby extreme cooling is produced under suitable conditions.

Of these methods, the first two. viz., freezing mixtures and cooling by evaporation are of practical importance, as they are utilized in the modern refrigerating machines. The third and fourth methods have been used for the liquefaction of gases. These are very important, since these are intimately connected with the production of very low temperatures and are of great interest on account of the theoretical principles involved as well of the many practical applications both in science and industry. The method of cooling due to desorption has its use in aiding to a further cooling of an already very much cooled system. For example, charcoal absorbs helium at 5 atmospheres and 10^oK and when it is allowed to escape at a lower pressure of 0.1 mm. cooling results due to the heat absorption by a process similar to the evaporation of the liquids and the temperature is lowered to 4^oK which is sufficient to liquefy Helium. The last method of adiabatic demagnetization is of special interest to the physicist, as it has enabled him to reach almost the absolute zero, where matter, so to say, is born, and any information concerning the nature of substances in that region is bound to be of great value. It is to be noted that most of these methods are not exclusive of each other by complementary, in the sense that they can be used conductively reach the desired low temperature. Here, we have discussed only Joule Thomson Effect.

2.3 Joule-Thomson Effect

If a gas initially at a constant high pressure is allowed to suffer throttle expansion through the porous plug of silk wool or cotton wool having a number of fine pores, to a region of constant lower pressure adiabatically, a change of temperature of the gas (either cooling or heating) is observed. This effect is called Joule-Thomson or Joule-Kelvin effect.

Joule in collaboration with William Thomson [Lord Kelvin] devised a very sensitive technique known as **porous plug experiment** and performed number of experiments from 1852 to 1862 and established beyond doubt the existence of intermolecular attraction. In this effect, the total heat function $H = U + PV$ remains constant.

At ordinary temperatures, all gases except hydrogen show cooling effect on passing through the porous plug, but hydrogen shows a heating effect. At sufficiently low temperatures all gases show a cooling effect.

2.3.1 Joule-Thomson Porous Plug Experiment

The experimental set up for the porous plug experiment to study the Joule-Thomson effect is shown in Fig.2.3.1. It consists of the following main parts:

1. The apparatus consists of a porous plug having two perforated brass discs *D, D.*
2. The space between *D, D* is packed with cotton wool or silk fibers.
3. The porous plug is fitted in a cylindrical box-wood *W* which is surrounded by a vessel containing cotton wool (Fig. 2.3.1). This is done to avoid loss or gain of heat from the surroundings.
4. T_1 and T_2 are two sensitive platinum resistance thermometers and they measure the temperatures of the incoming and outgoing gas.
5. The gas is compressed to a high pressure with the help of piston

P and it is passed through a spiral tube immersed in water bath maintained at a constant temperature. If there is any heating of the gas due to compression, this heat is absorbed by the circulating water in the water bath.

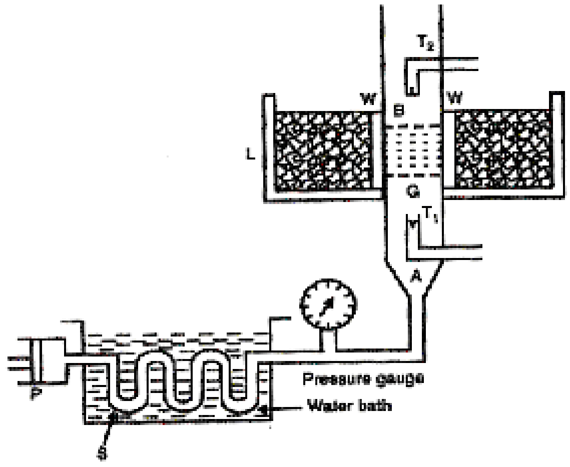


Fig. 2.3.

Experimental procedure

By means of the compression pump *P,* the experimental gas is passed slowly and uniformly through the porous plug keeping the high pressure constant, read by pressure gauge. During its passage through the porous plug, the gas is throttled, *i.e.* the separation between the molecules increases. On passing through the porous plug, the volume of the gas increases against the atmospheric pressure. As there is no loss or gain of heat during the whole process, the expansion of the gas takes place adiabatically.

The initial and final temperatures are noted by means of the platinum resistance thermometers T_1 and T_2.

2.4 Experimental Results

Inversion Temperature

The behaviour of a large number of gases was studied at various inlet temperatures of the gas and the results obtained are as follows:

(i) At sufficiently low temperatures, all gases show a cooling effect.

(ii) At ordinary temperatures, all gases except hydrogen and helium show cooling effect. Hydrogen and helium show heating instead of cooling at room temperature.

(iii) The fall in temperature is directly proportional to the difference in pressure on the two sides of the porous plug.

(iv) The fall in temperature for a given difference of pressure decreases with rise in the initial temperature of the gas. It was found that the cooling effect decreased with the increase of initial temperature and became zero at a certain temperature and at a temperature higher than this temperature, instead of cooling, heating was observed. This particular temperature at which the Joule-Thomson effect changes sign is called the *temperature of inversion.*

Definition

The temperature at which Joule-Thomson effect is zero and changes sign is known as the temperature of inversion.

It is denoted by T_i and at this temperature

$$\frac{2a}{RT_i} = b$$

or $$T_i = \frac{2a}{Rb}$$

when $T < T_i$, cooling takes place, and

$T > T_i$, heating takes place.

for helium $T_i = 35$ K, and for hydrogen $T_i = 193$ K.

Regenerative Cooling

In case of hydrogen and helium, heating was observed at room temperature because it was at a temperature far higher than its temperature of inversion. The temperature of inversion for hydrogen is –80°C and for helium is –258°C. If helium is passed through the porous plug at a temperature lower than –258°C, it will also show cooling effect. It means any gas below its temperature of inversion shows a cooling effect when it is passed through the porous plug or a throttle valve. This is called regenerative cooling or Joule-Kelvin cooling. This principle is used in the liquefaction of the so called permanent gases like nitrogen, oxygen, hydrogen and helium.

Estimates of J-T Cooling (Theory of Porous Plug Experiment)

Theory

The simple arrangement of the Joule-Thomson porous plug experiment is shown in Fig. 2.4. The gas is allowed to pass through the porous plug from the high pressure side to the low pressure side. Due to this large difference of pressure, the gas flows through the fine pores and becomes throttled or wire drawn, *i.e.* the molecules of the gas are further drawn apart from one another.

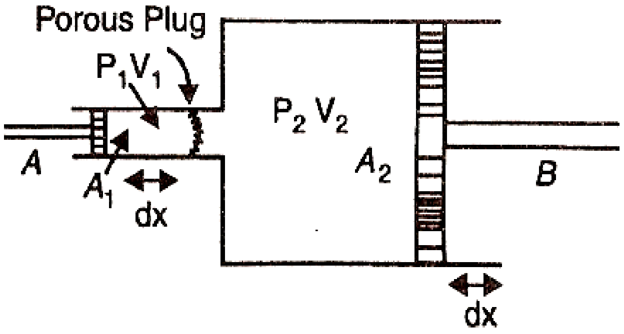


Fig. 2.4

Consider one gram molecule of a gas to the left and to right of the porous plug. Let P_1, V_1 and P_2, V_2 represent the pressure and volume on the two sides of the porous plug. When the piston A is moved through a certain distance dx, the piston B also moves through the same distance dx. The work done on the gas by the piston $A = P_1 A_1 dx = P_1 V_1$. The work done by the gas on the piston $B = P_2 A_2 dx = P_2 V_2$. Thus, the net external work done by the gas is $P_2 V_2 - P_1 V_1$.

If w is the work done by the gas in separating the molecules against their intermolecular attractions, the total amount of work done by the gas is

$$(P_2 V_2 - P_1 V_1) + w$$

No heat is gained or lost to the surroundings. There are three possible cases:

(i) Below the Boyle temperature,

$$P_1 V_1 < P_2 V_2$$

and $P_2 V_2 - P_1 V_l$ is $+ve$, w must be either positive or zero. Thus a net $+ve$ work is done *by the gas* and there must be cooling when the gas passes though the porous plug.

(ii) At the Boyle temperature if P_1 is not very high

$$P_1 V_1 = P_2 V_2$$

and $$P_2 V_2 - P_1 V_1 = 0$$

The total work done by the gas in this case is w. Therefore cooling effect at this temperature is only due to the work done by the gas in overcoming intermolecular attractions.

(iii) Above the Boyle temperature

$$P_1 V_1 > P_2 V_2$$

or $$P_2 V_2 - P_1 V_1 \text{ is } -ve$$

Thus, the observed effect will depend upon whether $(P_2 V_2 - P_1 V_1)$ is greater or less than w.

If $w > (P_l V_l - P_2 V_2)$, cooling will be observed.

If $w < (P_l V_l - P_2 V_2)$, heating will be observed.

Thus, the cooling or heating of a gas due to free expansion through a porous plug from a high pressure to a low pressure side will depend on *(i)* the deviation from Boyle's law and *(ii)* work done in overcoming intermolecular attractions.

Joule-Kelvin Effect-Temperature of Inversion

Assuming that the Van der Waals equation is obeyed, the attractive forces between the molecules are equivalent to an internal pressure $\frac{a}{V^2}$.

When the gas expands from V_1 to V_2, the work done in overcoming intermolecular attractions

$$w = \int_{V_1}^{V_2} P.dV$$

But $$P = \frac{a}{V^2}$$

$\therefore$ $$w = \int_{V_1}^{V_2} \frac{a}{V^2} dV$$

$$= -\frac{a}{V_2} + \frac{a}{V_1}$$

If V_1 and V_2 represent the gram molecular volumes on the high and the low pressure sides respectively, the external work done by the gas is

$$(P_2V_2 - P_1V_1)$$

Hence the total work done by the gas

$$W = (P_2V_2 - P_1V_1) + w$$

$$= (P_2V_2 - P_1V_1) - \frac{a}{V_2} + \frac{a}{V_1}$$

Van der Waals equation of state for a gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

or $$PV + \frac{a}{V} - bP - \frac{ab}{V^2} = RT$$

or $$PV = RT + bP - \frac{a}{V}$$

$$\left[\frac{ab}{V^2}\text{ is negligible}\right]$$

$$W = \left[RT + bP_2 - \frac{a}{V_2}\right] - \left[RT + bP_1 - \frac{a}{V_1}\right] - \frac{a}{V_2} + \frac{a}{V_1}$$

$$= b\left[P_2 - P_1\right] + 2a\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

But $$V_1=\frac{RT}{P_1} \quad \text{and} \quad V_2=\frac{RT}{P_2}$$

$$\therefore \qquad W=b\left[P_2-P_1\right]+2a\left[\frac{P_1}{RT}-\frac{P_2}{RT}\right]$$

or $$W=-b\left[P_1-P_2\right]+\frac{2a}{RT}\left[P_1-P_2\right]$$

or $$W=\left(P_1-P_2\right)\left(\frac{2a}{RT}-b\right) \qquad \ldots\text{(i)}$$

Suppose the fall in temperature is δT

$$W=JH$$

$$=J\,[MC_P\delta T]$$

where M is the gram-molecular weight of the gas

$$\therefore \qquad JMC_P\delta T=\left(P_1-P_2\right)\left(\frac{2a}{RT}-b\right)$$

or $$\delta T=\left(\frac{P_1-P_2}{JMC_P}\right)\times\left(\frac{2a}{RT}-b\right) \qquad \ldots\text{(ii)}$$

(i) Since P_1-P_2 is + ve

δT will be + ve if $\left(\frac{2a}{RT}-b\right)$ is + ve

i.e., $$\frac{2a}{RT}>b \quad \text{or} \quad T<\frac{2a}{Rb} \qquad \ldots\text{(iii)}$$

Therefore, cooling will take place if the temperature of the gas is less than $\frac{2a}{Rb}$.

(ii) For δT to be zero, from equation (ii),

$$\frac{2a}{RT}-b=0$$

or $$T=\frac{2a}{Rb}$$

This temperature is called the temperature of inversion and is represented by T_i

$$T_i=\frac{2a}{Rb} \qquad \ldots\text{(iv)}$$

(iii) δT will be negative, if

$$\left(\frac{2a}{RT}-b\right) \text{ is –ve}$$

i.e., $$b > \frac{2a}{RT}$$

or $$T > \frac{2a}{Rb}$$

$$T > T_i$$

Therefore, heating will take place if the temperature of the gas is more than the temperature of inversion.

Results

(i) If the gas is at the temperature of inversion, then no cooling or heating is observed when it is passed through the porous plug.

(ii) If the gas is at a temperature lower than the temperature of inversion, cooling will take place when it is passed through the porous plug. This is called regenerative cooling or Joule-Kelvin cooling. This principle has been used in the liquefaction of the so-called permanent gases like nitrogen, oxygen, hydrogen and helium.

(iii) If the gas is at a temperature higher than the temperature of inversion, instead of cooling, heating is observed when the gas passed through the porous plug.

2.5 Relation between Boyle Temperature, temperature of Inversion and Critical Temperature

Temperature of inversion,

$$T_i = \frac{2a}{Rb} \qquad \ldots(1)$$

Boyle temperature,

$$T_B = \frac{a}{Rb} \qquad \ldots(2)$$

Critical temperature,

$$T_c = \frac{8a}{27Rb} \qquad \ldots(3)$$

From (1) and (2), we have

$$T_i = 2T_B$$

From (1) and (3), we have

$$\frac{T_i}{T_c} = \frac{2a}{Rb} \cdot \frac{27Rb}{8a}$$

$$= \frac{27}{4} = 6.75 \qquad \ldots(4)$$

The experimental value of $\frac{T_i}{T_c}$ for actual gases is just less than 6.

It means that the temperature of inversion is very much higher than the critical temperature. For hydrogen $T_i = 190$ K and $T_c = 33$ K. As $T_i \geq T_c$, the methods employing regenerative cooling (Joule-Kelvin cooling) are preferred to those employing the initial cooling of the gas below the critical temperature.

Since ratio $\frac{T_i}{T_c}$ is a number (i.e., 6.75) from equation (4), the ratio of temperature of inversion and critical temperature does not depend upon the nature of the gas.

LIQUEFACTION OF GASES

Introduction

A gas goes into liquid and solid forms as the temperature is reduced. Thus the processes of liquefaction of gases and solidification of liquids are intimately involved in the production of low temperatures. Andrews experiment showed that if a gas is to be liquefied by merely applying pressure on it, it has to be cooled below its critical temperature. Critical temperatures of carbon dioxide, ammonia are sulphur dioxide are higher than room temperature. Hence these gases can be liquefied at room temperature without pre-cooling, simply by increasing the pressure. So, the simple process would be to cool the gas below its critical temperature by some coolant and then to liquefy it by applying a pressure on it. This method does not work always, in the case of some gases like oxygen, nitrogen, hydrogen and helium; no pre-cooling can bring the temperature below their critical temperatures. Thus, they cannot liquefied by this method.

Joule–Thomson expansion is a very important technique to liquefy gases. The cooling produced in J-T expansion of a gas depends on the difference of pressure on the two sides of the porous plug and the initial temperature. For most of the gases, the J-T cooling is very small. However, the cooling effect can be intensified by employing the process called regenerative cooling.

Regenerative Cooling

This process consists in cooling the incoming gas by a portion of the gas which has already been cooled by J-T expansion. That is, the cooling effect is made cumulative. By maintaining a continuous cycle of such operations, the initial temperature can be rapidly and progressively lowered. This happens due to the fact that, lower the temperature the greater is the cooling in J – T process.

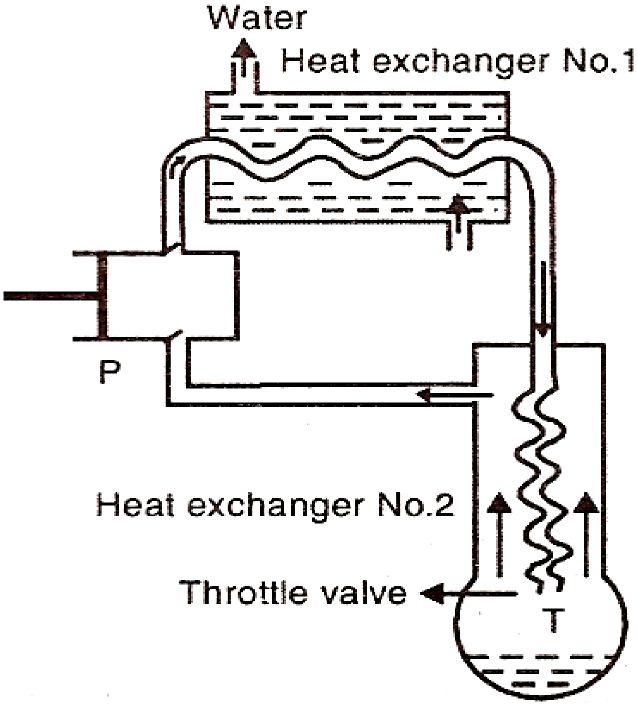


Fig. 2(ii)

The schematic arrangement of regenerative principle is shown in Fig. 2(ii). The gas is compressed to a high pressure by the compressor P. The heat of compression is removed by passing the gas in a water- cooled jacket (Heat exchanger No. 1). Then the gas undergoes J-T expansion at needle valve T so that it cools by a small amount. This cooled gas is allowed to go back to the pump P. while going up, it cools the incoming gas in heat exchanger No. 2. This gas gets further cooled after undergoing J-T expansion at T. This regenerative process continues till the gas gets liquefied. This process was first used by Linde to liquefy air.

2.6 Liquefaction of hydrogen

Temperature of inversion of hydrogen is – 80°C. But for practical purposes, it has to be pre-cooled to – 177° C to obtain appreciable cooling due to J-T effect. The apparatus is shown in Fig. 2.6. The entire apparatus is enclosed in an outer Dewar flask L for complete insulation. Hydrogen is compressed to a pressure of about 200 atmospheres by the pump P. Then it is passed through a coil immersed in solid CO_2 and alcohol. Then it enters the coil a in the chamber A where it is further cooled by the outgoing hydrogen. The hydrogen coming from chamber A is then passed through the coil b in chamber B. The chamber B contains liquid air and cools hydrogen in the coil b.

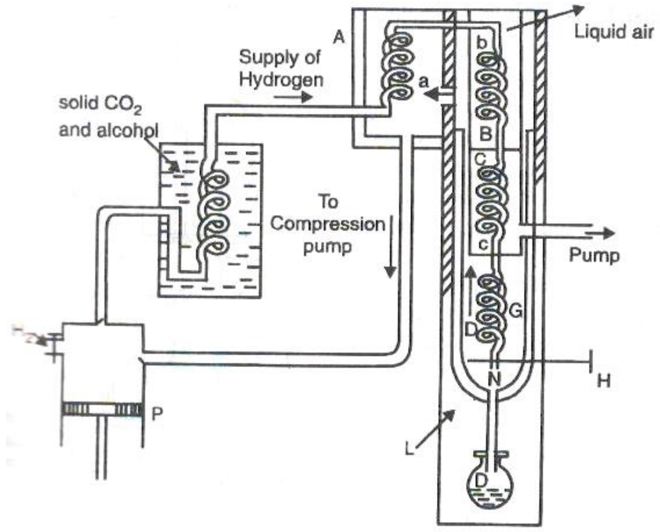


Fig. 2.6

This hydrogen is then passed through coil c in the chamber C. In chamber C, liquid air is allowed to boil under reduced pressure. Hence in this chamber hydrogen is cooled to a temperature of – 200^o C. After the pre-cooling, the hydrogen passes through the coil D. It suffers throttle expansion at the nozzle valve N which is operated by the handle H. Hence hydrogen is cooled further due to J-T effect. The cooled hydrogen passes up through the chambers C, B and A, cooling the gas from compress the gas coming through the coils in those chambers. Thus, the incoming gas from the compressor is cooled by the gas flowing up the chambers after expansion. After a few cycles, the temperature of the gas reaching N falls to -250^o C. Hydrogen gas under these conditions suffering J-T expansion is liquiefied and collected in the Dewar flask D.

The boiling point of liquid hydrogen at atmospheric pressure is 252.8^o C. By making it boil at a lower pressure, the liquid can be made to freeze into a solid. Solid hydrogen is white in colour.

2.6.1 Liquefaction of helium.

Helium, the last remaining gas, could not be liquefied for a long time. The attempts made by Dewar and Olszewski by adiabatic expansion method proved unsuccessful. At last, in 1908, Kamerlingh Onnes, after a tenacious and systematic study of the problem for well over ten years, succeeded to liquefy helium by the Joule-Thomson process. He first cleared the ground by determining the various constants that were implicated in the experiment. Such were the critical temperature $T_c = -268^o C = 5^o K$, the critical pressure $p_c = 2.3$ atmospheres, the normal boiling point = $-268.8^oC = 4.2^o K$, the inversion temperature = $-238^o C = 35^o K$ and the Boyle temperature = $- 256^o C = 17^o K$. He then concluded that if helium was to be liquefied by the Joule-Thomson expansion it had to be pre-cooled to 17^o K.

He succeeded in obtaining the temperature by passing compressed helium through liquid hydrogen boiling at reduced pressure. The apparatus employed is very complicated due to the use of air and hydrogen liquefiers for pre-cooling processes. A simple form of the apparatus for final stages is as shown in Fig. 2.6.1 pre-cooled helium gas at a pressure of about 40 atmospheres enters the spiral tube at C and flows partly through the spiral tubes S_1 and partly through S_2.

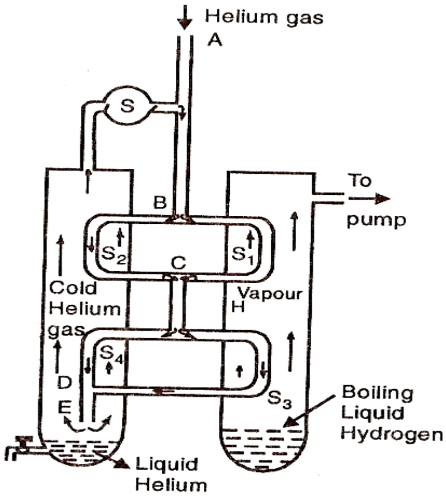


Fig. 2.6.1.

The spiral S_1 is surrounded by hydrogen boiling under reduced pressure and portion passing through the spiral S_2 is surrounded by cooled out going helium gas. Similar process takes place in the spirals S_3 and S_4 which are also surrounded by boiling liquid hydrogen and cooled helium respectively. The circulation process is repeated and when temperature of helium is sufficiently low it gets liquefied in passing through the nozzle N due to Joule-Thomson effect. The outgoing helium is compressed again by the compressor P and fed back to the spirals S_1 and S_2 and then S_3 and S_4. Liquid helium is collected in the Dewar flask through the tap T. To have perfect heat insulation the whole apparatus is surrounded by Dewar flasks.

Kapteza has liquefied helium pre-cooled by passing through cooling liquid nitrogen. Simon using the fact that the thermal capacity of metal container is extremely small at temperatures near the absolute zero, has liquefied helium in small quantities. By adiabatically expanding helium (with subsequent cooling) contained in a small metal cylinder immersed in liquid hydrogen the gas of liquefied. Simon also making use of the fact that heat is evolved when a gas is adsorbed on charcoal and when it is desorbed the temperature of the gas falls, liquefied helium by using considerably large quantities of activated charcoal immersed in liquid hydrogen. To minimize heat exchanges with the surroundings the apparatus is enclosed in an evacuating vessel. On pumping off helium, the temperature considerably falls and thus it is liquefied.

The storage of liquid hydrogen and liquid helium presents a more difficult problem than in the case of liquid air as these have a very low temperatures and low latent heats. Liquid hydrogen is usually stored in a double-Dewar vessel, the space between the evacuated containers is filled with liquid air.

To store liquid helium a triple Dewar vessel is generally used, the liquid helium being surrounded by liquid hydrogen which in turn is surrounded by liquid air. Liquid helium is a colourless mobile liquid which boils at 4.2K. It has a density which is about one-eighth that of water and its very flat meniscus shows that it has a very small surface tension.

OPTICS

Interference

2.7 Introduction

The phenomenon of interference of light has proved the validity of the wave theory of light. Thomas successfully demonstrated his experiment on interference of light, in 1802. when two or more wave trains act simultaneously on any particle in a medium, the displacement of the particle at any instant is due to the superposition of all the wave trains.

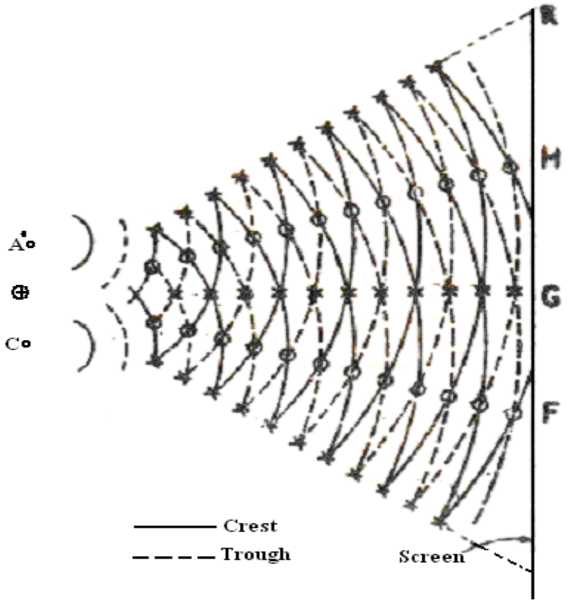


Fig. 2.7

Also after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent. This principle was explained by Huygens in 1678.

The phenomenon of interference of light is due to the superposition of two wave trains within the region of cross over. Let us consider the waves produced on the surface of water. In Fig. 2.7 points A and B are the two sources which produce waves of equal amplitude and constant phase difference. Waves spread out on the surface of water which are circular in shape. At any instant, the particle will be under the action of the displacement due to both the waves. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The points shown by crosses in the diagram will have maximum displacements because, either the crest of one will combine with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of with each other. As the intensity (energy) is directly proportional to the square of the amplitude $(I \alpha A^2)$ the intensity at these points is four times the intensity due to one wave. It should be remembered that there is no loss of energy due to interference. The energy is only transferred from the points of minimum displacement to the points of maximum displacement.

2.7.1 Young's Experiment

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes A and B (fig 2.7.1)

A and B are equidistant from S and are close to each other. Spherical waves spread out from S. spherical waves also spread out from A and B. these waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as E are bright because the crest due to one wave coincides with the crest due to the other and therefore they reinforce each other. The points such as F are dark because the crest of one falls on the trough of the other and they neutralize the effect of each other. Points, similar to E, where the trough of one falls on the trough of the other, are also bright because the two waves reinforce.

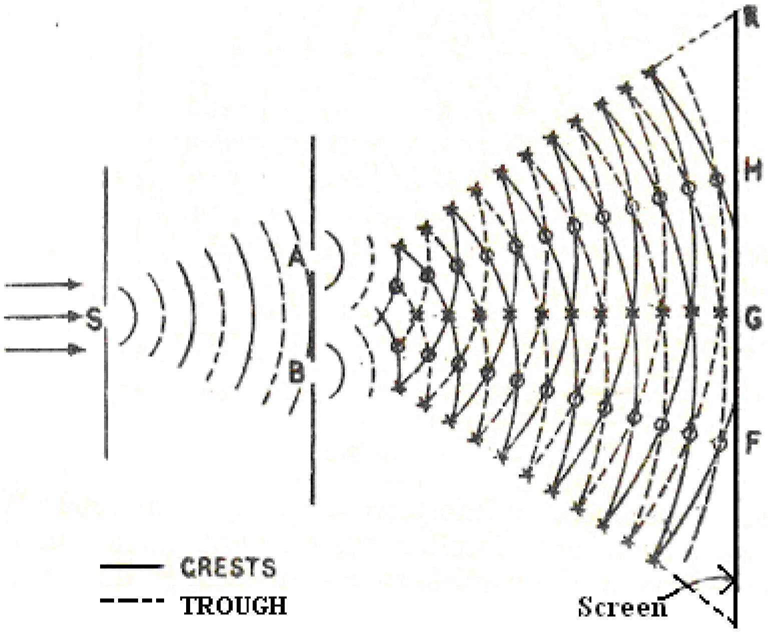


Fig. 2.7.1

It is not possible to show interference due to the independent sources of light because a large number of difficulties are involved. The two sources may emit light waves of largely different amplitude and wavelength and the phase difference between the two may change with time.

Coherent Sources

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. Methods have been devised where (i) interference of light takes place between the waves from the real source and a virtual source (ii) interference of light takes place between waves from two virtual sources formed due to a single source. In all such cases, the two sources will act, as if they are perfectly similar in all respects.

Since the wavelength of light waves is extremely small (of the order of 10^{-5} cm), the two sources must be narrow and must also be close to each other. Maximum intensity is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of 2π or the path difference between the two waves is a whole number multiple of wavelength. For minimum intensity at a point, the phase difference between the two waves reaching the point should be an odd number multiple of π or the path difference between the two waves should be an odd number multiple of half wavelength.

Phase Difference and path difference

If the path difference between the two waves is λ, the phase difference $= 2\pi$

Suppose for a path difference x, the phase difference is δ

For a path difference λ, the phase difference $= 2\pi$

$\therefore$ for a path difference x, the phase difference

$$= \frac{2\pi x}{\lambda}$$

Phase difference $\qquad \delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} x$ (path difference)

2.7.2 Analytical treatment of interference

Consider a monochromatic source of light S emitting waves of wavelength λ and two narrow pinholes A and B (Fig. 2.7.2) A and B are equidistant from S and act as two virtual coherent sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant, is δ

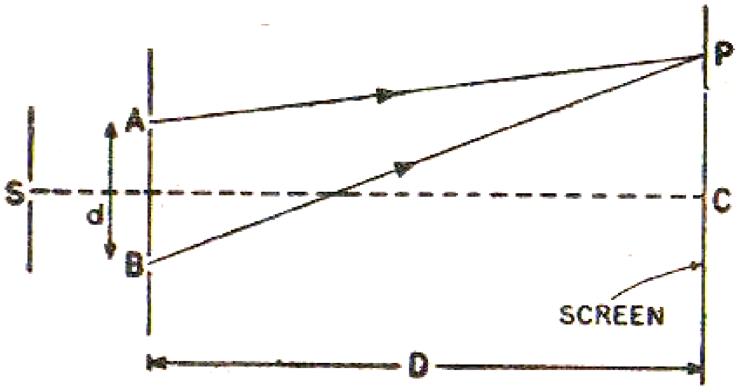


Fig. 2.7.2

If y_1 and y_2 are the displacement

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin(\omega t + \delta)$$

$$y = y_1 + y_2 = a\sin\omega t + a\sin(\omega t + \delta)$$

$$\therefore \qquad y = a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$

$$= a\sin\omega t(1+\cos\delta) + a\cos\omega t\sin\delta$$

Taking $\qquad a(1+\cos\delta) = R\cos\theta \qquad$ …(i)

And $\qquad a\sin\delta = R\sin\theta \qquad$ …(ii)

$$y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$$

$$y = R\sin(\omega t + \theta) \qquad \text{…(iii)}$$

Which represents the equation of simple harmonic vibration of amplitude R.

Squaring (i) and (ii) and adding.

$$R^2 \sin^2\theta + R^2\cos^2\theta = a^2\sin^2\delta + a^2(1+\cos\delta)^2$$

(or) $$R^2 = a^2\sin^2\delta + a^2(1+\cos^2\delta + 2\cos\delta)$$

$$R^2 = a^2\sin^2\delta + a^2 + a^2 + \cos^2\delta + 2a^2\cos\delta)$$

$$= 2a^2 + 2a^2\cos\delta$$

$$= 2a^2(1+\cos\delta)$$

$$R^2 = 2a^2.2\cos^2\frac{\delta}{2} = 4a^2\cos^2\frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$\therefore$ $$I = R^2$$

(or) $$I = 4a^2\cos^2\frac{\delta}{2} \qquad \text{...(iv)}$$

Special cases: (i) When the phase difference $\delta = 0, 2\pi, 2(2\pi), ...n(2\pi),$ or the path difference $x = 0, \lambda, 2\lambda, ...n\lambda.$

$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(ii) When the phase difference, $\delta = \pi, 3\pi, ...(2n+1)\pi,$ or the path difference

$$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ...(2n+1)\frac{\lambda}{2},$$

$$I = 0$$

Intensity is minimum when the path difference is an odd number multiple of half wavelength.

(iii) Energy distribution. From equation (iv) it is found that the intensity at bright points is $4a^2$ and at dark points it is zero. According to the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. For at bright point, the intensity due to the two waves should be $2a^2$ but actually it is $4a^3$. As shown in Fig. 2.7.2(iii)., the intensity varies from 0 to $4a^2$, and the average in still $2a^2$. It is equal to the uniform intensity $2a^2$ which will be present in the absence of the interference phenomenon due to the two waves. Therefore the formation of interference fringes is in accordance with the law of conservation of energy.

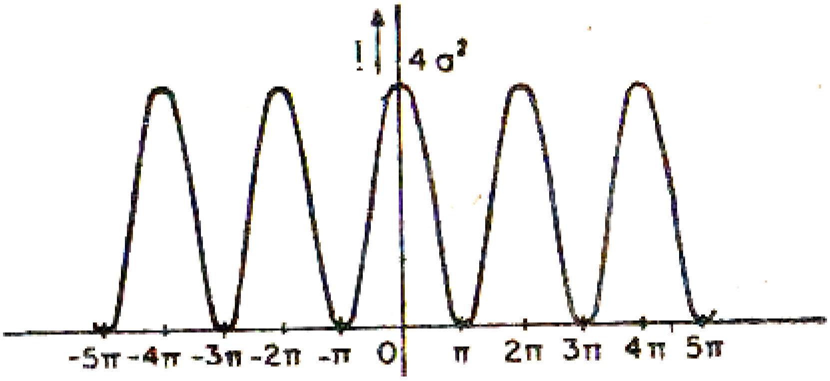


Fig. 2.7.2(iii)

2.7.3 Theory of interference fringes

Consider a narrow monochromatic source S and two pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen by placed at a distance D from the coherent source.

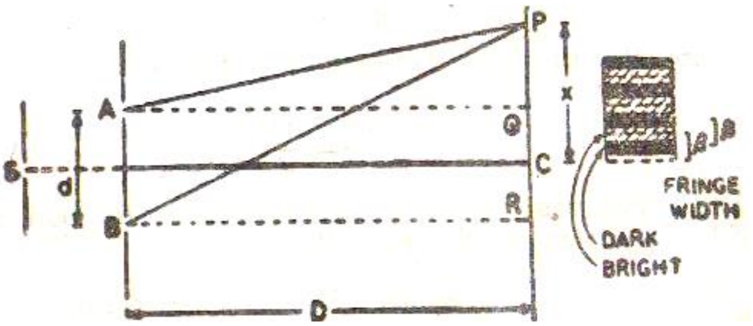


Fig. 2.7.3

The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus the point C has maximum intensity.

Consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here, $PQ = x - \frac{d}{2}, \quad PR = x + \frac{d}{2}$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right]$$

$$(BP)^2 - (AP)^2 = 2xd$$

$$BP - AP = \frac{2xd}{BP + AP}$$

But $BP = AP = D$ (approximately)

$\therefore$ Path difference $= BP - AP = \frac{2xd}{2D} = \frac{xd}{D}$ …(i)

Phase difference $= \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$ …(ii)

(i) **Bright Fringes.** If the path difference is a whole number multiple of wavelength λ, the point P is bright.

$$\therefore \quad \frac{xd}{D} = n\lambda$$

Where $n = 0, 1, 2, 3...$

(or) $x = \frac{n\lambda D}{d}$ …(iii)

This equation gives the distances of the bright fringes from the point C. At C, the path difference is zero and a bright fringe is formed.

When $n = 1,$ $x_1 = \frac{\lambda D}{d}$

$n = 2,$ $x_2 = \frac{2\lambda D}{d}$

$n = 3,$ $x_2 = \frac{3\lambda D}{d}$

$x_n = \frac{n\lambda D}{d}$

Therefore the distance between any two consecutive bright fringes

$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$ …(iv)

(ii) **Dark Fringes.** If the path difference is an odd number multiple of half wavelength, the point P is dark.

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2} \, where \, n = 0, 1, 2, 3...$$

(or) $x = \frac{(2n+1)\lambda D}{2d}$ …(v)

This equation gives the distances of the dark fringes from the point C.

When, $n = 0,$ $x_0 = \frac{\lambda D}{2d}$

$n = 1,$ $x_1 = \frac{3\lambda D}{2d}$

$$n=2, \qquad x_2=\frac{5\lambda D}{2d}$$

and $$x_n=\frac{(2n+1)\lambda D}{2d}$$

The distance between any two consecutive dark fringes,

$$x_2-x_1=\frac{5\lambda D}{2d}-\frac{3\lambda D}{2d}=\frac{\lambda D}{d} \qquad \ldots\text{(vi)}$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. Moreover, from equations (v) and (vi) it is clear that the width of the dark fringe. All the fringes are equal in width and are independent of the order of the fringe. The breadth of a bright or a dark fringe is, however, equal to half the fringe width and is equal to $\frac{\lambda D}{2d}$. The fringe width $\beta=\frac{\lambda D}{d}$.

Therefore, (i) the width of the fringe is directly proportional to the wavelength of light, $\beta \propto \lambda$. (ii) The width of the fringe is directly proportional to the distance of the screen from the two sources, $\beta \propto D$. (iii) the width of the fringe is inversely proportional to the distance between the two sources, $\beta \propto \frac{1}{d}$. Thus, the width of the fringe increases (a) with increase in the distance D and (c) by bringing the two sources A and B close to each other.

Example 1. Green light of wavelength 5100Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2cm, find the slit separation.

$$\beta=\frac{\lambda D}{d}$$

Here $\lambda=5100\times10^{-8}\,cm, \qquad d=?$

$$D=200cm$$

$$10\beta=2\,cm$$

(or) $\beta=0.2\,cm$

$$d=\frac{\lambda D}{\beta}$$

$$d=\frac{5100\times10^{-8}\times200}{0.2}$$

$$d=0.051cm$$

Example 2. Two coherent sources are 0.18 mm apart and the fringes are abserved on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.

Here $D = 80cm, \quad d = 0.18mm = 0.018cm$

$n = 4, \quad x = 10.8mm = 1.08cm, \lambda = ?$

$$x = \frac{nD}{d}$$

(or) $$\lambda = \frac{xd}{nD} = \frac{1.08 \times 0.018}{4 \times 80} = 6075 \times 10^{-8}\, cm$$

$$= 6075 \text{Å}$$

Example 3. In Young's double slit experiment the separation of the slits is 1.9. mm and the fringe spacing is 0.31 mm at a distance of 1 metre from the slits. Calculate the wavelength of light.

Here $\beta = 0.31mm = 0.031cm$

$d = 1.9mm = 0.19cm$

$D = 1m = 100cm$

$$\beta = \frac{\lambda D}{d}$$

(or) $$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.031 \times 0.19}{100}$$

$$\lambda = 5890 \times 10^{-8}\, cm$$

$$\lambda = 5890 \text{Å}$$

Example 4. Two straight and narrow parallel slits 1 mm apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of 100 cm from the slits are 0.50 mm apart. What is the wavelength of light?

Here $\beta = 0.50mm = 0.05cm$

$d = 1.9mm = 0.19cm$

$D = 1m = 100cm$

$$\beta = \frac{\lambda D}{d}$$

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.05 \times 0.1}{100}$$

$$\lambda = 5 \times 10^{-5}\ cm$$

(or) $$\lambda = 5000\ \text{Å}$$

2.7.4 Fresnel's Biprism

Fresnel used a biprism to show interference phenomenon. The biprism abc consists of two acute angled prisms placed base to base. Actually, it is constructed as a single prism of obtuse angle of about 179° (Fig. 2.7.4). The acute angle α on both sides is about 30'. The prism is placed with its refracting edge parallel to the line source S (slit) such that Sa is normal to the face bc of the prism. When light falls from S on the lower portion of the biprism it is bent upwards and appears to come from the virtual source B. similarly light falling from S on the upper portion of the prism is bent downwards and appears to come from the virtual source A. Therefore A and B act as two coherent sources. Suppose the distance between A and B=d.

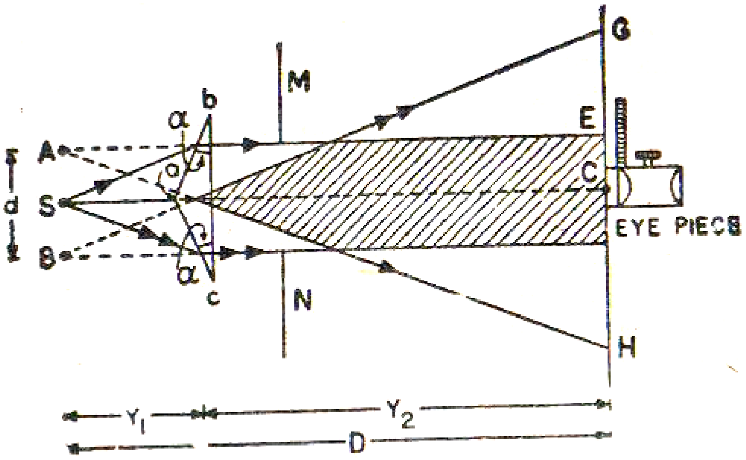


Fig. 2.7.4

If a screen is placed at C, interference fringes of equal width are produced between E and F but beyond E and F fringes of large width are produced which are due to diffraction. MN is a stop to limit the rays. To observe the fringes, the screen can be replaced by an eye-piece or a low power microscope and fringes are seen in the field of view. If the point C is at the principle focus of the eyepiece, the fringes are observed in the field of view.

Theory.

The point C is equidistant from A and B. Therefore, is has maximum intensity. On both sides of C, alternately bright and dark fringes are produced, the width of the bright fringe of dark fringe, $\beta = \frac{\lambda D}{d}$. Moreover, any point on the screen will be at the centre of a bright fringe if its distance from C is

$\frac{n\lambda D}{d}$, where n =0,1,2,3…etc. The point will be at the centre of a dark fringe if its distance from C is $\frac{2(n+1)\lambda D}{2d}$,

Where n=0,1,2,3,… etc.

2.7.5 Determination of wavelength of light. Fresnel's biprism can be used to determine the wavelength of a given source of monochromatic light.

A fine vertical slit S is adjusted just close to a source of light and the refracting edge is also set parallel to the slit S such that bc is horizontal (Fig. 2.7.5). They are adjusted on an optical bench.

A mircrometer eyepiece is placed on the optical bench at some distance from the prism, to view the fringes in its focal plane (at its cross wires).

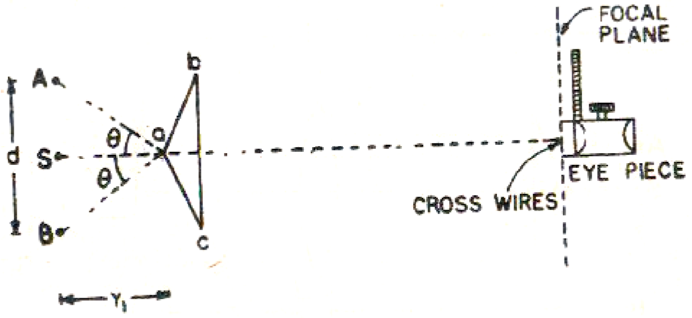


Fig. 2.7.5

Suppose the distance between the source and the eyepiece – D and the distance between the two virtual sources A and B=d. The eyepiece is moved horizontally (perpendicular to the length of the bench) to determine the fringe width. Suppose, for crossing 20 bright fringes from the field of view, the eyepiece has moved through a distance l.

Then the fringe width, $\beta = \frac{l}{20}$

But the fringe width $\beta = \frac{\lambda D}{d}$

$$\therefore \qquad \lambda = \frac{\beta d}{D} \qquad \text{…(i)}$$

In equation (i), β and D are known. If d is also known, λ can be calculated.

2.7.6 Determination of the distance between the two virtual sources (d). For this purpose, we make use of the displacement method. A convex lens is placed between the biprism and the eye-piece in such a position, that the images of the virtual sources A and B are seen in the field of view of the eyepiece. Suppose the lens is in the position L_1 (Fig. 2.7.6). Measure the distance between the image of A and B as seen in the eyepiece. Let it be d_1.

In this case, $\frac{d_1}{d}=\frac{v}{u}=\frac{n}{m}$

…(ii)

Now move the lens towards the eyepiece and bring it to some other position L_2, so that again the image of A and B are seen clearly in the field of view of the eyepiece. Measure the distance between the two images in this case also. Let it be equal to d_2.

Here, $v=m \text{ and } u=n,$

$\therefore$ $\frac{d_2}{d}=\frac{v}{u}=\frac{m}{n}$ …(iii)

From equations (ii) and (iii)

$$\frac{d_1 d_2}{d^2}=1$$

(or) $d=\sqrt{d_1 d_2}$

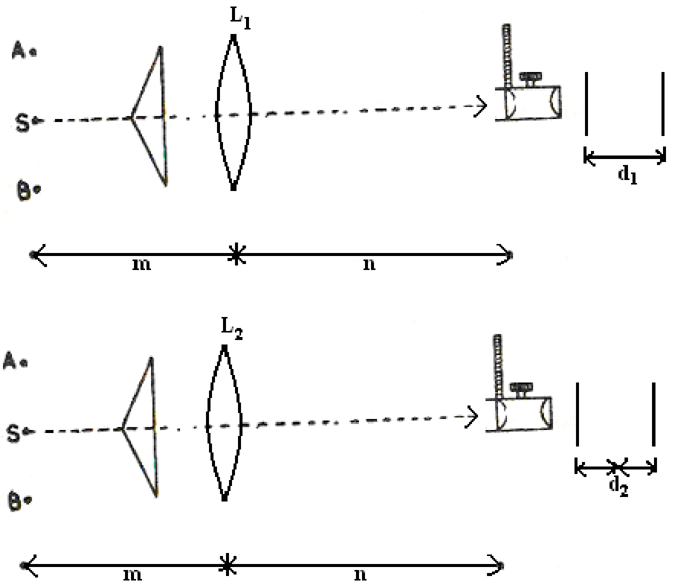


Fig. 2.7.6

Here d_1 will be greater than d_2 and d is the geometrical mean of d_1 and d_2. Therefore d can be calculated. Substituting the value of d, β and D in equation (i), the wavelength of the given monochromatic light can be determined.

The second method to find d is to measure accurately the refracting angle α. As the angle is small, the deviation produced $0=(\mu-1)\alpha.$ Therefore the total angle between Aa and Ba is $2\theta=2(\mu-1)\alpha.$ If the distance between the prism and the slit S is y_1 then $d=2(\mu-1)\alpha y_1$.Therefore d can be calculated.

Fringes with while light using A biprism

When white light is used, the centre of the fringe at C is white while the fringes on both sides of C are coloured because the fringe width (β) depends upon wavelength. Moreover, the fringes obtained in the case of a biprism using while light are different from the fringes obtained with Fresnel's mirrors. In a biprism, the two coherent virtual sources are produced by refraction and the distance between the two sources depends upon the refractive index, which in tern depends upon the wavelength of light. Therefore, for blue light the distance between the two apparent sources is different to that with red light. The distance of the nth fringe from the centre (with monochromatic light).

$$x = \frac{n\lambda D}{d}, where\, d = (2\mu - 1)\alpha y_1$$

$$\therefore \qquad x = \frac{n\lambda D}{2(\mu - 1)\alpha y_1}$$

Therefore for blue and red rays, the nth fringe will be,

$$x_b = \frac{n\lambda_b D}{2(\mu_b - 1)\alpha y_1} \qquad \ldots\text{(i)}$$

$$x_r = \frac{n\lambda_r D}{2(\mu_r - 1)\alpha y_1} \qquad \ldots\text{(ii)}$$

Example 5. A biprism is placed 5 cm from a slit illuminated by sodium light ($\lambda = 5890$Å) the width of the fringes obtained on a screen 75 cm from the biprism is 9.424 x 10^{-2} cm. What is the distance between the two coherent sources.

Here $\lambda = 5890 \times 10^{-8}\, cm$

$$d = ?\ \beta = 9.424 \times 10^{-2}\, cm$$

$$D = 5 + 75 = 80\, cm$$

$$\beta = \frac{\lambda D}{d}$$

(or) $$d = \frac{5890 \times 10^{-8} \times 80}{9.424 \times 10^{-2}}$$

(or) $$d = 0.05 cm$$

2.7.7 Fringes produced by a wedge-shaped thin film

Consider two plane surfaces OA and OB inclined at an angle θ and enclosing a wedge-shaped air-film. The thickness of the air film increases from O to A (Fig. 2.7.7). When the air-film is viewed with reflected monochromatic light, a system of equidistant interference fringes as observed which are parallel to the line of intersection of the two surfaces.

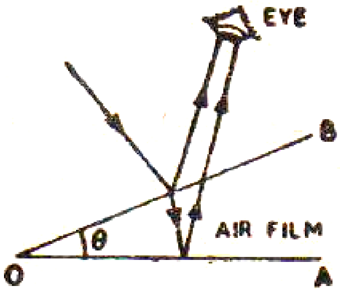


Fig. 2.7.7

The interfering rays do no enter the eye parallel to each other but they appear to diverge from a point near the film. The effect is best observed when the angle of incidence is small.

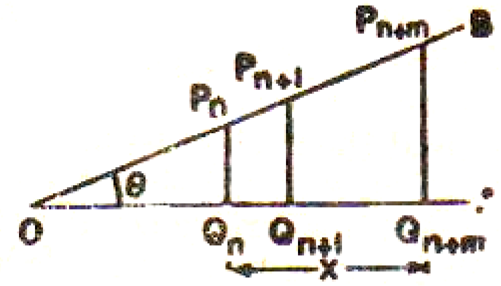


Fig. 2.7.7(a)

Suppose the nth bright fringe occurs at P_n (Fig. 2.7.7(a)). The thickness of the air-film at $P_n = P_nQ_n$. As the angle of incidence is small, $\cos r = 1$

Applying the relation for a bright fringe, $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

Here, for air $\mu = 1$

And Cos r=1

And t=P_nQ_n

$$\therefore \quad 2P_nQ_n = (2n+1)\frac{\lambda}{2}$$

The next bright fringe (n+1) will occur at P_{n+1}, such that

$$2P_{n+1}Q_{n+1} = \left[2(n+1)+1\right]\frac{\lambda}{2}$$

(or) $2P_{n+1} + Q_{n+1} = (2n+3)\frac{\lambda}{2}$

…(ii)

Subtracting (i) from (ii)

$$P_{n+1}+Q_{n+1}-P_nQ_n=\frac{\lambda}{2}$$

…(iii)

Thus, the next bright fringe will occur at the point where the thickness of the air film increases by $\frac{\lambda}{2}$. Suppose the (n+m)th bright fringe it at P_{n+m}. Them, there will be m bright fringes between P_n and P_{n+m} such that

$$P_{n+m}+Q_{n+m}-P_nQ_n=\frac{m\lambda}{2}$$

…(iv)

If the distance $Q_{n+m}+Q_n=x$

$$\theta=\frac{P_{n+m}Q_{n+m}-P_nQ_n}{Q_nQ_{n+m}}=\frac{m\frac{\lambda}{2}}{x}=\frac{m\lambda}{2x}$$

…(v)

(or) $$x=\frac{m\lambda}{2\theta}$$

…(vi)

Therefore, the angle of inclination between OA and OB can be known. Hence, x is the distance corresponding to m fringes. The fringe width.

$$\beta=\frac{x}{m}=\frac{\lambda}{20}$$

…(vii)

2.7.8 Testing the planeness of surfaces

If the two surfaces OA and OB are perfectly plane the air-film gradually varies in thickness from O to A.

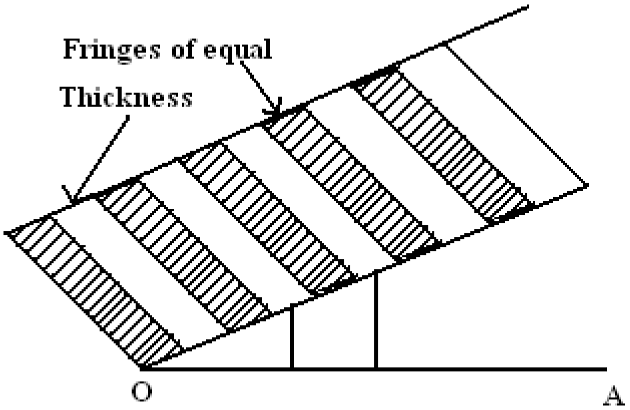


Fig. 2.7.8

The fringes are of equal thickness because each fringes is the locus of the points at which the thickness of the film has a constant value (Fig. 2.7.8). This is an important application of the phenomenon of interference. If the fringes are not of equal thickness it means the surfaces are not plane. The standard method is to take an optically plane surface OA and the surface to be tested OB. The fringes are observed in the field of view and if they are of equal thickness, the surface OB is plane. If not, the surface OB is not plane. The surface OB is polished and the process is repeated. When the fringes observed are of equal width, it means that the surface OB is plane.

2.8 Jamin's Refractometer

It is used to determine the refractive index of a gas at different pressures. A and B are two glass plates silvered at their back surfaces. The two plates are sufficiently thick and two identical glass tubes T_1 and T_2 are placed in the path of the beams 1 and 2 respectively (Fig. 2.8). A source S is placed at the focal plane of the lens L and a parallel beam of light is incident on the front surface at the plate A. It is divided into two beams by the plate A.

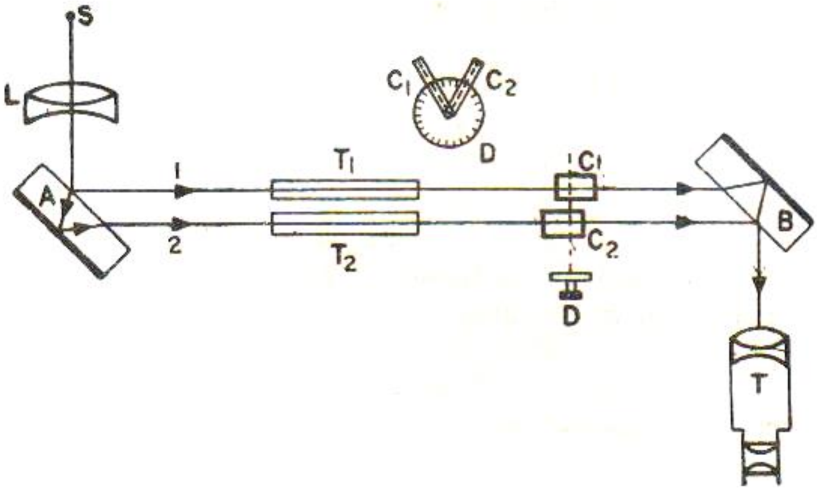


Fig. 2.8 Jamin's Refractometer

The beam 1 is reflected by the front surface and the beam 2 is reflected by the back surface. The two beams are incident on the plate B and the beam 2 is reflected by the front surface and the beam 1 is reflected by the back surface. The emergent beams interfere and they are viewed by a telescope T which is focused at infinity. Interference fringes are obtained. Here, the plants of A and B are inclined at a small angle.

The tubes T_1 and T_2 are evacuated and the fringes are observed in the field of view of the telescope. The gas is allowed to enter one of the tubes and the number of fringes that cross the centre of the field of view is counted. Suppose, n fringes have crossed the field of view. It the length of the tube is l, the path difference introduced = $(\mu = 1)l$

$$\therefore \qquad (\mu - 1)l = n\lambda$$

Therefore, the refractive index of the gas at a desired pressure can be determined.

In order to avoid the counting of fringes every time, two compensating plates C_1 and C_2 of equal thickness cut from the same piece, are introduced in the beams 1 and 2 as shown in fig. 2.8. The plates C_1 and C_2 can be rotated about a common horizontal axis (at a fixed angle θ between them) with the help of a calibrated circular disc D. When the disc D is rotated, the interfering beams passing through C_1 and C_2 are affected such that in one case the path increases and in the other case it decreases. The circular disc is calibrated by counting the number of fringes directly and is marked in terms of the refractive index and the number of wavelengths. Here, the tubes T_1 and T_2 are evacuated and using while light the telescope is focused such that the central white fringe is in the field of view. The gas is introduced at a desired pressure and temperature, into the tube T_1. The central fringe shifts. With the help of the circular disc D, the plate C_2 is rotated to bring the central fringe back to its original position. The reading on the calibrated circular disc directly gives the refractive index of the gas.

2.9 Rayleigh's Refractometer

Light from a monochromatic source S after passing the slit is incident on the lens L_1. The parallel beam is split up into two beams by the slits S_1 and S_2. After passing through the tubes T_1 and T_2 and the compensating plates. C_1 and C_2, the beams pass through the lens L_2 and interference fringes are observed with the help of the eyepiece or a telescope. This is mainly used to determine the refractive index of gases, slight variation in the refractive index of a gas and also the slight difference in the refractive index of solutions and gases. The circular disc D attached to the compensating plates C_1 and C_2 is previously calibrated in terms of wavelength and refractive index. Initially both the tubes T_1 and T_2 are evacuated and the central white fringe is observed in the field of view of the eyepiece using white light. The gas at a known pressure and temperature is introduced in the tube T_1 and the central white fringe shifts from the field of view.

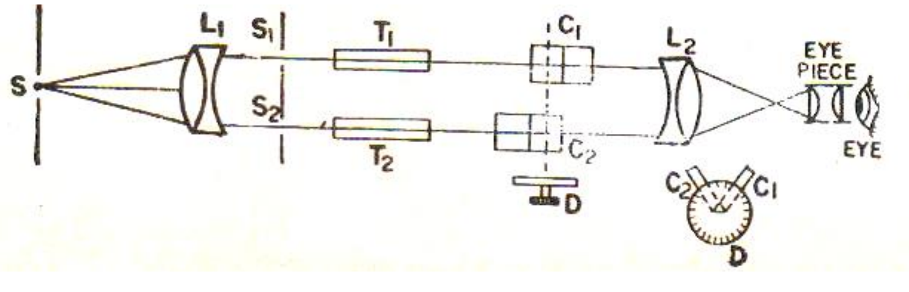


Fig. 2.9

By rotating the circular disc D, and thus the plates C_1 and C_2 the central white fringe is brought back to the centre of the field of view. The number of wavelengths graduated or calibrated on the circular disc are noted. The change in the path difference $(\mu-1)l = n\lambda$. Hence, μ can be calculated.

2.10 Specific Rotation

Liquids containing an optically active substance e.g., sugar solution, camphor in alcohol etc. Rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the solution or density of the active substance in the solvent (3) wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimeter (10cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution. Therefore,

$$S_{\lambda}^{t} = \frac{10\theta}{lC}$$

Where S_{λ}^{t}, represents the specific rotation at temperature t^{o}C for a wavelength λ, θ is the angle of rotation, *l* is the length of the solution in cm through which the plane polarized light passes and C is the concentration of the active substance in g/cc in the solution.

The angle through which the plane of polarization is rotated by the optically active substance is determined with the help of a polarimeter. When this instrument is used to determine the quantity of sugar in a solution, it is known as a saccharimeter.

2.10.1 Laurent's Half shade polarimeter

It consists of two nicol prisms N_1 and N_2, N_1 is a polarizer and N_2 is an analyzer. Behind N_1, there is a half wave plate of quartz Q which covers one half of the field of view, while the other half G is a glass plate. The glass plate G absorbs the same amount of light as the quartz plate Q. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube.

Light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G.

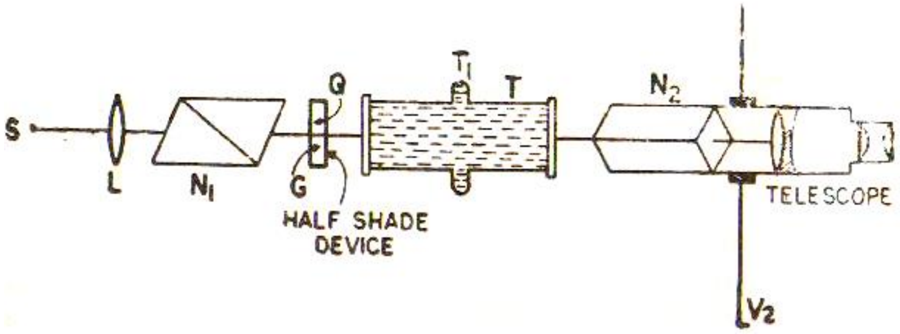


Fig. 2.10.1

Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB (Fig. 2.10.1(a)).

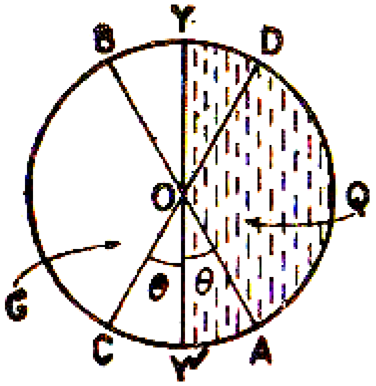
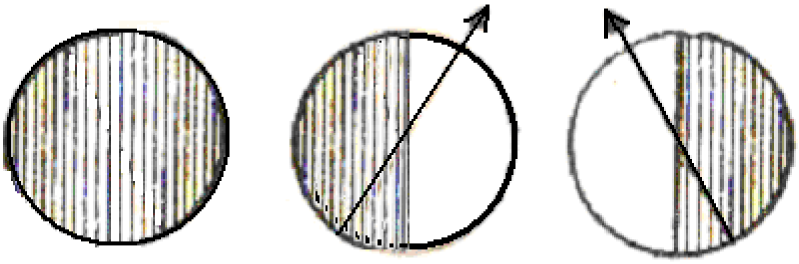


Fig. 2.10.1(a)

Here AB makes an angle θ with YY'. On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced between them.

The vibrations of the beam emerging out of quartz will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser N_2 has its principal plane or section along YY' i.e., along the direction which bisects the angle AOC, the amplitudes of light incident on the analyser N_2 from both the halves (i.e., quartz half and glass half) will be equal. Therefore, the field of view will be equally bright [Fig. 2.10.1(i)].

If the analyser N_2 is rotated to the right of YY', then the right half will be brighter as compared to the left half [Fig. 2.10.1(ii)]. On the other hand, if the analyser N_2 is rotated to the left of YY', the left half is brighter as compared to the right half [Fig. 2.10.1 (iii)].



(i)
Equality
Bright

(ii)

(iii)

Fig. 2.10.1

Therefore, to find the specific rotation of an optically active substance (say, sugar solution), the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube T. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position so that the whole field of view is equally bright. The new positions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentrations of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation θ. The graph is a straight line (Fig. 2.10.1(b)).

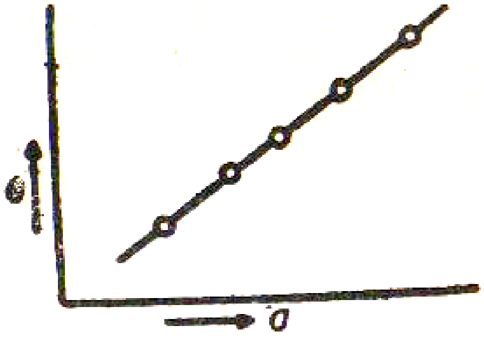


Fig. 2.10.1(b)

Then from the relation

$$S_\lambda^t = \frac{10\theta}{lC}$$, the specific rotation of the optically active substance is calculated.

Example 1. Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.

Here, $\theta = 13.2^o$

$C = 10\% = 0.1 g/cm^3$

$l = 20 cm$

$$S_\lambda^t = \frac{10 \times 13.2}{20 \times 0.1} = 66^o$$

Example 2. On introducing a polarimeter tube 25 cm long and containing sugar solution of unknown strength, it is found that the plane of polarization is rotated through 10^o. Find the strength of the sugar solution in g/cc^o (Given that the specific rotation of sugar solution is 60^o per decimeter per unit concentration.)

Here, $\theta = 10^o$

$S = 60^o$

$l = 25 cm$

$$S = \frac{10\theta}{lC}$$

$$C = \frac{10\theta}{lS} = \frac{10 \times 10}{25 \times 60} = \frac{1}{15}$$

$C = 0.067 g/cc$

Example 3. Calculate the specific rotation if the plane of polarization is turned 26.4^o, traversing 20 cm length of 20% sugar solution.

$$S_\lambda^t = \frac{10\theta}{lC}$$

Here, $\theta = 26.4^o, l = 20 cm$

$C = 20\% = 0.2 g/cm^3$

$\therefore$ $$S_\lambda^t = \frac{10 \times 26.4}{20 \times 0.2}$$

$= 66^o$

Example 4. A 20 cm long tube containing sugar solution rotate the plane of polarization by 11^o. If the specific rotation of sugar is 66^o, calculate the strength of the solution.

Here $\theta = 11^o$

$l = 20 cm,$ $S = 66^o$

$$S = \frac{10\theta}{lC} \qquad C = \frac{10\theta}{lS}$$

or $$C = \frac{10 \times 11}{20 \times 66}$$

$$C = 0.0833 g / cm^3$$

Questions

1. Derive van der Waals equation for gases and calculate the theoretical values of the critical constants.
2. Derive van der Waals equation of state and use it to obtain the expressions for the critical constants in terms of the constants of the van der Waals equation.
3. What consideration led van der Waals to modify the perfect gas equation? Deduce can der Waals equation of state for a gas. Also obtain the expression for the critical constants of a gas in terms of van der Waals constants.
4. What are the critical constants of a gas? State and explain van der Waals equation. Calculate the critical constants of a gas in terms of the constants of this equation.
5. Calculate the van der Waals constants for dry air, given that T_c = 132 K, P_c = 37.2 atmospheres, R per mole = 82.07×10^{-6} atmospheres m^3; b = $36.40 \times 10^{-6} m^3$.
6. Explain Joule-Thomson effect.
7. Explain the principle of regenerative cooling. How is the principle applied in the liquefaction of gases?
8. Explain the principle involved in the liquefaction of liquid helium.
9. Describe the porous plug experiment. Sketch a liquid air plant and explain its action.
10. Describe the methods for the liquefaction of hydrogen and helium using Joule – Thomson effect.
11. Write an essay on the industrial and scientific applications of “Low Temperatures”.
12. Describe Joule-Kelvin effect and give its theory. How is it utilized in the liquefaction of gases?
13. What is Joule-Thomson effect? Obtain an expression for the cooling produced in this process in case of van der Waal’s gas. Why do hydrogen and helium show a heating effect at ordinary temperature?
14. Describe Joule-Thomson effect and give its theory. How has it been utilised in the liquefaction of gases?

15. Describe Joule-Thomson porous plug experiment. What are the important inferences from this experiment?
16. Give an account of the methods of liquefying gases and discuss the principles on which they depend.
17. Define Joule-Thomson effect for liquefaction of gases. Prove that any gas below its inversion temperature will cool on suffering Joule-Thomson expansion. What is regenerative cooling.
18. Describe Jamin's interferometer.
19. Define specific rotation.
20. Describe Fresenel’s biprism. Explain how the wavelength of light can be determined with tits help.
21. Explain how interference fringes may be obtained with the help of a Fresnel’s biprism.
22. Short notes on:
    - (i) Rayleigh’s refractometer.
    - (ii) Interference fringes.
    - (iii) Testing of Optical planes.
23. Find the specific rotation of a given sample of sugar solution if the plane of polarization is turned through 26.4^o. The length of the tube containing 20% sugar solution is 20 cm.
24. Describe Laurent’s Half shade Polarimeter.

NOTES

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UNIT – III

ELECTRICITY, MAGNETISM & ATOMIC PHYSICS

ELECTRICITY & MAGNETISM

3.1 CAREY FOSTER BRIDGE

Description. The Carey Foster bridge is a form of Wheat stone's bridge. It consists of a uniform wire *AB* of length 1 metre stretched on a wooden board. (Fig. 3.1).

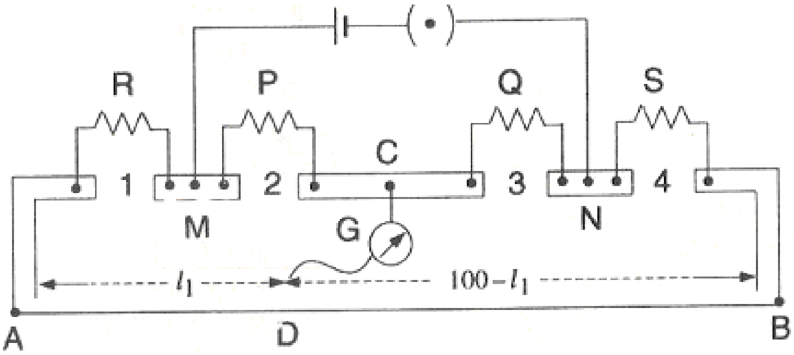


Fig. 3.1

Two equal resistances *P* and *Q* are connected in gaps 2 and 3. The unknown resistance *R* is connected in gap 1. A standard resistance S, of the same order of resistance as *R,* is connected in gap 4. A Leclanche cell is connected across *MN.* A galvanometer *G* is connected between the terminal *C* and a sliding contact maker *D.*

Theory. The contact maker is moved until the bridge is balanced. Let l_1 be the balancing length as measured from end *A*. Let α and β be the end resistances at *A* and *B.* Let ρ be the resistance per unit length of the wire.

From the principle of Wheat stone's bridge,

$$\frac{P}{Q} = \frac{R+\alpha + l_1\rho}{S + \beta + (100-l_1)\rho} \qquad \ldots(1)$$

The resistances *R* and *S* are interchanged and the bridge is again balanced. The balancing length l_2 is determined from the same end *A*.

Then,

$$\frac{P}{Q} = \frac{S+\alpha + l_2\rho}{R + \beta + (100-l_2)\rho} \qquad \ldots(2)$$

Figs. 3.1(a) and 3.1(b) and represent the equivalent Wheatstone's bridge circuit in the two cases.

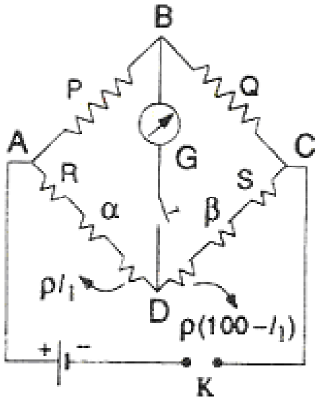


Fig. 3.1 (a)

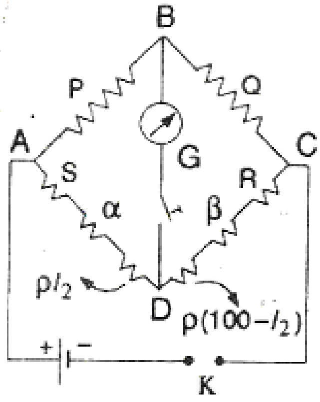


Fig. 3.1 (b)

From Eqns. (1) and (2),

$$\frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho}=\frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho} \qquad \ldots(3)$$

Adding 1 to both sides of Eq. (3),

$$\frac{R+\alpha+l_1\rho+S+\beta+100\rho-l_1\rho}{S+\beta+(100-l_1)\rho}=\frac{S+\alpha+l_2\rho+R+\beta+100\rho-l_2\rho}{R+\beta+(100-l_2)\rho}$$

$$\therefore \quad \frac{R+S+\alpha+\beta+100\rho}{S+\beta+(100-l_1)\rho}=\frac{R+S+\alpha+\beta+100\rho}{R+\beta+(100-l_2)\rho}$$

Since the numerators are equal, the denominators must be equal.

$$\therefore \quad S+\beta+100\rho-l_1\rho=R+\beta+100\rho-l_2\rho \qquad \ldots(4)$$

or $$S-l_1\rho=R-l_2\rho$$

$$\therefore \quad R=S+\rho(l_2-l_1) \qquad \ldots(5)$$

To find ρ. A standard resistance of 0.1 Ω is connected in gap 1. A thick copper strip is connected in gap 4 *i.e.,* R = 0.1 Ω and S = 0. The balancing length l_1' is determined. The standard resistance and the thick copper strip are interchanged. The balancing length l_2' is determined.

From Eq. (5), $$0.1=S+\rho(l_2'-l_1')$$

or $$\rho=\frac{0.1}{(l_2'-l_1')}$$

Thus by knowing S and ρ, the unknown resistance R is calculated.

3.2 Determination of the temperature coefficient of resistance

Let R_0 and R_1 be the resistances of a wire at temperatures 0°C and t^oC. Then,

$$R_t = R_0(1+\alpha t)$$

or

$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{1}{R_0}\frac{dR}{dt}$$

where α is the temperature coefficient of resistance of the material.

The increase of resistance per unit original resistance per degree rise of temperature is called temperature coefficient of resistance.

The given wire is wound non-inductively in the form of a double spiral on a glass tube. It is immersed in a beaker containing ice at 0^{o}C. The resistance of the wire is determined as above. The resistance of the wire is determined at 10^o, 20^o, 30^o, …100^{o}C. A graph is drawn with temperature along the X-axis and resistance along the X-axis and resistance along the Y-axis. A straight line is obtained.

Slope of the line = tan θ = $\frac{dR}{dt}$

Y intercept = R_0.

α is calculated using the formula, $\alpha = \frac{1}{R_0}\frac{dR}{dt}$

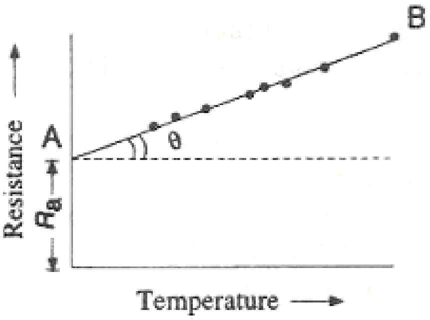


Fig. 3.2

Note. Let R_1 and R_2 be the resistances at t_1^{o}C and t_2^{o}C respectively.

$$R_1 = R_0[1+\alpha t_1] \quad \text{and} \quad R_2 = R_0[1+\alpha t_2]$$

$$\therefore \quad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

Example 1. *In an experiment with Carey Foster bridge, the shift in the balance point is 5.4 cm when a thick copper strip and one ohm resistance are interchanged. The one ohm resistance is then replaced by an unknown resistance. Now the balance point shifts by 10 cm on interchanging. Calculate the unknown resistance.*

Sol. $\rho = \frac{1}{5.4}$ ohm/cm

$$R = S + \rho(l_2 - l_1) = 0 + \frac{1}{5.4} \times 10 = 1.85 \text{ ohm.}$$

3.3 Potentiometer

Principle. A potentiometer is a device for measuring or comparing potential differences. A potentiometer can be used to measure any electrical quantity which can be converted into a proportionate D.C. potential difference.

It consists of a uniform wire *AB* of length 10 m stretched on a wooden board. A steady current is passed through the wire *AB* with the help of a cell of EMF *E*. Let

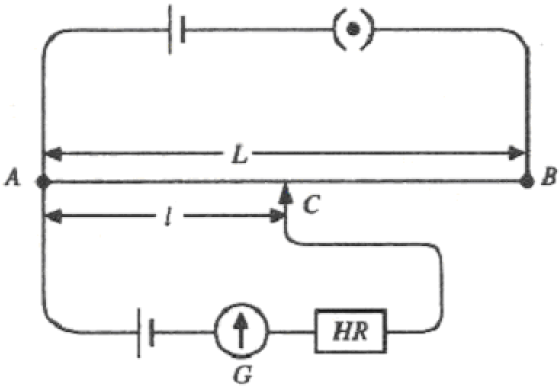


Fig. 3.3

r = resistance per unit length of potentiometer wire, and

l = steady current passing through the wire.

Let *C* be a variable point.

Let $AB = L$ and $AC = l$.

PD across $AB = L\,r\,I$, and

PD across $AC = l\,r\,I$

$$\frac{PD \text{ across } AB}{PD \text{ across } AC} = \frac{L\rho I}{l\rho I} = \frac{L}{l}$$

$$\therefore \quad PD \text{ across } AC = \frac{l}{L} \times PD \text{ across } AB$$

i.e., for a steady current passing through the potentiometer wire AB, the PD across any length is proportional to the length of the wire.

If a D.C. voltmeter is connected between *A* and the variable point *C*, it will be noted that the voltmeter registers greater values of PD's as the point *C* slides from *A* to *B*.

3.4 Calibration of voltmeter (Low range)

The connections are made as shown in Fig. 3.4. The voltmeter is connected parallel to *R*. Let *l* be the balancing length for the standard cell. The PD across *R* is balanced against the potentiometer. Let l_1, be the balancing length when the voltmeter reads V_1.

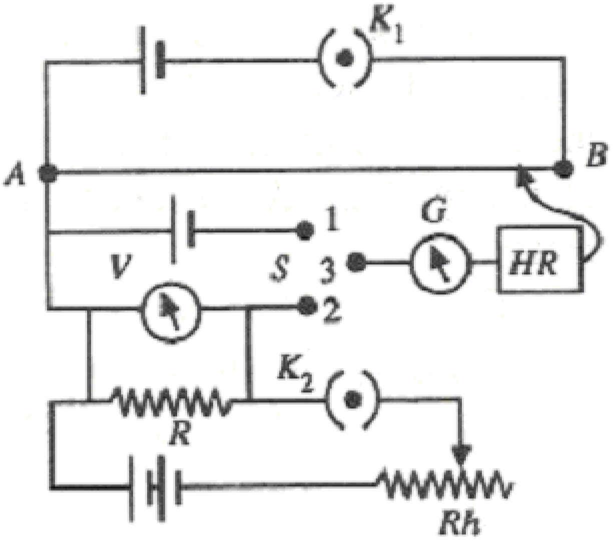


Fig. 3.4

PD across $R = El_1/l$

Correction to voltmeter = $(El_1/l) - V_1$

The experiment is repeated for various readings of the voltmeter and a calibration graph is drawn.

3.4.1 Calibration of voltmeter (High range)

Connections are made as shown in Fig. 3.4.1. Take suitable high resistances in *P* and *Q* such that the PD across *P* does not exceed the PD across the potentiometer. The balancing length *l* for the standard cell is determined first. Then the PD across *P* is balanced against the potentiometer and the balancing length l_1 is determined.

PD across $P = El_1/l$

PD across $P + Q = \left(\frac{P+Q}{P}\right)\left(\frac{El_1}{l}\right)$

Correction to voltmeter = $\left(\frac{P+Q}{P}\right)\left(\frac{El_1}{l}\right) - V_1$

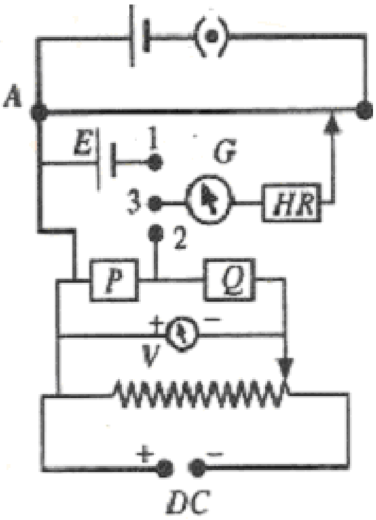


Fig. 3.4.1

The experiment is repeated for various readings of the voltmeter. A calibration curve is plotted for voltmeter, taking voltmeter readings on *X*-axis and corrections on *Y*-axis.

3.4.2 Calibration of Ammeter

Connect the ends of the potentiometer wire to the terminals of a storage cell through a key K_1. (Fig. 3.4.2).

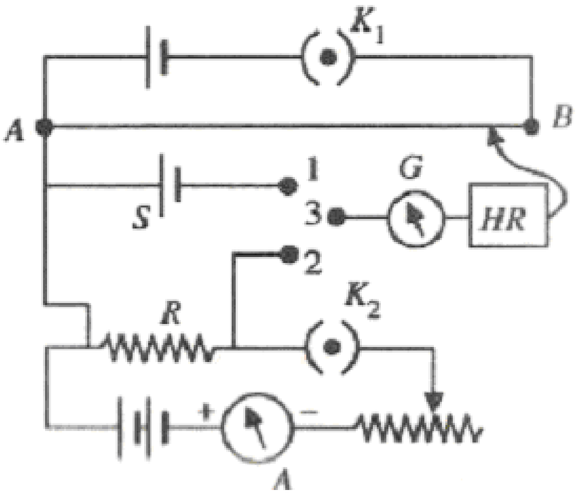


Fig. 3.4.2

S is a standard cell. Connect the ammeter *(A)* to be calibrated in series with a battery, key K_2, a rheostat and a standard resistance *R.* When a current / passes through the standard resistance *R*, the PD across *R* is *IR.* This potential drop is measured with the help of potentiometer.

Connect 1 and 3 and balance the EMF of the standard cell against the potentiometer. Find the balancing length from *A.* The PD per cm of the potentiometer = E/l.

Connect 2 and 3. Adjust the rheostat so that the ammeter reads a value A_1. Balance the PD across *R* against the potentiometer and find the balancing length l_1.

PD across $R = El_1/l$

Current through $R = El_1/(lR)$

Correction to ammeter reading = $(El_1/lR) - A_1$

Similarly, the corrections for other ammeter readings are determined. A calibration curve is plotted for ammeter, taking ammeter readings on *X*-axis and corrections on *Y*-axis.

3.5 Moving Coil Ballistic Galvanometer

Principle. *When a current is passed through a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.*

Construction. It consists of a rectangular coil of thin copper wire wound on a non-metallic frame of ivory. It is suspended by means of a phosphor bronze wire between the poles of a powerful horse-shoe magnet. A small circular mirror is attached to the suspension wire.

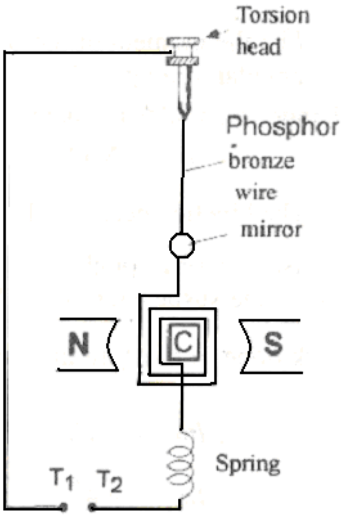


Fig. 3.5 (a)

Lower end of the coil is connected to a hair-spring. The upper end of the suspension wire and the lower end of the spring are connected to terminals T_x and T_2.

A cylindrical soft iron core (C) is placed symmetrically inside the coil between the magnetic poles which are also made cylindrical in shape. This iron core concentrates the magnetic field and helps in producing *radial field.*

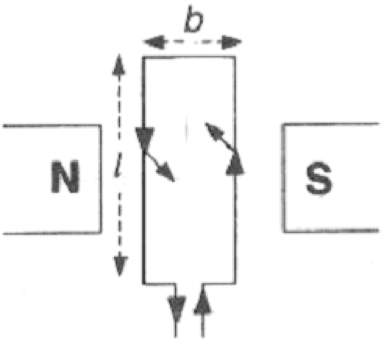


Fig. 3.5 (b)

The B.G. is used to measure *electric charge.* The charge has to pass through the coil as quickly as possible and before the coil starts moving. The coil thus gets an impulse and a throw is registered. To achieve this result, a coil of high moment of inertia is used so that the period of oscillation of the coil is fairly large. The oscillations of the coil are practically undamped.

Theory, (i) Consider a rectangular coil of N turns placed in a uniform magnetic field of magnetic induction B. (Fig. 3.5(b)) Let l be the length of the coil and b its breadth.

Area of the coil = $A = lb$.

When a current i passes through the coil,

torque on the coil = $\tau = NiBA$. …(1)

If the current passes for a short interval dt, the angular impulse produced in the coil is

$$\tau\, dt = NiBA dt \qquad \ldots(2)$$

If the current passes for t seconds, the total angular impulse given to the coil is

$$\int_0^t \tau\, dt = NBA\int_0^t i\, dt = NBAq \qquad \ldots(3)$$

Here, $\int_0^t i\, dt$ = q = total charge passing through the galvanometer coil.

Let I be the moment of inertia of the coil about the axis of suspension and ω its angular velocity. Then,

change in angular momentum of the coil = $I\omega$ …(4)

$\therefore$ $I\omega = NBAq.$ …(5)

(ii) The kinetic energy of the moving system $\frac{1}{2}I\omega^2$ is used in twisting the suspension wire through an angle θ. Let c be the restoring torque per unit twist of the suspension wire. Then,

work done in twisting the suspension wire by an angle $\theta = \frac{1}{2}c\theta^2$

$$\therefore \quad \frac{1}{2}I\omega^2 = \frac{1}{2}c\theta^2.$$

$$\text{or} \quad I\omega^2 = c\theta^2 \qquad \text{…(6)}$$

(iii) The period of oscillation of the coil is

$$T = 2\pi\sqrt{\left(\frac{I}{c}\right)} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{c}$$

$$\therefore \quad I = \frac{T^2 c}{4\pi^2} \qquad \text{…(7)}$$

Multiplying Eqs. (6) and (7), $I^2\omega^2 = \frac{c^2T^2\theta^2}{4\pi^2}$

$$\text{or} \quad I\omega = \frac{cT\theta}{2\pi} \qquad \text{…(8)}$$

Equating (5) and (8), $NBAq = \frac{cT\theta}{2\pi}$

$$\text{or} \quad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta \qquad \text{…(9)}$$

This gives the relation between the charge flowing and the ballistic throw θ of the galvanometer. $q \propto \theta$.

$\left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)$ is called the *ballistic reduction factor (K).*

$\therefore$ $q = K\theta$ …(10)

Correction for Damping in Ballistic Galvanometer

We have assumed that the whole of the kinetic energy imparted to the coil is used in twisting the suspension of the coil. In actual practice, the motion of the coil is damped by air resistance and the induced current produced in the coil. The first throw of the galvanometer is, therefore, smaller than it would have been in the absence of damping. The correct value of first throw is however obtained by applying damping correction.

Let θ_1, θ_2, θ_3, ... be the successive maximum deflections from zero position to the right and left. Then it is found that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = d \qquad \ldots(1)$$

The constant d is called the *decrement* per half vibration.

Let $d = e^{\lambda}$ so that λ = $\log_e d$

Here λ is called the *logarithmic decrement.*

For a complete vibration,

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}$$

Let θ be the true *first* throw in the absence of damping.

$\theta > \theta_1$. The first throw θ_1 is observed after the coil completes a quarter of vibration. In this case, the value of the decrement would be $e^{\lambda/2}$.

$$\therefore \qquad \frac{\theta}{\theta_1} = e^{\lambda/2} = \left(1 + \frac{\lambda}{2}\right)$$

$$\text{or} \qquad \theta = \theta_1 \left[1 + \frac{\lambda}{2}\right] \qquad \ldots(2)$$

We can calculate λ by observing the first throw θ_1 and the eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \times \frac{\theta_4}{\theta_5} \times \frac{\theta_5}{\theta_6} \times \frac{\theta_6}{\theta_7} \times \frac{\theta_7}{\theta_8} \times \frac{\theta_8}{\theta_9} \times \frac{\theta_9}{\theta_{10}} \times \frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

$$\text{or} \qquad \lambda = \frac{1}{10} \log_e \frac{\theta_1}{\theta_{11}} = \frac{2.3026}{10} \log_{10} \frac{\theta_1}{\theta_{11}}. \qquad \ldots(3)$$

$$\therefore \qquad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \ldots(4)$$

Dead-beat and Ballistic galvanometers

Galvanometers are classified as (i) dead-beat or aperiodic and (ii) ballistic galvanometers.

A moving coil galvanometer in which the coil is wound on a metallic conducting frame is known as a dead-beat galvanometer. It is called "dead-beat" because it gives a steady deflection without producing any oscillation, when a steady current is passed through the coil.

Conditions for a moving coil galvanometer to be dead beat:

(i) *Moment of inertia of the system should be small.*

(ii) *Coil should be mounted on a conducting frame.*

(iii) *Suspension fibre should be comparatively thicker.*

Conditions for a moving coil galvanometer to be ballistic:

(i) *The moment of inertia of moving system should be large.*

(ii) *Suspension fibre should be very fine.*

(iii) *Air resistance should be small.*

(iv) *The damping should be small i.e., the coil should be wound on a non-conducting frame.*

3.5.1 Current and Voltage Sensitivities of a moving-coil galvanometer

The figure of merit or current sensitivity (S_c) of a moving coil mirror galvanometer is the current that is required to produce a deflection of 1 mm on a scale kept at a distance of 1 metre from the mirror.

It is expressed in μA/mm.

The voltage sensitivity (S_v) is the p.d. that should be applied to the galvanometer to produce a deflection of 1 mm on a scale at a distance of 1 metre.

It is expressed in μV/mm.

To determine the current and voltage sensitivities of a galvanometer, the circuit shown in Fig.3.5.1 is used. Two resistance boxes *P* and *Q* and a key *K* are connected in series with a lead accumulator of emf *E*. Between the ends of *P*, a resistance box *R* and the M.G., through a commutator, are connected.

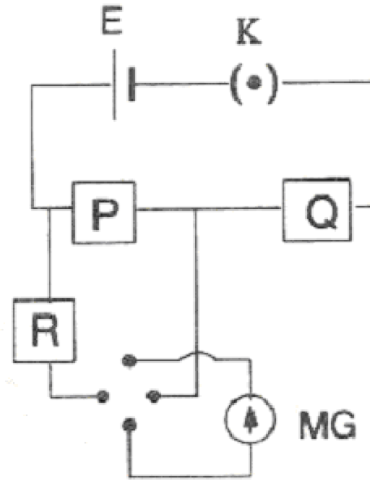


Fig. 3.5.1

A low resistance (say 1 Ω) is introduced in *P* and a high resistance (say 9999 Ω) in *Q*. With no resistance in *R*, the deflection 6 of the galvanometer is found.

$$\therefore \quad \text{Voltage sensitivity, } S_V = \frac{EP}{(P+Q)\theta} \times 10^6 \, \mu V/mm$$

The resistance in *R* is adjusted such that the deflection becomes θ/2. The resistance in *R* is equal to the galvanometer resistance R_g.

$$\therefore \quad \text{Current sensitivity, } S_C = \frac{EP}{(P+Q)R_g\,\theta} \times 10^6 \, \mu A/mm$$

The experiment is repeated for various values of *P* keeping *P* + *Q* constant.

3.6 Measurement of charge sensitiveness (Figure of merit of a B.G.)

The charge passing through a B.G., is given by

$$q = \frac{T}{2\pi}\frac{c}{NBA}\theta_1\left(1+\frac{\lambda}{2}\right) = K\theta_1\left(1+\frac{\lambda}{2}\right)$$

Here *K* is charge sensitiveness or figure of merit of the galvanometer. It is also known as the 'ballistic reduction factor' of the galvanometer.

The charge that should circulate through the coil to produce an undamped throw of 1 mm in the spot of light on a scale placed at a distance of 1 metre from the mirror is called the *charge sensitiveness K* of the ballistic galvanometer.

Two resistance boxes *P* and *Q* and a key *K* are connected in series with an accumulator of emf *E (Fig. 3.6)*. A capacitor of known capacitance C is connected to *P* through the vibrator *V* and charging terminal *Ch* of the charge-discharge key. The capacitor is charged with the p.d. across *P*. The charge on the capacitor can be discharged through the B.G. included in the circuit through the vibrator and discharge terminal of the charge-discharge key. A commutator *Cr* is included in the circuit to reverse the charge in the B.G.

1000 Ω in *P* and 9000Q in *Q* are included. The capacitor is charged and immediately discharged through the B.G. The first throw θ_1 is noted. The experiment is repeated with *P* = 2000 Ω, 3000 Ω etc., keeping $P + Q = 10{,}000$ Ω. Mean value of P/θ_1 is calculated.

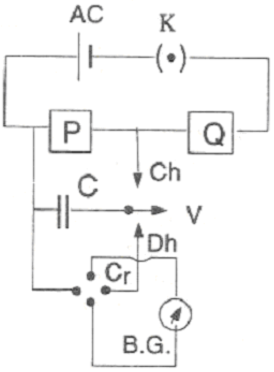


Fig. 3.6

Let the capacitance of the capacitor be $C\mu$F.

Charge on the capacitor $q = \frac{EP}{(P+Q)} \times C\ \mu C$.

This charge produces a throw θ_1.

Undamped throw $\theta = \theta_1\left(1 + \frac{1}{2}\lambda\right)$

Charge required to produce unit deflection = *K*.

$$\therefore \qquad K\theta_1\left(1 + \frac{1}{2}\lambda\right) = \frac{EP}{(P+Q)} \times C$$

$$\text{or} \qquad K = \frac{EC}{P+Q} \times \frac{P}{\theta_1\left(1 + \frac{1}{2}\lambda\right)}\ \mu C/div.$$

The value of λ is obtained by observing the first throw θ_1 and then the eleventh throw θ_{11} and using the relation

$$\lambda = \frac{1}{10}\log_e \frac{\theta_1}{\theta_{11}} = \frac{1}{10} \times 2.3026 \times \log_{10} \frac{\theta_1}{\theta_{11}}.$$

USES OF BALLISTIC GALVANOMETER

Absolute capacitance of a capacitor

(i) Two resistance boxes *P* and *Q* are connected in series with an accumulator of emf *E*. A small resistance (≈ 0.1 Ω) is taken in *P* and a large resistance (9999.9 Ω) in *Q* so that $P + Q = 10{,}000\ \Omega$. The galvanometer *(MG)* and a resistance box *R* are connected across *P*. With no resistance in *R*, the steady deflection *d* of the galvanometer is found. A suitable resistance is taken in *R* till the deflection becomes *half*. The resistance in *R* is the galvanometer resistance R_g. The experiment is repeated for various values of *P* keeping $P + Q$ constant.

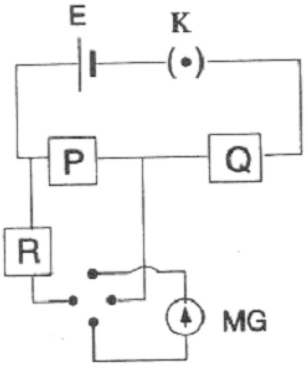


Fig. 3.6(i)

Current through galvanometer = $\dfrac{EP}{P+Q}\times\dfrac{1}{R_g}$ …(1)

Current through the galvanometer is also = $\dfrac{c}{BAN}d$ …(2)

From Eqns. (1) and (2),

$$\frac{c}{BAN}d = \frac{EP}{P+Q}\times\frac{1}{R_g}$$

$$\therefore \qquad \frac{c}{BAN} = \frac{E}{(P+Q)}\left(\frac{P}{d}\right)\frac{1}{R_g} \qquad \text{…(3)}$$

The mean value of *P/d* is found out from this part of the experiment.

(ii) The galvanometer coil is set oscillating freely in open circuit. The time for 10 oscillations is found and the period *T* is calculated.

(iii) Connections are made as shown in Fig. 3.6(iii). Resistances P_1(1000 Ω) and Q_1 (9000 Ω) are included in the boxes *P* and *Q* respectively.

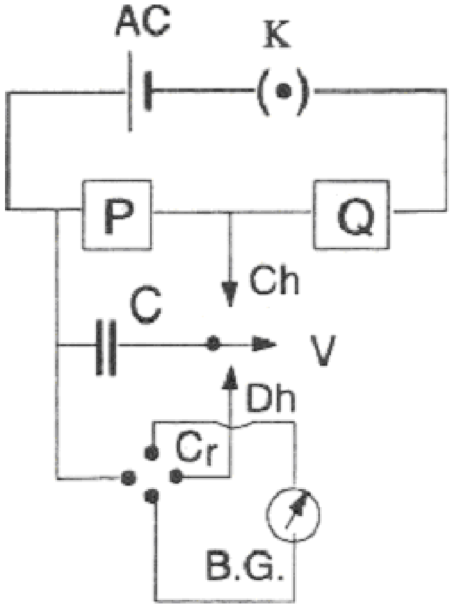


Fig. 3.6 (iii)

Potential difference across

$$P_1 = V = \frac{EP_1}{P_1 + Q_1}$$

The drop of potential across P_1 is used to charge the capacitor, by connecting the terminals *Ch* and *V* of the charge-discharge key.

Charge on the capacitor = $q = CV = C \times \frac{EP_1}{P_1 + Q_1}$ …(4)

The terminals *Dh* and *V* are now connected so that the capacitor gets discharged through the galvanometer. The first throw θ_1 is noted.

$$q = \frac{T}{2\pi}\frac{c}{BAN}\theta_1\left(1+\frac{1}{2}\lambda\right) \quad \text{…(5)}$$

$$\therefore \qquad C \times \frac{EP_1}{P_1 + Q_1} = \frac{T}{2\pi}\frac{c}{BAN}\theta_1\left(1+\frac{1}{2}\lambda\right)$$

[From Eqns. (4) and (5)]

or
$$C = \frac{T}{2\pi}\frac{c}{BAN}\left(\frac{\theta_1}{P_1}\right)\frac{P_1 + Q_1}{E}\left(1+\frac{1}{2}\lambda\right) \quad \text{…(6)}$$

Substituting the value of (*c*/*BAN*) from Eq. (3) in Eq. (6),

$$C = \frac{T}{2\pi}\frac{E}{(P+Q)}\left(\frac{P}{d}\right)\frac{1}{R_g}\left(\frac{\theta_1}{P_1}\right)\frac{P_1+Q_1}{E}\left(1+\frac{1}{2}\lambda\right)$$

But $P + Q = P_1 + Q_1$

$$\therefore \qquad C = \frac{T}{2\pi}\cdot\frac{1}{R_g}\cdot\left(\frac{P}{d}\right)\cdot\left(\frac{\theta_1}{P_1}\right)\left(1+\frac{1}{2}\lambda\right) \qquad \ldots(7)$$

The experiment is repeated for various values of P_1 keeping $(P_1 + Q_1)$ the same as $P + Q$. The mean value of θ_1 / P_1 is calculated.

(iv) To find λ, the coil is set oscillating. The first throw θ_1, and the eleventh throw θ_{11} are noted. Then,

$$\lambda = \frac{2.3026}{10}\log_{10}\frac{\theta_1}{\theta_{11}}$$

Substituting the values of T, R_g, (P/d), (θ_1 / P_1), and λ in Eq (7), C (the value of capacitance of the given capacitor) is determined.

3.7 Comparison of two Capacitances using B.G.

Connections are made as shown in Fig. 3.7. Let C_1 and C_2 be the capacitances of the two given capacitors. These capacitors are connected to the end terminals of the DPDT key. A resistance of 1000 Ω. is introduced in P and 9000 Ω. in Q.

The capacitor C_1 is charged to the p.d. across P. The charge on C_1 is then discharged through the B.G. The throws in the B.G. are noted before and after reversing the commutator. The mean throw θ_1 is found out.

With the same resistances in P and Q, the handle of the DPDT key is thrown on the side of C_2. C_2 is charged to the same potential across P. The charge on C_2 is then discharged through the *B.G.* The mean throw θ_2 is found out.

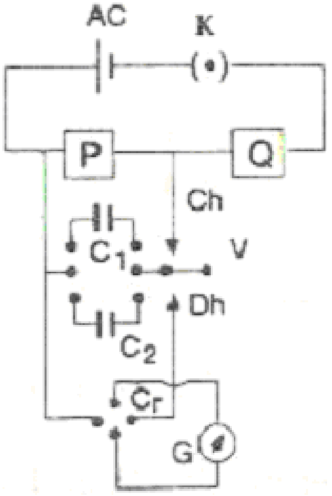


Fig. 3.7

Let V be the p.d. across the terminals of P. Then,

In the first case, $$q_1 = C_1 \times V = K\theta_1\left(1+\frac{1}{2}\lambda\right)$$

In the second case, $$q_2 = C_2 \times V = K\theta_2\left(1+\frac{1}{2}\lambda\right)$$

$$\therefore \qquad \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

The experiment is repeated for different values of *P* keeping *(P + Q)* constant.

P+Q ohms	P ohms	Throw due to C_1			Throw due to C_2			$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$
		Left	Right	Mean θ_1	Left	Right	Mean θ_2	
Mean $\frac{C_1}{C_2}$ =…								

3.8 The vector Atom Model

Introduction.

1 Bohr's theory was able to explain only the series spectra of the simplest hydrogen atom. It could not explain the multiple structure of spectral lines in the simplest hydrogen atom. Sommerfeld's theory was able to give an explanation of the fine structure of the spectral lines atom. However, Sommerfeld's theory could not predict the correct number of the fine structure lines. Moreover, it gave no information about the relative intensities of the lines. Sommerfeld's theory could not explain the complex spectra of alkali metals like sodium.

2. These older theories were inadequate to explain new discoveries like Zeeman Effect and Stark Effect in which the spectral lines could be split up under the influence of magnetic and electric fields.

3. Another drawback of the Bohr model was that it could not explain how the orbital electrons in an atom were distributed around the nucleus.

Therefore, in order to explain the complex spectra of atoms and their relation to atomic structure, the vector atom model was introduced. The two distinct features of the vector atom model are:

1. the conception of spatial quantization, and
2. the spinning electron hypothesis

3.9 Spatial quantization. According to Bohr's theory, the orbits are quantized as regards their magnitude (i.e. their size and form) only. But according to quantum theory, the direction or orientation of the orbits in space also should be quantized. To specify the orientation of the electron orbit in space, we need a fixed reference axis. This reference line is chosen as the direction of an external magnetic field that is applied to the atom. The difference permitted orientations of an electron orbit are determined by the fact that the projections of the quantized orbits on the field direction must themselves be quantized. The idea of space quantization leads to an explanation of Zeeman effect. The Stern –Garlach experiment provided an excellent proof of the space quantization of atom.

3.10 Spinning electron. To account for the observed find structure of spectral lines and to explain the anomalous Zeeman effect, the concept of spinning electron was introduced by Uhlenbeck and Goudsmit in 1926. According to their hypothesis, the electron spins about an axis of its own, while it also moves round the nucleus of the atom in its orbit. The spin of the electron is analogous to the spinning of a planet about its own axis, as it moves in an elliptical orbit around the sun. In other words, the electron is endowed with a spin motion over and above the orbital motion. According to the quantum theory, the spin of the electron also should be quantized. Hence a new quantum number called the spin quantum number (s) is introduced. Since the orbital and spin motions are both quantized in magnitude and direction according to the idea of spatial quantization, they are considered as quantized vectors. Hence the atom model based on these quantized vectors is called the “vector atom model”, to which vector laws apply.

According to the older theories, the electron was supposed to have only orbital motion round the nucleus. Hence, only the orbital angular momentum and orbital magnetic moment were considered. The spin endows the electron with a spin angular momentum sħ and a spin magnetic moment. Hence the total angular momentum of an atom should be the vector sum of the orbital angular momentum and spin angular momentum. Similarly, the total magnetic moment of an atom should be the vector sum of the orbital and spin magnetic moments.

3.11 Quantum Numbers Associated with the Vector atom Model

(1) The principal quantum number (n). This is identical with the one used in Bohrsommerfeld theory. The serial number of the shells starting from the innermost is designated as its principal quantum number (n). It can take only integral values excluding zero. i.e., n=1,2,3,4,….

(2) The orbital quantum number (l). This may take any integral value 0, 1,2,3,4,….(n-1). Thus, if n=4, l can take of four values 0,1,2,3. By convention, an electron for which l=0, is called as electron; if *l*=1.p electron ; *l*=2, d electron; *1*=3, f electron etc. The orbital angular momentum P_l of the electron is given by $p_l = l\hbar$.

According to the wave mechanics $p_l = \left[l\left(l+1\right)\right]^{1/2}\hbar$

3) **The spin quantum number (s).** This has only values, $=\frac{1}{2}$. The spin angular momentum $p_s = s\hbar$ where $s=\frac{1}{2}$. According to wave mechanics, $p_s = \left[s\left(s+1\right)\right]^{1/2}\hbar$

4) **Total angular momentum quantum number (j).** It represents the total angular momentum of the electron which is the sum of the orbital angular momentum and spin angular momentum. The vector $\vec{j}$ is defined by the equation $\vec{j} = \vec{l} + \vec{s}$ with the restriction that $\vec{j}$ is positive. The spin angular momentum $s = \pm\frac{1}{2}$. $\therefore \vec{j} = l \pm s$, plus sign when s is parallel to *l* and minus sign when s is antiparallel. Thus for l=2 and $s=\frac{1}{2}$, *j* can have the values $\frac{5}{2}$ *and* $\frac{3}{2}$ (Fig. 3.11 (4)).

The total angular momentum of the electron $= p_j = j\hbar$.

According to wave mechanics, $p_j = \sqrt{j(j+1)}\hbar$.

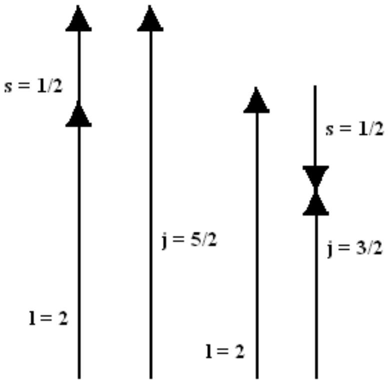


Fig. 3.11(4)

To explain the splitting of spectral lines in a magnetic field, three more magnetic quantum numbers are introduced.

5) **Magnetic orbital quantum number (m_l).** The projection of the orbital quantum number *l* on the magnetic field direction is called the magnetic orbital quantum number m_l. The possible values of m_l, are $l, l-1, l-2, ..., 0, -1, -2, ...-l$. *i.e.*, there are $(2l+1)$ possible values of m_l. This is illustrated in Fig. 3.11(5) for $l = 3$. The angle θ between l *and* B is given by cos $\theta = \frac{m_1}{l}$. Conversely, the permitted orientations of the *l* vector relative to the field direction B is also $(2l+1)$. For example, if $l = 3$, the permitted orientations of l are 7 directions shown in Fig. 3.11(5). l cannot be inclined to B at any other angle. This is known as 'spatial quantization'.

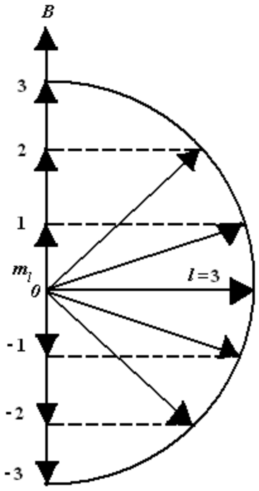


Fig. 3.11(5)

6) **Magnetic spin quantum number (m_s).** this is the projection of the spin vector s along the direction of the magnetic field. The spin angular momentum (s) can assume only two possible positions with respect to the

magnetic field: it may be parallel to it or antiparallel. m_s can have only two values $+\frac{1}{2} or -\frac{1}{2}$, as illustrated in Fig. 3.11(6)

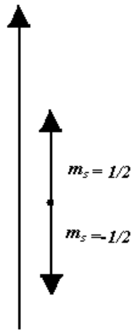


Fig. 3.11(6)

7) **Magnetic total angular momentum quantum number (m_j).** This is the projection of total angular momentum vector j on the direction of the magnetic field. Since we are dealing with a single electron, j can have only odd half-integral values $(\because j = l \pm \frac{1}{2})$. Hence, m_j must have only odd half-integral values. m_j can have only $(2j+1)$ values, from + j to – j zero excluded. Fig. 3.11(7) shows the possible value of m_j for $j = 3/2$.

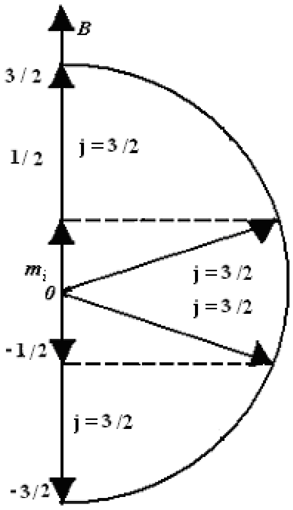


Fig. 3.11(7)

Notes. (1). The state of an electron in an atom is completely specified by the four quantum number, n, l, m_l and m_s.

(1) In spectroscopic notation, small letters l, s, j and $s, p, d, f, g,$ etc., are used to describe the state of the electron and capital letters, L, S, J and S, P, D, F, G are used to describe the state of the atom as a whole.

3.12 Coupling Schemes

In an atom having two or more electrons, the orbital and spin angular momentum of all its electrons can be added together in two ways. The methods of combination depends on the impaction or coupling between the orbital and spin angular momenta. Two types of schemes have been developed. They are : (1) L-S coupling or the Russel - Saunders coupling and (2) the $j-j$ coupling.

(1) **L-S coupling**. The type of coupling which occurs most frequently is the L-S coupling. In this type, all the orbital angular momentum vectors of the various electrons combine to form a resultant L and independently, all their spin angular momentum vectors combine to form the total angular momentum j of the atom (Fig. 3.12(1)). This scheme may be summarised as follows:

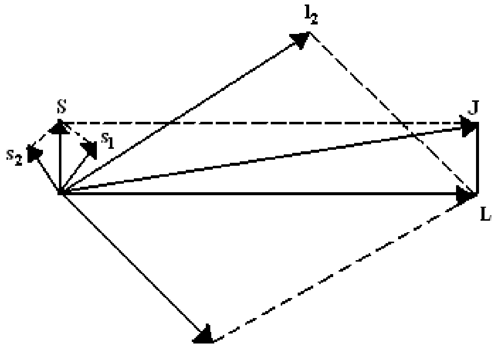


Fig. 3.12(1)

$L=\Sigma l_i; S=\Sigma s_i\,; J=L+S. L$ is always an integer including zero. S is an integer for an even number of electrons, and odd multiple of $\frac{1}{2}$ for an odd number of electrons (Fig. 3.12(a)). Hence J must be an integer, if S is an integer and J must be an odd multiple of $\frac{1}{2}$ if S is an odd multiple of $\frac{1}{2}$. It can be shown that, when L>S, J can have (2S+1) values and when L<S,J can have $(2L+1)$ values. In particular, if L=0, J can have only one value namely J=S.

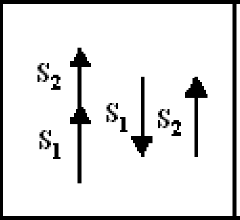
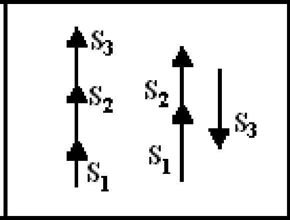
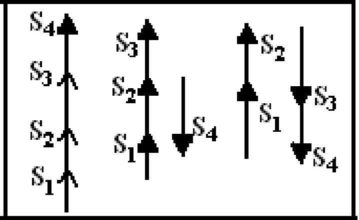
2 Electrons	3 Electrons	4 Electrons
		
S = 1,0	3/2, 1/2	2, 1, 0

Fig. 3.12(a)

(2) **The j-j coupling.** This method is employed when the interaction between the spin and orbital vectors of each electron is stronger than the interaction between either the spin vectors or the orbital vectors of the different electrons. The orbital and spin angular momenta of each electron in the atom are added to obtain the resultant angular momentum of the electron (Fig 3.12(2))

Thus $\vec{j}_i = \vec{l}_i + \vec{s}_i$.

The vector sum of all the individual j vectors gives the total angular momentum J of the atom. Thus $J = \Sigma j_i$. This type of coupling exists mainly in heavy atoms.

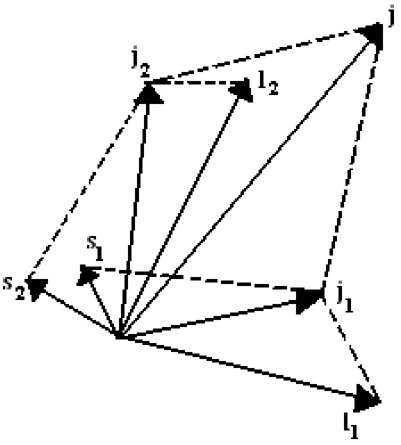


Fig. 3.12(2)

Application of spatial quantisation. The resultant vectors L,S and J, representing the atom, can be obtained by the above coupling schemes. According to quantum theory L,S and J are quantized in magnitude and direction. Hence the number of permitted orientations of L,S and J with respect to a given field direction are $(2L+1), (2S+1) \, and \, (2J+1)$ respectively. The corresponding magnetic quantum numbers $m_L = \sum m_l \; m_s = \sum m_s \; and \, m_J = \sum m_j$, can have only $(2L+1), (2S+1)$ and $(2J+1)$ values respectively.

Note : For a one electron atom with only a single effective electron, the state of the atom as a whole is identical with the state of the electron i.e., $L = l.S = s \, and \, J = j$.

3.13 The Pauli Exclusion Principle

Statement. No two electrons in an atom exist in the same quantum state. The four quantum numbers *n, l,* m_1 and m_3 determine the state of an electron completely. Hence the principle may be stated as “No two electrons in an isolated atom may have the same four quantum numbers.

Explanation. The principle implies that each electron in an atom must a different set of quantum numbers n, l, m1, and m3. If two electrons have all their quantum numbers identical, then one of those two electrons would be excluded from entering into the constitution of the atom. Hence the name "exclusion principle".

Application. This principle enables us to calculate the number of electrons that can occupy a given subshell.

(1) Consider the K-shell with n=1, l=0 and hence ml=0 since $s=\frac{1}{2}, m_s$ can be either $+\frac{1}{2} or -\frac{1}{2}$. Hence, the K-shell can have two electrons: electron 1 with quantum numbers $n=1, l=0, m_1=0,\ m_s=\frac{1}{2}$; and electron 2 with quantum numbers $n=1, l=0, m_l=0 m_s=-\frac{1}{2}$. If there were a third electron, its quantum numbers will be identical with those of the first or second electron, which is against Pauil's exclusion principle. The K-shell is therefore completed or closed with two electrons.

(2) For the L-shell, $n=2 and\, l=0\ or 1$. For the subshell $n=2, l=0, m_1$ must be zero and $m_s=+\frac{1}{2} or -\frac{1}{2}$. Hence there can be only 2 electrons in this subshell. For the subshell $n=2, l=1, m_l$ must be zero and $m_s=+\frac{1}{2} or -\frac{1}{2}$. Hence there can be only 2 electrons in this subshell. For the subshell $n=2, l=1, m_l$ can have three values -1, 0, +1. For each of these three values of m_l, m_s may be either $+\frac{1}{2} or -\frac{1}{2}$. Hence there will be six possible set of values for the quantum numbers characterizing the electrons. Therefore, the maximum number of electrons in this subshell is 6. The L shell with two subshells $\left[(n=2, l=0)\ and\ (n=2, l=1)\right]$ is, therefore, completed when it contains 2+6=8 electrons.

(3) For the M-shell with $n=3$ there can be three subshells with $l=0,1,2$. The first and second subshells are completed by 2 and 6 electrons as explained above. The third subshell is completed with $2(2l+1), i.e.,$ 10 electrons since $l=2$. Hence the total number of electrons required to complete the M-shell is 18.

(4) Similarly, the N-shell can have a maximum of 32 electrons.

In general, we get the two following conclusions:

(i) In the nth shell there are n sub-shells corresponding to the values 0,1,2,3…….(n-1) of *l*.

The maximum number of electrons in a sub-shall with a given value of l is $2(2l+1)$

Orbital quantum number (l)

0 1 2 3 4 ....

Number of possible electron states

2 6 10 14 18 ....

Subshell symbol

s p d f g ....

(ii) The number of electrons that can be accommodated in a shell with principal quantum number n=Sum of the electrons in the constituents subshells.

$$= \sum_{l=0}^{l=n-1} 2(2l+1) = 2\sum_{l=0}^{l=n-1}(2l+1)$$

$$= 2[1+3+5+7+.......\{2(n-1)+1\}] = 2n^2$$

The following table shows the distribution of electrons according to this scheme.

Shell symbol

K L M N O

Quantum number (n)

1 2 3 4 5

Number of electrons $(2n^2)$

2 8 18 32 50

The distribution of electrons in the various states (shells and sub-shells) according to the exclusion principle is given in the following table (Table 3.13)

Table 3.13

n	l	m_1	m_s	Number of electrons in sub-shell with spectroscopic notation	Total number of electrons in shell = $2n^2$
1	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2 $1s^2$	2
2	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2 $2s^2$	8
2	1	-1, 0, +1	$+\frac{1}{2}, -\frac{1}{2}$	6 $2p^6$	

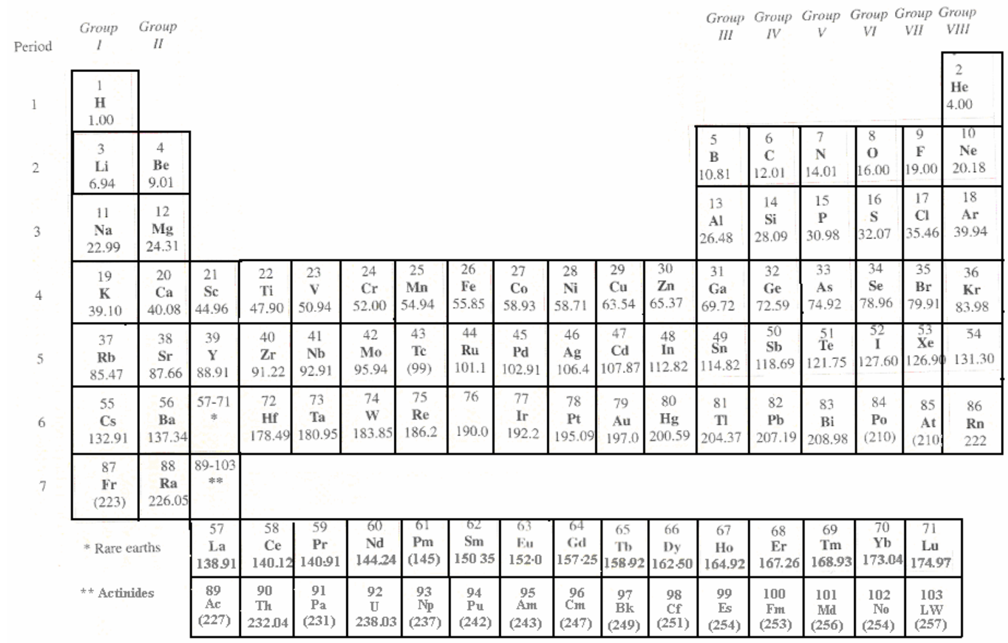
3	0	0	$+\frac{1}{2},-\frac{1}{2}$	2 $3s^2$	18
3	1	-1, 0, +1	$+\frac{1}{2},-\frac{1}{2}$	6 $3p^6$	
3	2	-2, -1, 0 +1, +2	$+\frac{1}{2},-\frac{1}{2}$	10 $3d^{10}$	
4	0	0		2 $4s^2$	32
4	1	-1, 0, +1	$+\frac{1}{2},-\frac{1}{2}$	6 $4p^6$	
4	2	-2, -1, 0 +1, +2	$+\frac{1}{2},-\frac{1}{2}$	10 $4d^{10}$	
4	3	-3, -2, -1, 0, +1, +2, +3		14 $4f^{14}$	

3.14 The Periodic Classification of Elements

The periodic table. The periodic table is an arrangement of different elements that exist in nature, based on their chemical properties and atomic numbers. Table 3.14. shows the simplest form of the periodic table.

Elements with similar properties form the groups shown as vertical columns in the table. Thus group I consists of hydrogen plus the alkali metals, all of which are

Table 3.14



Period	Group I	Group II											Group III	Group IV	Group V	Group VI	Group VII	Group VIII
1	1 **H** 1.00																	2 **He** 4.00
2	3 **Li** 6.94	4 **Be** 9.01											5 **B** 10.81	6 **C** 12.01	7 **N** 14.01	8 **O** 16.00	9 **F** 19.00	10 **Ne** 20.18
3	11 **Na** 22.99	12 **Mg** 24.31											13 **Al** 26.48	14 **Si** 28.09	15 **P** 30.98	16 **S** 32.07	17 **Cl** 35.46	18 **Ar** 39.94
4	19 **K** 39.10	20 **Ca** 40.08	21 **Sc** 44.96	22 **Ti** 47.90	23 **V** 50.94	24 **Cr** 52.00	25 **Mn** 54.94	26 **Fe** 55.85	27 **Co** 58.93	28 **Ni** 58.71	29 **Cu** 63.54	30 **Zn** 65.37	31 **Ga** 69.72	32 **Ge** 72.59	33 **As** 74.92	34 **Se** 78.96	35 **Br** 79.91	36 **Kr** 83.98
5	37 **Rb** 85.47	38 **Sr** 87.66	39 **Y** 88.91	40 **Zr** 91.22	41 **Nb** 92.91	42 **Mo** 95.94	43 **Tc** (99)	44 **Ru** 101.1	45 **Pd** 102.91	46 **Ag** 106.4	47 **Cd** 107.87	48 **In** 112.82	49 **Sn** 114.82	50 **Sb** 118.69	51 **Te** 121.75	52 **I** 127.60	53 **Xe** 126.90	54 131.30
6	55 **Cs** 132.91	56 **Ba** 137.34	57-71 *	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.85	75 **Re** 186.2	76 190.0	77 **Ir** 192.2	78 **Pt** 195.09	79 **Au** 197.0	80 **Hg** 200.59	81 **Tl** 204.37	82 **Pb** 207.19	83 **Bi** 208.98	84 **Po** (210)	85 **At** (210	86 **Rn** 222
7	87 **Fr** (223)	88 **Ra** 226.05	89-103 **															
	* Rare earths		57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140·91	60 **Nd** 144.24	61 **Pm** (145)	62 **Sm** 150 35	63 **Eu** 152·0	64 **Gd** 157·25	65 **Tb** 158·92	66 **Dy** 162·50	67 **Ho** 164.92	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.04	71 **Lu** 174.97	
	** Actinides		89 Ac (227)	90 Th 232.04	91 Pa (231)	92 U 238.03	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (249)	98 Cf (251)	99 Es (254)	100 Fm (253)	101 Md (256)	102 No (254)	103 LW (257)	

extremely active chemically and all of which have valence of +1. Group VII consists of the halogens that have valence of -1. Group VIII consists of the inert gases which are chemically inactive.

The horizontal rows are called periods. As we go from left to right in the same period, the chemical and physical properties of the elements vary gradually as the atomic number increases. Since the atomic number gives also the number of electrons in the atom, it follows that the atoms of successive elements in the periodic table are formed by the addition of one more electron at each step.

We have already seen the arrangement of electrons in an atom by applying Pauli's exclusion principle. The notion of electron shells and subshells fits perfectly into the pattern of the periodic table. The total orbital and spin angular momenta of the electrons in a closed subshell are zero. The electrons in a closed shell are all very tightly bound, since the positive nuclear charge is large relative to the negative change of the inner shielding electrons. Since an atom containing only closed shells has no dipole moment, it does not attract other electrons, and its electrons, cannot be readily detached. We expect such atoms to be passive chemically, like the inert gases and the inert gases all turn out to have closed shell electron configurations.

Those atoms which have a single electron in their outermost shell, tend to lose this electron. Hydrogen and the alkali metals are in this category and accordingly have valences of +1. Atoms whose outer shells lack a single electron for being closed, tend to acquire such an electron, which accounts for the chemical behaviour of the halogens. It is clear that the chemical and physical properties of an atom are determined by the number and arrangement of the electrons in the outermost-shell and not by the total number of electrons in the atom. In this manner the similarities of the members of the various groups of the periodic table may be accounted for.

3.15 Some Examples of Electron Configurations with their modern symbolic Representations.

The electronic configuration of an atom is the distribution of electrons in various subshells around the nucleus of the atom. In describing the electron configuration, small letters are used to represents the values of l as follows:

$$\begin{array}{cccccccc} l=0, & 1, & 2, & 3, & 4, & 5, & ..... \\ s, & p, & d, & f, & g, & h, & ..... \end{array}$$

i.e., if an electron is in a shell for which $l=0,$ it is called an s electron; for $l=1,$ a p electron and so on. The value of the principle quantum number n is written as a prefix to the letter representing its l value. For example, a state in which $n=2, l=0$ is a $2s-state; n=4, l=2 is\, a\, 4d$ -state and so on. The number of electrons having the same n and l values is indicated by an index written at the upper right of the letter representing their l value. Thus the 11 electrons of sodium in the normal state are designated as follows: $1s^2\, 2s^2\, 2p^6\, 3s. i.e.,$ there are two 1s electrons, two 2s electrons, six 2p electrons, and one 3 s electrons. We shall now consider electron configurations of a few elements.

(1) Hydrogen (Z=1). The normal state of an atom is one in which all the electrons are in the lowest possible energy levels. In hydrogen, the normal state is characterized by the quantum numbers $n=1, l=0, m_1=0\; and\, m_s$ may be either $+\frac{1}{2} or -\frac{1}{2}$. The symbolic representation is ls. The K- shell requires one more electron to be completed. Hence atomic hydrogen is very active chemically.

(2) Helium (Z=2). It has both its electrons in the shell $n=1, l=0, m_s=\frac{1}{2}$ for one electron and $-\frac{1}{2}$ for the second electron. The symbolic representations is $\boxed{ls^2}$. This shell is now completed or closed. The rectangular enclosure indicates that the electrons are interlocked in a closed shell. Therefore, helium may be expected to have a very stable configuration. This should also be true of all the other inert gases.

(3) Lithium (Z=3). It has three electrons. Two electrons can be put in the shell $n=1, l=0.$ The third electron must be put into a new shell $n=2, l=0$. So the neutral lithium atom is represented by $ls^2 2s.$ Lithium is one of the alkali elements and has a valence of unity. This means that the valence 2s electron can be detached easily from the atom to from the lithium ion Li^+. This is indicated by the fact that its ionization potential is only 5.39 volts, whereas for He it is 24.58 volts. Lithium is chemically quite active and is monovalent. Similarly, all alkali metals (Na, K, Rb, Cs) have one electron in their outermost shell and hence are monovalent having similar chemical properties.

(4) Beryllium $(Z=4)$. It has two electrons in the completed K-shell $(n=1)$. It has two additional electrons in the $(n=2, l=0)$ subshell. It is represented by $ls^2 2s^2$. Beryllium is one of the alkaline earth elements with a valence of 2. Its optical spectrum is that of a two electron atom. The atoms of the other elements of the group (Mg, Ca, Sr, Ba, Ra) should have similar structures. They have two electrons outside an inert gas or closed shell configuration. They have very similar chemical and physical properties.

Similarly the electronic configurations from boron (Z=5) to neon (Z=10) are:

(i) Boron : $ls^2 2s^2 2p$

(ii) Carbon $(Z=6): ls^2\, 2s^2\, 2p^2$

(iii) Nitrogen $(Z=7): 1s^2\, 2s^2\, 2p^3$

(iv) Oxygen $(Z=8): 1s^2\, 2s^2\, 2p^4$

(v) Flourine $(Z=9): 1s^2\, 2s^2\, 2p^5$

(vi) Neon $(Z=10): 1s^2\, 2s^2\, 2p^6$

In neon, both the n=1 and n=2 shells are completed. Neon is one of the inert gases and has a very stable configuration

The next eight elements from sodium (Z=11) to argon (Z=18) are formed by adding the additional electrons to the M shell for which n=3.

(i) Sodium $(Z=11): \boxed{1s^2\, 2s^2\, 2p^6}\, 3s$. Sodium has an electron (3s electron) outside a closed shell. This single electron, like that in lithium, is easily ionized; the valence is 1; the spectrum is that of one-electron atom.

(ii) Magnesium $(Z=12): \boxed{1s^2\, 2s^2\, 2p^6}\, 3s^2$. The two electrons in the outermost incomplete M shell (n=3) are the valence electrons marking Mg divalent.

(iii) Aluminum $(Z=13): \boxed{1s^2\, 2s^2\, 2p^6}\, 3s^2\, 3p$ Al is trivalent.

Questions

1. Describe the theory of low range ammeter using potentiometer.
2. Describe the theory of low range voltmeter using potentiometer
3. Describe the theory of Carey Foster bridge, find the resistance of the two coils separately and hence calculate the specific resistance. Find the length of the third coil of the same material without unwinding it.
4. Explain the theory of Moving coil Ballistic galvanometer?
5. How do you determine the charge sensitiveness of Ballistic galvanometer?
6. Describe the comparison of two capacitors using BG.
7. Describe the vector model of the atom and explain the different quantum numbers associated with it. Write down the electron configuration for Cu(29) employing modern symbolism and explain it.
8. State and explain Pauli's exclusion principle as applied to electrons in atoms. Describe how this principle assists in the interpretation of the periodic system of the elements.
9. What is Pauli's exclusion principle? On the basis of this principle explain the configuration of electrons in atoms.

NOTES

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UNIT - IV

NUCLEAR PHYSICS & SOLID STATE PHYSICS

NUCLEAR PHYSICS

4. Meson Theory of Nuclear Forces

According to the meson theory of nuclear forces, all nucleons consist of identical surrounded by a "cloud" of one or more mesons. Mesons may be neutral or may have a positive or negative charge. The sole difference between neutrons and protons is supposed to lie in the composition of their respective meson clouds. Yukawa assumed that π meson is exchanged between the nucleons and that this exchange is responsible for the nuclear binding forces. The forces that act, between one neurons and another, and between one proton and another, are the result of the exchange of neutral mesons (π^0) between them. The force between a neutron and a proton is the result of the exchange of charged masons $(\pi^+ \; and \; \pi)$ between them.

Thus a neutron emits a π meson and it converted into a proton:

$$n \rightarrow p + \pi^-$$

The absorption of the π^- meson by the proton (with which the neutron was interacting) converts it into a neutron:

$$p + \pi^- \rightarrow n.$$

In the reverse process, a proton emits a π^+ meson, becoming a neutron and the neutron, on receiving the π^+ meson, becomes a proton:

$$p \rightarrow n + \pi^+$$

$$n + \pi^+ \rightarrow p$$

Thus is the nucleus of an atom, attractive forces exist between (1) proton and proton (2) proton and neutron and (3) neutron and neutron. These forces of attraction are much larger then the electrostatic force of repulsion between the protons, thus giving a stability to the nucleus.

Just as a photon is a quantum of electromagnetic field, a meson is a quantum of nuclear field. Yukawa considered the equation for particle of mass m as,

$$\left(\nabla^2 - \frac{m^2 c^2}{(hl\,2\pi)^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0 \qquad \ldots(1)$$

This is a relativistic equation valid only for spinless particles.

Separating the time dependent part, the equation for the radial part is

$$(\nabla^2 - \mu^2)\phi(r) = 0 \qquad \ldots.(2)$$

Where $\mu = mcl\,(h/2\pi)$

The solution of Eq. (2) is $\phi(r) = -g\dfrac{e^{-\mu r}}{r}$ ...(3)

Here g is a constant, which plays the same role as the change q in electromagnetic theory. In analogy with electromagnetism, the potential between two nucleons is then given by

$$V(r) = -g^2\frac{e^{-\mu r}}{r} \qquad \text{...(4)}$$

Here g^2 is called the Coupling constant'. The argument made Yukawa predict the existence of pion as a quantum of nuclear force field.

The range of the pion field is $\dfrac{h/2\pi}{m_\pi C} \approx 1.4\, fm$

The form of V(r) given by Eq. (4) is known as the one-pion exchange potential (OPEP).

On the basis of the range of nuclear force and the uncertainty principle, it is possible to estimate the mass of the meson. According to uncertainty principle $\Delta E\, X\, \Delta t = h/2\pi\, where\, \Delta E\, and\, \Delta t$ are the uncertainties in energy and time. The range of nuclear force is $R \approx 1.4\, X\, 10^{-15} m.$ Let us assume that the meson travels between nuclei at approximately the speed of light c. Let Δt be the time interval between the emission of meson from one nucleon and the absorption by the other nucleon.

Then $\Delta t = R/c.$ $\therefore \Delta E = \dfrac{(h/2\pi)}{\Delta t}$

$\therefore$ The minimum meson mass is specified by $m \geq \dfrac{(h/2\pi)}{Rc}.$

It terms of the electronic mass m_e, the mass of the meson is

$$\frac{m}{m_e} = \frac{h/2\pi}{m_e\, Rc} = \frac{1.054\, X\, 10^{-34}}{(9.108\, X\, 10^{-31})(1.4\, X\, 10^{-15})(3X\, 10^{8})} = 275$$

i.e., mass of the meson ≈ 275 X mass of electron.

In 1947, Powell discovered π meson of mass about 273 m_e. This particle showed strong interaction with nucleons and was recognized as the Yukawa particle.

The discovery of the meson of mass of about 273 electrons mass and the existence of positive, negative and neutral mesons, lends some support to this theory. The experimental values of the magnetic moments of a free proton and of a free neutron also lend some support to the "Yukawa's meson field theory" of nuclear forces. A free proton is, for a part of its life-time, a neutron with a closely bound meson. Hence the magnetic moment of a free proton can be the resultant of the true magnetic moment of the proton and the magnetic moment of the meson. Thus the net magnetic moment of a free proton will exceed that given by the simple theory. Similarly, a neutron is for a fraction of its life-time dissociated into a proton and a negative meson. This combination will have a negative magnetic moment. It follows that, though uncharged, a neutron will have a negative magnetic moment.

4.1 Models of Nuclear Structure

The precise nature of the forces acting in the nucleus is unknown. Hence, nuclear models are resorted to for investigation and theoretical prediction of its properties. Such models may be based on (i) the extrinsic analogy between the properties of atomic nuclei and those of a liquid drop (ii) the electron shell of an atom etc. The corresponding models are called the liquid –drop model, shell model etc.

4.2 The Liquid Drop Model

In the liquid model, the forces acting in the nucleus are assumed to be analogical to the molecular forces in a droplet of some liquid. This model was proposed by Neils Bohr who observed that there are certain marked similarities between an atomic nucleus and a liquid drop. The similarities between the nucleus and a liquid drop are the following:

(i) The nucleus is supposed to be spherical in shape in the stable state, just as a liquid drop is spherical due to the symmetrical surface tension forces.

(ii) The force of surface tension acts on the surface of the liquid-drop. Similarly, there is a potential barrier at the surface of the nucleus.

(iii) The density of a liquid –drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

(iv) The intermolecular forces in a liquid are short range forces. The molecules in a liquid drop interact only with their immediate neighbours. Similarly, the nuclear forces are short range forces. Nucleons in the nuclear forces and a constant binding energy per nucleon.

(v) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

(vi) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

Semi-empirical mass formula. The liquid-drop model can be used to obtain an expression for the binding energy of the nucleus. Weizacker proposed the semi-empirical nuclear binding energy formula for a nucleus of mass number A, containing Z protons and N neutrons. It is written as

$$B.E. = aA - bA^{2/3} - \frac{cZ(Z-1)}{A^{1/3}} - \frac{d(N-Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

Where a,b,c and δ are constants.

Explanation of the terms. (1) The first terms is called the volume energy of a nucleus $(E_v = aA)$. The larger the total number of nucleons A, the more difficult it will be to remove the individual protons and neutrons from the nucleus. The B.E. is directly proportional to the total number of nucleons A.

2) The nucleons, at the surface of the nucleus, are not completely surrounded by other nucleons. Hence energy of the nucleon on the surface is less then that in the interior. The number of surface nucleons depends upon the surface effect reduces the B.E. by $E_s = bA^{2/3}$. A nucleus of radius R has an area of $4\pi R^2 = 4\pi r_0^2 A^{2/3}$. Hence the surface effect reduces the B.E. by $E_s=bA^{2/3}$. The negative energy E_s is called the surface energy of a nucleus. It is most significant for the lighter nuclei, since a greater fraction of their nucleons are on the surface.

3) The electrostatic repulsion between each pair of protons in a nucleus also contributes towards decreasing its B.E. The Coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a volume equal to that of the nucleus. Hence $E_c \propto Z(Z-1)/2$ (the number of proton pairs in a nucleus containing Z protons) and E_c is inversely proportional to the nuclear radius $R = r_0 A^{1/3}.E_c$ is negative because it arises from a force that opposes nuclear stability.

4) The fourth term $E_a = \frac{d(N-Z)^2}{A}$ originates from the lack of symmetry between the number of protons (Z) and the number of neutrons (N) in the nucleus. The maximum stability of a nucleus occurs when N=Z. Any departure from this introduces an asymmetry N-Z, which results in a decrease in stability. The decrease in the B.E. arising from this is called the asymmetric energy (E_a). this is also negative.

5) The final correction term δ allows for the fact that even-even nuclei are more stable than odd-odd nuclei. δ is positive for even-even nuclei, is negative for odd-odd nuclei and $\delta = 0$ for an odd A.

The best values of the constants, expressed in MeV, are a=15.760; b=17.810, c=0.711, d=23.702, $\delta = 34$.

The contributions of the various effects is Weizacker's empirical formula are represented schematically in the graph of fig. 4.2

Merits. (1) The liquid drop model accounts for many of the salient features of nuclear matter, such as the observed binding energies of nuclei and their stability against α and β disintegration as well as nuclear fission.

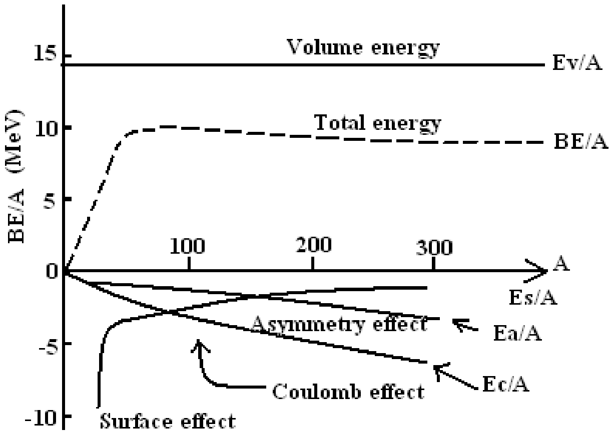


Fig. 4.2

2). The calculation of atomic masses and binding energies can be done with good accuracy with the liquid drop-model.

However, this model fails to explain other properties, in particular the magic numbers. It fails to explain the measured spins and magnetic moments of nuclei.

Example. Calculate the atomic number of the most stable nucleus for a given mass number A.

Sol. The most stable nucleus with a given mass number A is that which has the maximum value of the B.E. thus we have to compute $\frac{\partial (B.E)}{\partial Z}$ with A constant, equate it to zero. In the formula for B.E., we can write $Z(Z-1) = Z^2 \text{ and } N - Z = A - 2Z. \text{Then}$

$$B.E. = aA - bA^{2/3} - \frac{cZ^2}{A^{1/3}} - \frac{d\,(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

$$\frac{\delta E_b}{\delta Z} = -2cZA^{-1/3} + 4d\,(A-2Z)A^{-1} = 0$$

or, introducing the numerical values of c and d, we have

$$Z = \frac{A}{2 + 0.0157\,A^{2/3}}.$$

For light nuclei; having small A, we can neglect the second term in the denominator. $\therefore Z \approx A/2.$ This result is confirmed experimentally.

4.3 The Shell Model

The shell model of the nucleus assumes that the energy structure (energy levels of the nucleons) of the nucleus is similar to that of an electron shell in an atom. According to this model, the protons and neutrons are grouped in shells in the nucleus, similar to extra-nuclear electrons in various shells outside the nucleus. The shells are regarded as “filled” when they contain a specific number of protons or neutrons of both. The number of nucleons in each shell is limited by the Pauli exclusion principle. The shell model is sometimes referred to as the independent particle model because it assumes that each nucleon moves independently of all the other nucleon of all the other nucleons and is acted on by an average nuclear field produced by the action of all the other nucleons.

Evidence for shall model. It is known that a nucleus is stable if it has a certain definite number of either protons or neutrons. These number are known as magic numbers. The magic numbers are 2, 8, 20,50,82 and 126. Thus nuclei containing 2, 8, 20, 50, 82 and 126 nucleons of the same kind form some sort of closed nuclear shell structures. The main points in favour of this inference are:

i. The inert gases with closed electron shells exhibit a high degree of chemical stability. Similarly, nuclides whose nuclei contain a magic number of nucleons of the same kind exhibit more than average stability.

ii. Isotopes of elements having an isotopic abundance greater than 60% belong to the magic number category.

iii. Tin ($_{50}Sn$) has ten stable isotopes, while calcium ($_{20}Ca^{40}$) has six stable isotopes. So elements with Z=50, 20 are more then usually stable.

iv. The three main radioactive series (viz., the uranium series, actinium series and thorium series) decay to $_{82}pb^{208}$ with Z=82 and N=126. Thus lead $_{82}pb^{208}$ is the most stable isotope. This again shows that the numbers 82 and 126 indicate stability.

v. It has been found that nuclei having a number of neutrons equal to the magic number, cannot capture a neutron because the shells are closed and they cannot contain an extra neutron.

vi. It is found that some isotopes are spontaneous neutron emitters when excited above the nucleon binding energy by a preceding β-decay. These are $_{8}O^{17}$, $_{36}Kr^{87}$ and $_{54}Xe^{137}$ for which N = 9, 51 and 83 which can be written as 8 + 1, 50 + 1, and 82 + 1. If we interpret this loosely bound neutron, as a valency netutron, the neutron numbers 8, 50, 82 represent greater stability than other neutron numbers.

It is apparent from the above conclusions that nuclear behaviour is often determined by the excess or deficiency of nucleons with respect to closed shells of nucleons corresponding to the magic numbers. It was, therefore, suggested that nucleons revolve inside the nucleus just as electrons revolve outside in specific permitted orbits. The protons and neutrons move in two separate systems of orbits round the centre of mass of all the nucleons. The extra-nuclear electrons revolve in the coulomb field of a relatively distant heavy nucleus. But the nucleons move in orbits around a common centre of gravity of all the constituents of the nucleus. Each nucleon shell has a specific maximum capacity. When the shells are filled to capacity, they give rise to particular numbers (the magic numbers) characteristic of unusual stability.

The shell models able to account for several nuclear phenomena in addition to magic numbers.

(i) It is observed that even-even nuclei are, in general, more stable than odd-odd nuclei. This is obvious from the shell model. According to pauli's principle, a single energy sublevel can have a maximum of two nucleons only completed sublevels are present which means greater stability. On the other hand, an odd-odd nucleus contains incomplete sublevels for both kinds of nucleon which means lesser stability.

(ii) The shell model is able to predict the total angular momenta of nuclei. In even-even nuclei, all the protons and neutrons should pair off so as to cancel out one another's spin and orbital angular momenta. Thus even-even nuclei ought to have zero nuclear angular momenta, as observed. In even-odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momentum of the rest of nucleus for a half-integral total angular momentum. Odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta. Both these predictions are experimentally confirmed.

The collective Model

The collective model was proposed by A.Bohr, B.R. Mottleson and James Rainwater. The model combines the best features of the liquid drop model and the shell model. In this model it is assumed that the particles within the nucleus exert a centrifugal pressure on the surface of the nucleus. That results in the permanent deformation to non-spherical shape. As a result, the surface may undergo periodic oscillations.

The particles within the nucleus move in a non-spherical potential. Thus the nuclear distortion reacts on the particles and modifies the independent particle aspect. Thus the nucleus is considered as a shell structure capable of performing collections in shape and size. Thus the model can easily describe the drop like properties such as nuclear fission and at the same time it can retain the shell model characteristics. It is capable of explaining not only the large electric quadrupole moments but it can also predict the fine structure of nuclear level spectrum due to the energies associated with the vibrational and rotational motion of the core.

The total energy is expressed as

$$W = E_{rot} + E_{vib} + E_n$$

Where E$_{rot}$ is the energy due to rotational motion of the core, E$_{vib}$ is the energy due to the vibrational coordinates and E$_n$ is the energy due to the nucleonic coordinates. So the wave function is the product of three wave functions each containing the respective coordinates.

According to the collective model, all even-even nuclei such $as_2He^4, {}_8O^{16}, {}_{20}Ca^{40}, {}_{82}pb^{208}$ will have spherical shapes and zero electric quadrupole moment, while the even-odd, odd-even or odd-odd nuclei will have non-spherical shapes and finite electric quardrupole moment. The nuclear energy levels predicted by the model agree closely with the ones given by $\gamma - ray$ spectra of the nuclei.

Exercise

1. Find the energy release, it two $_1H^2$ nuclei can fuse together to from $_2He^4$ nucleus. The binding energy per nucleon of H^2 and He^4 is 1.1 MeV and 7.0 MeV respectively.

   (Hint. B.E. for $_2He^4$ = 28 MeV ; B.E., for $_1H^2$ nucleus = 2.2. MeV.

   $$\Delta E = \{mass\, of\ {}_1H^2\} - \{Mass\, of {}_2He^4\}$$
   $$= 2(M_p + M_n - 2.2) - (2M_p + 2M_n - 28.0) = 23.6\,MeV)$$

2. Calculate the binding energy per nucleon in $_6C^{12}$. Masses of proton, neutron and electron aer 1.007276. 1.0086655 u respectively. The mass of $_6C^{12}$ atom is 12.000000

   (Hint. Mass of 6 protons +mass of 6 neutrons = (6 x 1.007276+6 x 1.008665) u=12.095646 u.

   Mass of carbon nucleus = mass of carbon atom – mass of 6 electrons

   = (12.000000 – 6 x 0.00055) u= 11.996700 u

   Δm = (12.095646 – 11.996700) u = 0.098946 u

   B.E. = 0.098946 x 931.3 MeV = 92.148 MeV

   B.E. per nucleon = 92.148/12 = 7.69 MeV)

Nuclear detectors

4.4 Introduction

Most of the nuclear reactions are accompanied by the emission of changed particles like α -particles, protons, electrons and radiations like $\gamma - rays$. In order to understand these particles and their interaction with atomic nuclei, precise information about their mass, momentum, energy, etc., are necessary. We shall describe in this chapter some o f the common techniques employed for the detection of nuclear radiations and for analyzing their energies.

Several nuclear radiation detectors depend for their operation on the ionization that is produced in them by the passage of charged particles. This group of detectors includes ionization chambers, proportional counters. G-M counters, semiconductor radiation detectors, cloud chambers and spark chambers. In other detectors, excitation and sometimes molecular dissociation also play important roles. These phenomena, in combination with ionization, bring about the luminescence in scintillation detectors and the latent images in photographic emulsions.

Interaction between Energetic Particles and Matter

(a) Heavy Charged Particles. A heavy charged particle (like a proton, α-particle or fission fragment) has a fairly definite range in a range in a gas liquid, or solid. The particle loses energy primarily by the excitation and ionization of atoms in its path. The energy loss occurs in a large number of small increments. The primary particle has such a large momentum that its direction is usually not seriously changed during the slowing process. Eventually it loses all its energy and comes to rest. The distance traversed is called the range of the particle.

The energy loss per unit length $(-dE/dx)$ is called the stopping power. The rate dE/dx at which a heavy particle of change ze and speed v loses energy in an absorber of atomic number Z which contains N atoms per unit volume whose average ionization energy is I is given by

$$-\frac{dE}{dx}=\frac{z2e4NZ}{4\pi\varepsilon_0{}^2\, m_0v^2}\left[In\left(\frac{2m_0v^2}{l}\right)-In\left(1-\frac{v^2}{c^2}\right)-\frac{v^2}{c^2}\right] \qquad \ldots(1)$$

M_0 is electron rest mass

The range can be calculated by integrating Eq. (1) over the energies of the particle

$$R=\int_T^0\left(-\frac{dE}{dx}\right)^{-1} dE \qquad \ldots(2)$$

(b) Electrons. Electrons interact through coulomb scattering from atomic electrons, just like heavy charged particles. There are however, a number of important differences:

1. Electrons travel at relativistic speeds.

2. Electron will suffer large deflections in collisions with other electrons, and therefore will follow erratic paths. The range will therefore be very different from the length of the path that the electron follows.

3. Very energetic electrons (E > 1 MeV) lose an appreciable fraction of their energies by producing continuous X-rays (also called Bremsstrablung). The cross section for this process increases with increasing E.

(C)The absorption of $\gamma-Rays$. The interaction of $\gamma-rays$ with matter is markedly different from that of charged particles such as $\alpha\ or\ \beta$ particles. $\gamma-rays$ are extremely penetrating so that they are able to pass through considerable thicknesses of matter. $\gamma-rays$ show the exponential absorption in radiation absorbed dI is proportional both to dx and I. hence.

$dI = \mu I\, dx or\, I = I_0 e^{-\mu x}$.

Here, μ is a constant of proportionality which is a characteristic property of the medium, known as linear absorption coefficient. The mass absorption coefficient μ_m may be obtained by dividing μ by the density of the medium. $\mu_m = \mu / \rho$. The above relation gives the intensity (number of quanta per unit area per second) of the beam of initial intensity I_0, after traversing a thickness x of the homogeneous material. At low energies (0.1 MeV to 25 MeV) there are three important processes through which γ photons are absorbed by matter. (Fig. 4.4(c))

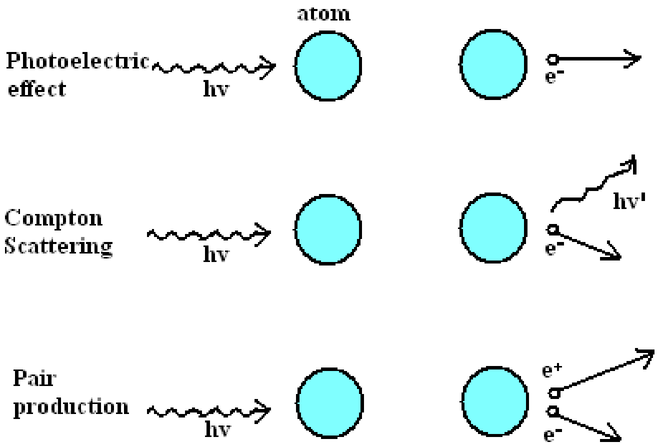


Fig. 4.4(c)

(i) Photoelectric effect. In this process, the $\gamma-rays$ knock out electrons from inside the atoms of the absorbing material. This results in the ionization of the atoms and the emission of fluorescent radiations. Einstein's equation for those photo-electrons will be

$$\frac{1}{2}mv_k^2 = hv - W_k, \frac{1}{2}mv_L^2 = hv - W_L\ etc.$$

Where hv is the photon energy, v_k, v_L represent the velocities of the photoelectrons arising in the K, L. shells, and $W_k, W_L....$ are the binding energies of K, L… shells.

(ii) Pair Production. In this process, the photon disappears and is converted to an electron positron pair. This process can take place only when the photon energy exceeds $2m_0c^2$. The pair production process cannot occur in free space and usually takes place in the presence of a nuclear field. The nucleus recoils in this process conserving momentum. But the K.E. carried away by the nucleus is negligibly small due to its large mass compared with that of the electron. Photon energy, if any, in excess of $2m_0c^2$ is shared as K.E. by the product particles.

(iii) **Compton effect.** It is elastic scattering process in which the photon imparts energy to an electron. When a photon of energy hv strikes the perfectly free electron (at rest), the photon with diminished energy hv' is scattered at an angle θ with the direction of incident photon and the electron recoils at an angle ϕ. The energy absorbed by these Compton electrons is only a small fraction of the total energy of the incident $\gamma - rays$, unlike in the case of photoelectrons.

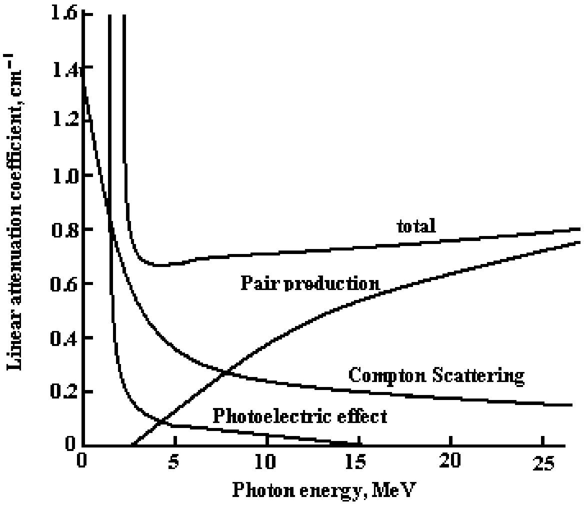


Fig. 4.4(iii)

At low photon energies, the photoelectric effect is the chief mechanism of energy loss. The importance of the photoelectric effect decreases with increasing energy. In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV, whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached. Pair production becomes increasingly likely the more the photon energy exceeds the threshold of 1.02 MeV. Fig. 4.4(iii) is a graph of the linear attenuation coefficient for photons in lead as a function of photon energy. The contributions to μ of the photoelectric effect, Compton scattering, and pair production are shown.

Example. The linear attenuation coefficient for 2-MeV gamma rays in water is about 5m-1. (a) Find the relative intensity of a beam of 2MeV gamma rays after it has passed through 0.1 m of water. (b) How far must such a beam travel in water before its intensity is reduced to 1 percent of its original value?

Sol. We have, $I = I_0 e^{-\mu x}$.

i. Here, $\mu = 5m^{-1}$; $x = 0.1$, m; $I/I_0 = ?$

$$\frac{I}{I_0} = e^{-\mu x} = e^{-0.5} = 0.61.$$

ii. Here $I_0/I = 100$, $\mu = 5m^{-1}$; $x = ?$

$$x = \frac{\log_e (I_0/I)}{\mu} = \frac{\log_e 100}{5} = 0.92m.$$

4.5 Ionization Chamber

The principle employed here is that charged sub-atomic particles can ionize gases. The number of ion-pairs produced gives us information not only on the nature of the incident particles, but even on their energy. The ionization chamber consists of a hollow metallic cylinder C, closed at both ends, with a window W at end for the entry of the ionizing particles or radiation (Fig. 4.5). A metal rod r, well insulated from the cylinder, is mounted coaxially within the cylinder. R is connected to a quadrant electrometer E. A p.d of several hundred volts is maintained between C and R. An earthed guard ring G prevents leakage of charge from the cylinder to the rod. The chamber contains some gas like sulphur dioxide or methyl bromide. When a charged particle enters the chamber, it produces a large number of ion pairs in the enclosed gas, along its path. Positive ions move towards R and negative towards C.

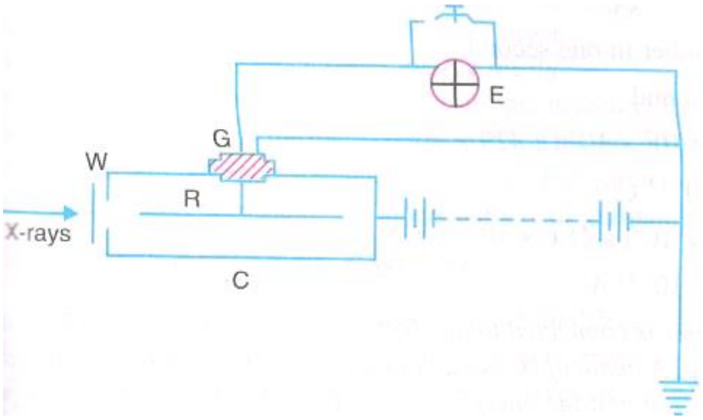


Fig. 4.5

The quadrant electrometer E measures the rate of deposition of positive charges on R. The ionisation currents produced are quite small $\approx 10^{-12} - 10^{-15}$ amperes. Special electrometers and D.C. amplifying devices have to be employed to measure such small currents.

If individual particles are to be counted, then the pulses of current produced are fed to a pulse amplifier, which is joined to the ionization chamber by a coupling capacitor (fig. 4.5(i)) Ionisation chambers have been used to study α -particles, β -particles, protons, electrons and nuclei of lighter elements. Ionisation chambers were extensively used in the early studies of cosmic ray phenomena. Ionisation chambers can also be used for measurements of X-rays and γ -rays. For neutron detection, the chambers walls are lined inside with a boron compound in the form of a paste.

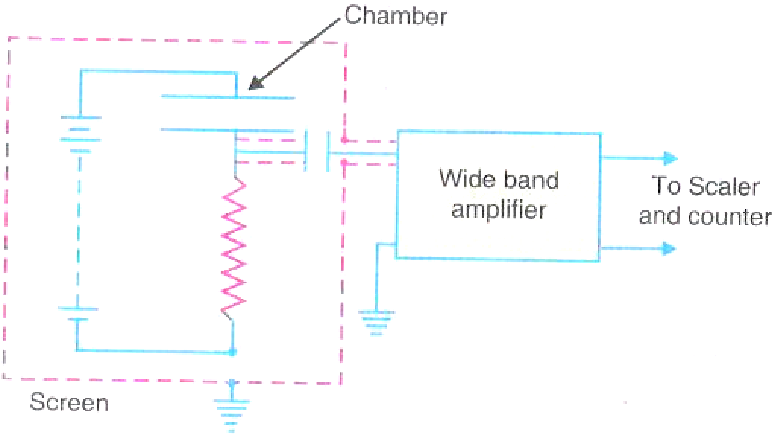


Fig. 4.5(i)

An ionization chamber is much less sensitive to β -particles (in comparison to α particles) because β -particles produce fewer pairs of ions in their passage through the chamber.

For detecting $\gamma - rays$, an ionization chamber of thick wall made of high atomic –number material (Pt, Bi) is employed. The $\gamma - rays$ impinging of the walls of the chamber eject high –speed electrons which produce ionization in the gas.

Example 1. $\alpha -$particles of energy 5MeV pass through an ionization chamber at the rate of 10 per second. Assuming all the energy is used in producing ion pairs, calculate the current produced. (35 eV is required for producing an ion pair and e=1.6 x 10^{-19}C).

Sol. Energy of $\alpha -$particles = 5 x 10^6 eV.

Energy required for producing one ion pair = 35 eV

No. of ion pairs produced by one $\alpha -$particle

$$= \frac{5\times10^6}{35} = 1.429\times10^5$$

Since 10 particles enter the chamber in one second,

No. of ion pairs produced per second

$= 1.429 \times 10^5 \times 10 = 1.429 \times 10^6$

Charge on each ion $= 1.6 \times 10^{-19}$ C

$\therefore$ Current $= (1.429 \times 10^6) \times (1.6 \times 10^{-19})$C/s

$= 2.287 \times 10^{-13}$ A.

Example 2. An ionization chamber is connected to an electrometer of capacitance 0.5 pF and voltage sensitivity of 4 divisions per volt. A beam of

α particles causes a deflection of 0.8 divisions. Calculate the number of ion pairs required and the energy of the α-particles. Given that 1 ion pair requires energy of 35 eV and e=1.6 x 10^{-19} coulomb.

Sol. Voltage sensitivity of electrometer = 4 divisions / volt

$\therefore$ Voltage required to produce a deflection of 0.8. divisions =

$$= \frac{0.8}{4} volt = 0.2 volt$$

$$Q = CV = (0.5 \times 10^{-12}) \times 0.2 (since\, C = 0.5\ pF = 0.5 \times 10^{-12} F)$$
$$= 10^{-13} C$$

$\therefore$ No. of ion pairs required = $\dfrac{10^{-13}}{1.6 \times 10^{-19}} = 6.25 \times 10^{5}$

1 ion pair requires 35 eV.

$\therefore$ Total energy required = 35 x (6.25 x 10^{5})eV

= 21.88 MeV.

4.6 Geiger-Muller Counter

It consists of a metal chamber C containing air or some other gas at a pressure of about 10 cm of Hg. A fine tungsten wire (W) is stretched along the axis of the tube and is insulated from it by ebonite pulgs EE (Fig. 4.6). The wire is connected to the positive terminal of a high tension battery (about 1000 to 3000 volts) through a high resistance R (about 100 megohms) and the negative terminal is connected to the chamber C. The D.C. Voltage is kept slightly less then that which will cause a discharge between the electrodes.

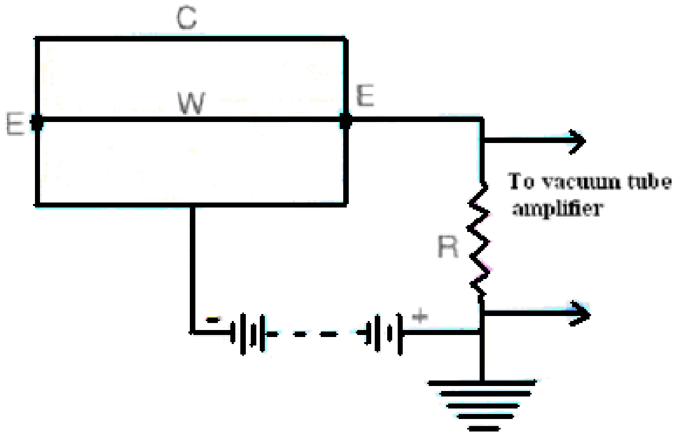


Fig. 4.6

When an ionizing particle (say an α particle) enters the counter, ionization takes place and a few ions are produced. If the applied P.D. is strong enough, these ions are multiplied by further collisions. An avalanche of electrons moves towards the central wire and this is equivalent to a small current impulse which flows through the resistance R. The critical potential is lowered momentarily, casing a sudden discharge through the resistance R. The p.d. thus developed across R is amplified by vacuum tube circuits and is made to operate a mechanical counter. In this way single particles can be registered. The sudden pulse of discharge sweeps away the ions from the chamber and the counter is ready to register the arrival of the next particle.

The voltage characteristics of a Geiger-Muller counter are shown in Fig. 4.6(i). This is a plot of the counting rate against the counter potential with a radioactive source placed near the counter. It is seen that there is a threshold below which the tube does not work. This can be several hundred volts. As the applied potential is increased, the counting begins and rises rapidly to a flat portion of the curve called the plateau. The is the region of the counter operation where the counting rate is, more or less, independent of small changes in p.d across the tube. Beyond the plateau the applied electric field is so high that a continuous discharge takes place in the tube as shown in fig. 4.6(i) and the count rate increases very rapidly. It does not require any ionizing event for to happen so that the tube must not be used in this region.

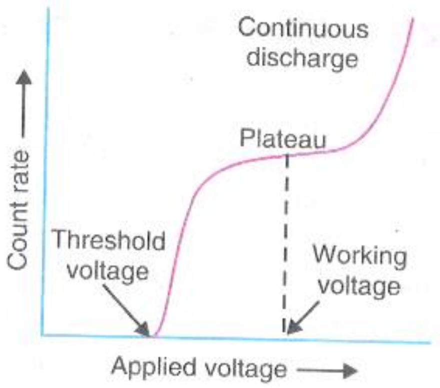


Fig. 4.6(i)

The efficiency of the counter is defined as the ratio of the observed counts/sec. to the number of ionizing particles entering the counter per second. Counting efficiency is defined as the ability of its counting, if at least one ion pair is produced in it.

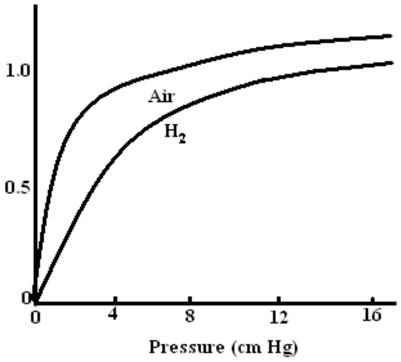


Fig. 4.6(ii)

Counting efficiency $= \in = 1 - e^{slp}$ where s= specific ionization at one atmosphere; p=pressure in atmospheres and *l*=path length of the ionization particle in the counter. The efficiency ε of a GM counter, as a pressure for air and hydrogen, is illustrated in Fig. 4.6(ii).

The counter set-up is portage (with the transistorized electronics) and serves best for mineral prospecting, apart from its several other applications in cosmic ray work. A virtue of the Geiger counter in that the pulse height is constant over a range of applied voltages, as in Fig 4.6(i). so the power supply does not have to be precisely regulated as it does for a proportional counter. Also, the pulses are several volts in height which amplifies unnecessary.

Disadvantages of the Geiger counter are : (i) it is insensitive for a period of 200 to 400 μs following each pulse, which prevents its use at very high counting rates. (ii) it cannot provide information about the particle or photon causing a pulse.

Example. A self-quenched G-M counter operates at 1000 volts and has a wire diameter of 0.2. mm. the radius of the cathode is 2 cm and the tube has a guaranteed lifetime of 10^9 counts. What is the maximum radial field and how long will the counter last if it is used on an average for 30 hours per week at 3000 counts per minute? Consider 50 weeks to a year.

Sol. The radial field at the central wire is

$$E_{\max} = \frac{V}{r \log_e (b/a)} = \frac{1000}{0.0001 \times 2.3026 \log_{10} \left(\frac{2 \times 10^{-2}}{10^{-4}} \right)}$$

$$= 1.89 \times 10^6 \; volts/metre$$

If the lifetime of the tube is N years. The total number of counts recorded will be

$$N \times 50 \times 30 \times 60 \times 3000 = 2.7 \times 10^8 N$$

$$\therefore \qquad 2.7 \times 10^8 \times N = 10^9$$

$$or \qquad N = 3.7 \, years$$

4.7 The Wilson Cloud Chamber

Principle. If there is a sudden expansion of saturated vapour in a chamber, supercooling of the vapour occurs. Tiny droplets will be formed by condensation over the dust particles present in the chamber. If, therefore, we have completely dust-free and saturated air, and if it is suddenly allowed to expand and thereby cool, condensation will not take place. But if ions are available in the chamber during the expansion, they serve as nuclei for condensation. Hence, if an ionising particle passes thought the chamber during an expansion. Ions are produced along its path and droplets condense on these ions. Hence the “track” of the particle becomes visible.

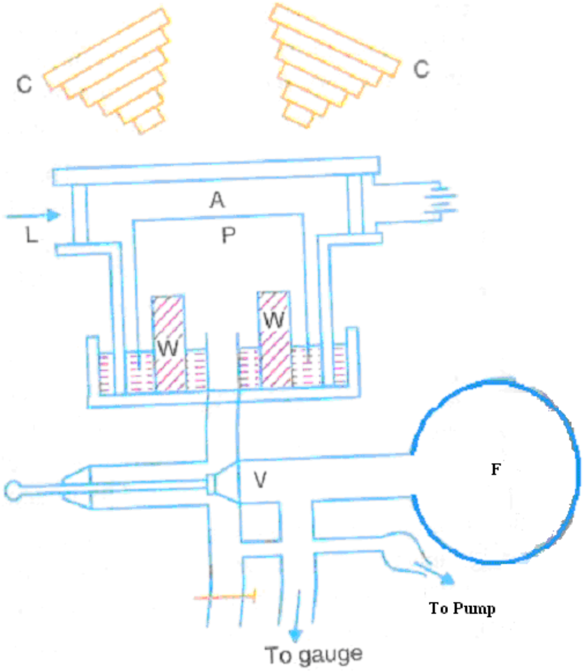


Fig. 4.7

Description. The apparatus consists of a large cylindrical chamber A, with walls and ceiling made of glass (fig. 4.7). It contains dust-free air saturated with water vapour. P is a piston working inside the chamber. When the piston moves down rapidly, adiabatic expansion of the air inside the chamber takes place. The piston is connected to a large evacuated vessel F through a valve V. when the valve is opened, the air under the piston rushes into the evacuated vessel F, thereby causing the piston to drop suddenly. The wooden blocks WW reduce the air space inside the piston. Water at the bottom of the apparatus ensures saturation in the chamber. The expansion ration can be adjusted by altering the height of the piston.

As soon as the gas in the expansion chamber is subjected to sudden expansion, the ionizing particles are shot into the chamber through a side window. A large number extremely fine droplets are formed on all the ions produced by the ionizing particles. These droplets form a track of the moving ionizing particles. At this stage, the expansion chamber is profusely illuminated by a powerful beam of light L. Two cameras CC are used to photograph the tracks. The process of expansion, shooting of the ionizing particles into the expansion, chamber, illuminating the chamber and clicking the camera must all be carried out in rapid succession in order to get satisfactory results.

The ionizing agent can be easily identified from its path in the cloud chamber. α-particles, being comparatively massive, go straight and their paths are thick, straight and sharply defined. β-particles being lighter, are easily deflected by collision and their paths are thin and crooked. The cloud chamber has led to the discovery of many elementary particles like positron, meson, etc.

Advantages (1). Cloud chambers can be used to study the variation of specific ionization along the track of a charged particle and the range of such particles.

(2) The sign of the electric change and the momentum p of the particle can be determined if the chamber is placed in a strong magnetic field. Let a particle of mass m and charge q move with a velocity v perpendicular to the direction of the magnetic field of flux density B. The particle will be forced by the field to follow a circular path of radius R. The magnetic force Bqv is exactly balanced by the centrifugal force mv^2 / R.

Thus $Bqv = Mv^2 / R \text{ or } mv = p = BRq.$

The K.E. of the particle can be calculated, if the rest mass energy m_0c^2 of the particle is known, by the relation.

$$K.E. = Ek = \sqrt{\left[p^2 c^2 + \left(m_0c^2\right)\right]} - m_0c^2$$

Limitations.

(i) One is not always sure of the sense of track photographed.

(ii) The range of the particle may exceed the dimensions of the chamber so that the whole track is not photographed.

(iii) There remains a certain amount of uncertainty about the nature of the nuclei constituting the arms of the forked tracks.

4.7.1 Diffusion Cloud Chamber

The disadvantage of the cloud chamber lies in the fact that it needs a definite time to recover after an expansion.

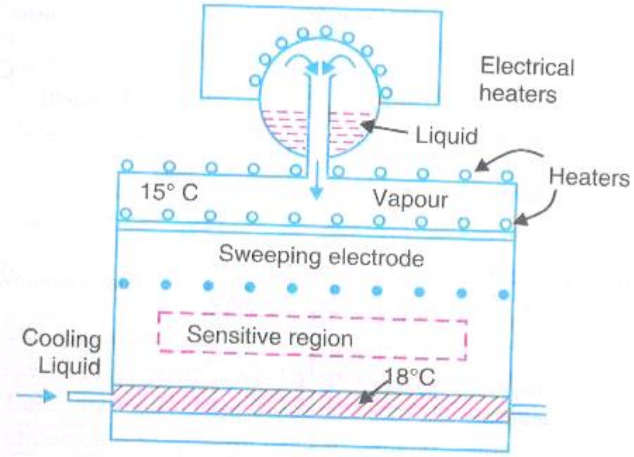


Fig. 4.7.1

Hence it is not possible to have a continuous record of events taking place in the chamber. This difficulty was removed by the introduction of the diffusion cloud chamber.

The outline of the apparatus is shown in Fig. 4.7.1.

It consists of a chamber containing a heavy gas which is kept warm at the top and cold at the bottom. Thermal and top of the chamber by external heating or cooling. The liquid (methyl alcohol) vaporises in the warm region, where the vapour pressure is high. The vapour a region near the base, the supersaturation factor is high and condensation takes place around the available ions. The chamber remains continuously sensitive to ionizing particles until the supply of volatile liquid is exhausted. The system is illuminated by a strong source of light and the track of the particle is photographed by camera.

4.8 Particle Accelerators

Introduction

A particle accelerator is a device for increasing the K.E. of electrically charged particles. Methods of acceleration can be classed into three groups: direct field, inductive, and resonance. According to the shape of the path of the particles, accelerators are classified as linear and cyclic. In linear accelerators, the paths of particles are approximately straight lines; in cyclic accelerators, they are circles or spirals. (i) In a direct field linear accelerator, a particle passes only once through an electric field with a high p.d. set up by electrostatic generators. (ii) The only accelerator of the inductive type is the betatron. (iii) In magnetic resonance accelerators, the particle being accelerated repeatedly passes through an alternating electric field along a closed path, its energy being increased each time. A strong magnetic field is used to control motion of particles and to return them periodically to the region of the accelerating electric field. The particles pass definite points of the alternating electric field approximately when the field is in the same phase (“in resonance”). The simplest resonance accelerator is the cyclotron.

4.9 The Cyclotron

Construction. The cyclotron (Fig. 4.9) consists of two hollow semicircular metal boxes, D_1, D_2 called “dees”. A source of ions is located near the mid-point of the gap between the “dees”. The “dees’’ are insulated from each other and are enclosed in another vacuum chamber. The “dees” are connected to a powerful radio-frequency oscillator. The whole apparatus is placed between the pole-pieces of a strong electromagnet. The magnetic field is perpendicular to the plane of the “dees”.

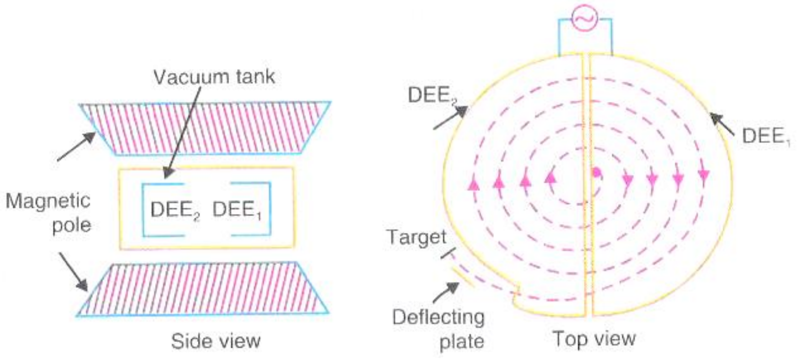


Fig. 4.9

Theory. Suppose a positive ion leaves the ion source at the centre of the chamber at the instant when the 'dees' D_1 and D_2 are at the maximum negative and positive A.C. potentials respectively. The positive ion will be accelerated towards the negative dee D_1 before entering it. The ions enter the space inside the dee with a velocity v given by $Ve = \frac{1}{2}mv^2$, where V is the applied voltage and e and m are the charge and mass of the ion respectively. When the ion is inside the "dee'' it is not accelerated since this space is fields free. Inside the dee, under the action of the applied magnetic field, the ions travel in a circular path of radius r given by

$$Bev = mv^2 / r \qquad \ldots(1)$$

Where B=the flux density of the magnetic field.

(or) $$r = mv / Be \qquad \ldots(2)$$

The angular velocity of the ion in its circular path

$$= \omega = \frac{v}{r} = \frac{Be}{m} \qquad \ldots(3)$$

The time taken by the ion to travel the semicircular

$$\text{path } = t = \frac{\pi}{\omega} = \frac{\pi m}{Be} \qquad \ldots(4)$$

Suppose the strength of the field (B) or the frequency of the oscillator (f) are so adjusted that by the time the ion has described a semicircular path and just enters the space between D_1 and D_2, D_2 has become negative with respect to D_1. The ion is then accelerated towards D_2 and enters the space inside it with a greater velocity. Since the ion is now moving with greater velocity, it will describe a semicircle of grater radius in the second 'dee'. But from the equation $t = \pi m / Be$ it is clear that the time taken by the ion to describe a semicircle is independent of both the radius of the path (r) and the velocity of the ion (v). Hence the ion describes all semicircles, whatever be their radii, in exactly the same time. This process continues until the ion reaches the periphery of the dees. The ion thus spirals round in circles of increasing radius and acquires high energy. The ion will finally come out of the dees in the direction indicated, through the window.

Energy of an ion. Let r_{max} be the radius of the outermost orbit described by the ion and v_{max} the maximum velocity gained by the ion in its final orbit. Then the equation for the motion of the ion in a magnetic field is

$$Bev_{\max} = \frac{mv_{\max}{}^2}{r_{\max}}$$

(or) $$v_{\max} = B\frac{e}{m}r_{\max} \qquad \ldots(5)$$

$\therefore$ The energy of the ion $\therefore$

$$E=\frac{1}{2}mv_{\max}{}^{2}=\frac{B^2r_{\max}{}^{2}}{2}\left[\frac{e^2}{m}\right] \qquad \ldots(6)$$

The condition for acceleration of the ion in the inter-dee gap is that

The time taken by the ion to travel the semicircular path = Half the time period of oscillation of the applied high frequency voltage

i.e., $$\frac{\pi m}{Be}=\frac{T}{2}\,or\,T=\frac{2\pi m}{Be}$$

$\therefore$ Frequency of the oscillator

$$f=\frac{Be}{2\pi m} \qquad \ldots(7)$$

Hence the energy of the ion is given by

$$E=2\pi^2r_{\max}{}^{2}\,f^2m \qquad \ldots(8)$$

The particles are ejected out of the cyclotron not continuously but as pulsed streams.

Limitations of the Cyclotron. The energies to which particles can be accelerated in a cyclotron are limited by the relativistic increase of mass with velocity. The mass of a particle, when moving with a velocity v is given by $m=\frac{m_0}{\sqrt{1-v^2/c^2}}$ where m_0 is the rest mass and c the velocity of light.

According to equation (4),

The time taken by the ion to travel the semicircular path $=t=\frac{\pi m}{Be}=\frac{T}{2}$

$\therefore$ Frequency of the ion $\}=n=\frac{1}{T}=\frac{Be}{2\pi m}\,or\,n=\frac{Be\sqrt{1-v^2/c^2}}{2\pi m_0}$

Therefore, the frequency of rotation of the ion decreases with increase in velocity. The ions take longer time to describe their semicircular paths than the fixed period of the oscillating electric field. Thus, the ions lag behind the applied potential and finally they are not accelerated further. Due to this reason, the energy of the ions produced by the cyclotron is limited can be overcome in the following two ways.

Now, the frequency of the ion $=Be\frac{\sqrt{1-v^2/c^2}}{2\pi m_0}$

(i) **Field variation.** The frequency of the ion can be kept constant by increasing the magnetic field (B) at such a rate that the product $B\sqrt{1-v^2/c^2}$ remains constant. For this purpose, the value of the magnetic field B should increase, as velocity of the ion increases, so that the product $B\sqrt{1-v^2/c^2}$ remains unchanged. This type of machine in which the frequency of electric field is kept constant and magnetic field is varied is called synchrotron.

(ii) **Frequency modulation.** In another from of apparatus, the frequency of the applied A.C. is varied so that it is always equal to the frequency of rotation of the ion. This type of machine in which magnetic field kept constant and the frequency of the applied electric field is varied is called a frequency modulated cyclotron or synchro-cyclotron.

Example 1. A crclotron in which the flux density is 1.4 weber/m^2 is employed to accelerate protons. How rapidly should the electric field between the dees be reversed? Mass of the proton =1.67x10^{-27} kg and charge=1.6x10^{-19}C.

Sol. Here, $B = 1.4\,weber/m^2; m = 1.67\times10^{-27}\,kg;\ e = 1.6\times10^{-19}\,C$

$$\therefore \qquad t = \frac{\pi m}{Be} = \frac{\pi(1.67\times10^{-27})}{1.4\times(1.6\times10^{-19})} = 2.342\times10^{-8}\ s.$$

Example 2. Deuterons in a cyclotron describe a circle of radius 0.32 m just before emerging from the dees. The frequency of the applied e.m.f. is 10 MHz. Find the flux density of the magnetic field and the velocity of deuterons emerging out of the cyclotron. Mass of deuterium = 3.32 x 10^{-27} kg; e =1.6 x 10^{-19} C.

Sol. We have $f = \dfrac{Be}{2\pi m} \qquad \therefore B = \dfrac{2\pi mf}{e}$

Here, $m = 3.32\times10^{-27}\,kg;\ f = 10MHz = 10^7\,Hz; e = 1.6\times10^{-19}\,C$

$$\therefore \qquad B = \frac{2\pi(3.32\times10^{-27})10^7}{1.6\times10^{-19}} = 1.303\,weber/m^2$$

We have $\dfrac{mv^2}{r_{\max}} = Bev \ \ or \ \ v = \dfrac{Ber_{\max}}{m}$

Here,

$B = 1.303\ weber/m^2;\ e = 1.6\times10^{-19}\ C; r_{\max} = 0.32\ m\ and\ m = 3.32\times10^{-27}\,kg$

$$\therefore \qquad v = \frac{Ber_{\max}}{m} = \frac{1.303(1.6\times10^{-19})0.32}{3.32\times10^{-27}}$$

$$= 2.009\times10^7\,ms^{-1}$$

4.9.1 The Synchrocyclotron

Synchrocyclotron is a modified from of the Lawrence cyclotron. This consists of only one dee placed in a vacuum chamber between the poles of an electromagnet (Fig. 4.9) Instead of the second dee, opposite the opening of the dee, there is metal sheet connected to the earth. The alternating P.D is applied between the dee and the metal plate (Fig. 4.9.1). The alternating potential applied to the ‘dee’ is made to rise and fall periodically, instead of remaining constant. The frequency is changed at such a rate that as the ion lags a little due to the increase in mass caused by increase in velocity, the electric field frequency also automatically lags in variation. Hence the particle always enters the dee at the correct moment, when it can experience maximum acceleration. An advantage of using one dee is that is leaves sufficient space in the vacuum chamber for the ion source and the target. The pole-pieces of the magnet are of suitable shape such that the field decreases outwards from the centre. This ensures good focusing of the accelerated ions.

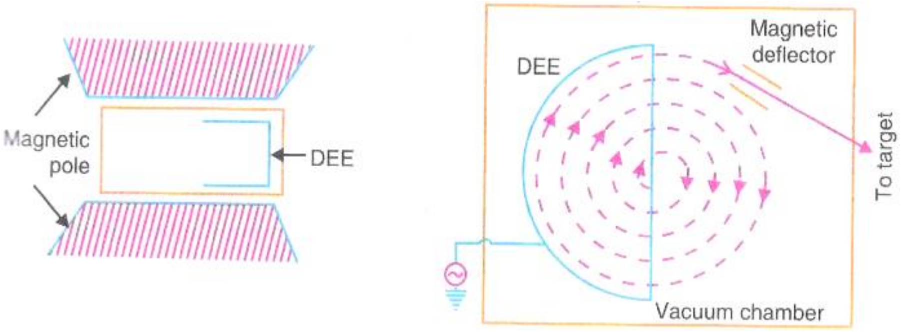


Fig. 4.9.1

4.10 The Betatron

Betatron is a device to accelerate electrons (beta particles) to very high energies. It was constructed in 1941 by D.W. Kerst. The action of this device depends on the principle of a transformer.

Construction.

It consists of a doughnut shaped vacuum chamber placed between the pole-pieces of an electromagnet. The electromagnet is energized by an alternating current. The electromagnet is energized by an alternating current. The magnet produces a strong magnetic field in the doughnut. The electrons are produced by the electron gun (FG) and are allowed to move in a circular orbit of constant radius in the vacuum chamber (Fig. 4.10(a)). The magnetic field varies very slowly compared with the frequency of revolution of the electrons in the equilibrium orbit.

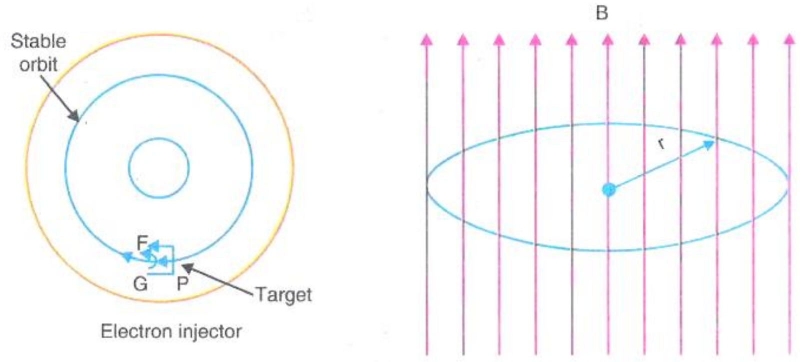


Fig. 4.10 (a) (b)

The varying magnetic field, parallel to the axis of the vacuum tube, produces two effects on the electrons viz., (i) The changing flux due to the electromagnet produces the induced e.m.f. which is responsible for the acceleration of the electrons. (ii) The field of the magnet serves at the same time to bend the electrons in a circular path in the chamber and confine them to the region of the changing flux.

Theory:

Consider the electron moving in an orbit of radius r(Fig. 4.10(b)) Let be ϕ the flux linked with the orbit. The flux increases at the rate $d\phi/dt$ and the induced e.m.f in the orbit is given by

$$E = -\frac{d\phi}{dt} \qquad \ldots(1)$$

The work done on an electron of charge e in one revolution $= Ee = -e\frac{d\phi}{dt}$

…(2)

Let f be the tangential electric force acting on the orbiting electron.

For one revolution, the path length is $2\pi r$. Then

The work done on the electron in one revolution $= F \times 2\pi r$

$$\therefore \qquad F \times 2\pi r = -e\frac{d\phi}{dt}$$

(or)

$$F = -\frac{e}{2\pi r}\frac{d\phi}{dt} \qquad \ldots(3)$$

When the velocity of the electron increases due to the above force, it will try to move into an orbit of larger radius. Because of the presence of the magnetic flux perpendicular to the plane of the electron orbit, the electron will experience a radial force inward given by

$$Bev = mv^2/r \qquad \ldots(4)$$

Here B is the value of the magnetic field intensity at the electron orbit of constant radius r, v=velocity of the electron and m=mass of the electron from (4).

The momentum of the electron = mv=Ber …(5)

From Newton's second law of motion.

$$F = \frac{d}{dt}(mv) = er\frac{dB}{dt} \qquad \ldots(6)$$

To maintain the constant radius of the values of F given in equations (3) and (6) must be numerically equal.

$$\therefore \qquad \frac{e}{2\pi r}\frac{d\phi}{dt} = er\frac{dB}{dt} \ \ or \ \ d\phi = 2\pi r^2\, dB$$

Integrating, $\int_0^{\phi} d\phi = \int_0^{B} 2\pi r^2 \, dB$

(or) $\phi = 2\pi r^2 B$ …(7)

If the uniform magnetic field B acts over an area πr^2, the magnetic flux $\phi = \pi r^2 B$. Therefore the flux through the orbit is twice the flux enclosed by the orbit, if the magnetic field were to be uniform over the area. Equation (7) represents the condition under which a betatron works and is called betatron condition. This distribution of magnetic flux is obtained by the special pole-pieces where the magnetic field is greater at the centre of the orbit than at its circumference.

Fig. 4.10(c) shows the variation of magnetic field with time. Electrons are injected into the chamber when magnetic field just begins to rise. The electrons are then accelerated by the increasing magnetic flux linked with the electron orbit. During the time the magnetic field reaches its peak value, the electrons make several thousand revolutions and get accelerated. If they are allowed to revolve any more, the decreasing magnetic field would retard the electrons. Hence, the electrons are extracted at this stage by using an auxiliary magnetic field to deflect them from their normal course. The high energy electron beam can be made to strike the target, generating X-rays. Alternately the electrons can be made to emerge out of the apparatus and used for transmutation work.

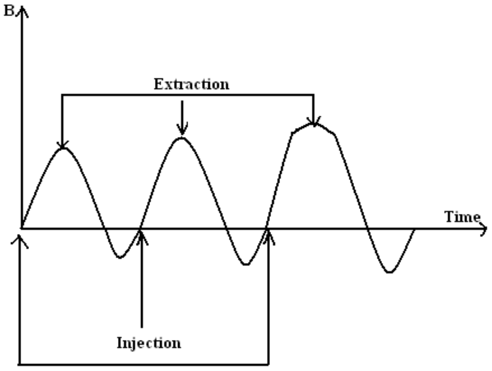


Fig. 4.10(c)

Example. In a certain betatron the maximum magnetic field at orbit was 0.4 Wb/m^2, operating at 50 Hz with a stable orbit diameter of 1.524 m. Calculate the average energy gained per revolution and the final energy of the electrons.

Sol. In the betatron, the electron velocities are nearly c.

$\therefore$ the total distance traveled in the acceleration time (i.e. one quarter cycle)

$$= c \times \frac{T}{4} = c \times \frac{\pi}{2\omega}$$

Total number of revolutions $= N = \dfrac{c\pi / 2\omega}{2\pi r} = \dfrac{c}{4\omega r}$

Here, frequency $= f = 50 Hz. \therefore \omega = 2\pi f = 2\pi \times 50 = 100\pi; r = 0.762 m, \, and \, c = 3\times 10^8 ms^{-1}$

$$\therefore \qquad N = \frac{3\times 10^8}{4(100\pi)0.762} = 3.132\times 10^5$$

Let E be the final energy acquired by the electrons. Since the electrons must be treated relative-istically

Momentum of the electrons =mv = E/c

But $mv^2 / r = Bev \; or \; mv = Ber \; or \; E = Berc$

$$\therefore \qquad E = \frac{0.4(1.6\times 10^{-19})(0.762)(3\times 10^8)}{1.6\times 10^{-13}} MeV$$

=91.45 MeV.

Average energy gained per revolution = $\dfrac{91.45\times 10^6}{3.132\times 10^5} = 291.9 eV.$

4.11 Bonding in Crystals

Most solids are crystalline, with the atoms, ions or molecules of which they are composed falling into regular, repeated three-dimensional patterns. The presence of long-range order is thus the defining property of a crystal. Crystals may be classified in terms of the dominant type of chemical binding force keeping the atoms together. All these bonds involve electrostatic forces, with the chief differences among them lying in the ways in which the outer electrons of the structural elements are distributed. The distinct types of bonds that provide the cohesive forces in crystals can be classified as follows: (i) the ionic bond (ii) the covalent bond (iii) the metallic bond (iv) the van der Waals bond and (v) the hydrogen bond. We briefly discuss the different types of bonds in crystals.

4.12 lonic Bond

Ionic bonds are formed when atoms that have low ionization energies, and hence lose electrons readily, interact with other atoms that tends to acquire excess electrons. The former atoms give up electrons to the latter.

Thus the atoms become positive and negative ions respectively. These ions come together in an equilibrium configuration in which the attractive forces between positive and negative ions predominate over the repulsive forces between similar ions.

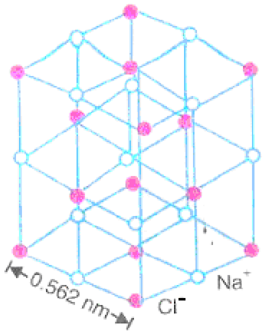


Fig. 4.12(a)

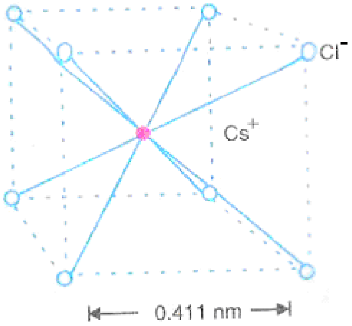
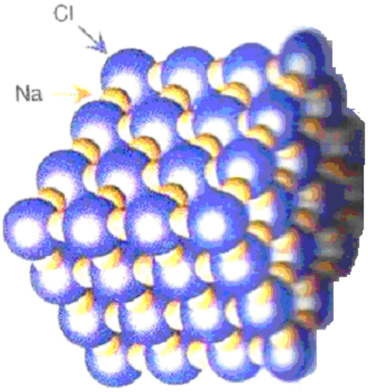


Fig. 4.12(b)

Consider the case if sodium chloride which is a typical example of an ionic crystal. Here, a single valence electron is transferred from the sodium atom to the chlorine atom. The Na^+ and Cl^- ions so formed are arranged in a face-centered cubic structure (fig. 4.12(a)) A different arrangement is found in cesium chloride crystals. The body-centered cubic structure of a CsCl crystal is shown in Fig. 4.12 (b)

Expression for the cohesive energy of an ionic crystal

The cohesive energy of an ionic crystal is the energy that would be liberated by the formation of the crystal from individual neutral atoms. The principal contribution to the cohesive energy of an ionic crystal is the electrostatic potential energy U_{coulomb} of the ions. Let us consider an Na^+ ion in NaCl. Its nearest neighbours are six Cl^- ions, each one the distance r away



Sodium chloride crystal structure

The potential energy of the Na^+ ion due to the 6Cl^- ions $U_1 = -\dfrac{6e^2}{4\pi\varepsilon_0 r}$

The next nearest neighbours are 12 Na^+ ions, each one the distance $\sqrt{2}\,r$ away.

The potential energy of the Na^+ ion due to the 12 Na^+ ions

$$U_2 = -\frac{12e^2}{4\pi\varepsilon_0\sqrt{2}r}$$

Then there are 8Cl$^-$ ions at $\sqrt{3}r$ distance, 6 Na$^+$ ions at 2r distance and so on. When the summation is continued over all the + and – ions a crystal of infinite size, the result is

$$U_{conlomb} = -\frac{6e^2}{4\pi\varepsilon_0 r}\frac{12e^2}{4\pi\varepsilon_0\sqrt{2}\,r} - \frac{8e^2}{4\pi\varepsilon_0\sqrt{3}r} + \frac{6e^2}{4\pi\varepsilon_0\,2r} + \ldots$$

$$= -\frac{e^2}{4\pi\varepsilon_0 r}\left(\frac{6}{\sqrt{1}} - \frac{12}{2} - + \frac{8}{\sqrt{3}} - \frac{6}{\sqrt{4}} + \ldots\right)$$

$$= -1.748\frac{e^2}{4\pi\varepsilon_0 r}$$

or in general $U_{conlomb} = -\alpha\frac{e^2}{4\pi\varepsilon_0 r}$ …(1)

This result holds for the potential energy of a Cl$^-$ ion also. α is called the modeling constant of the crystal. It has the same value for all crystals of the same structure. For simple crystal structures α lies between 1.6 and 1.8

Two ions cannot continuously approach each other under coulomb attraction on account of the exclusion principle. When they are at a certain small distance apart, they begin to repel each other with a force which increases rapidly with decreasing internuclear distance r. The potential energy contribution of the short-range repulsive forces can be expressed approximately in the form.

$$U_{repulsive} = \frac{B}{r^n} \qquad \ldots (2)$$

Where B is a constant and n is a number (n=9)

Therefore, the total potential energy U of each ion due to its interactions with all the other ions is

$$U = U_{conlomb} + U_{repulsive} = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^n} \qquad \ldots(3)$$

We must now evaluate the constant B. At the equilibrium separation r$_0$ of the ions, U is a minimum. So $((dU/dr) = 0$ *when* $r = r_0$

$$\left(\frac{dU}{dr}\right)_{r=r_0} = \frac{\alpha e^2}{4\pi\varepsilon_0\, r_0^{\,2}} - \frac{nB}{r_0^{\,n+1}} = 0$$

(or) $B = \dfrac{\alpha e^2}{4\pi\varepsilon_0\, n} r_0^{\,n-1}$ ….(4)

The total potential energy at the equilibrium separation is

$$U = -\frac{\alpha e^2}{4\pi\varepsilon_0 r_0} + \frac{\alpha e^2 r_0^{n-1}}{4\pi\varepsilon_0 n r_0^n}$$

$$\therefore \qquad U = -\frac{\alpha e^2}{4\pi\varepsilon_0 r_0}(1-\frac{1}{n}) \qquad \ldots(5)$$

This is the magnitude of the energy needed to separate an ionic crystal into individual ions (not into atoms)

In an NaCl crystal, the equilibrium distance r_0, between ions is $2.81 X 10^{-10} m, \alpha = 1.784 \; and \; n = 9, 1/4\pi\varepsilon_0 = 9X10^9 \; Nm^2C^{-2}$

The potential energy of an ion of either sign is

$$U = -\frac{\alpha e^2}{4\pi\varepsilon_0 r_0}(1-\frac{1}{n})$$

$$= -\frac{(9X10^9 \; Nm^2C^{-2})(1.748)(1.60X10^{-19} \; C)^2}{2.81 X 10^{-10} m}(1-\frac{1}{9})$$

$$= 1.27 \, X \, 10^{-18} \; J = -7.96eV$$

Because we may not count each ion more than once, only half this potential energy, or -3.98 eV, represents the contribution per ion to the cohesive energy of the crystal.

Some energy is needed to transfer and electron from a Na atom to a Cl atom to from a Na^+-Cl^- ion pair. This electron transfer energy is the difference between the +5.14 eV ionization energy of Na and the -3.61 eV electron affinity of Cl, or +1.53 eV. Each atom thus contributes +0.77 eV to the cohesive energy.

$\therefore$ the total cohesive every per atom in the NaCl crystal is

$$E_{cohesive} = (-3.98 + 0.77)eV/atom = -3.21eV/atom.$$

Properties : (i) Most ionic solids are hard, brittle and have high melting points.

(ii) They are soluble in polar liquids like water.

(iii) Their electrical conductivity is much smaller then that of metals at room temperature. But is contrast to metals, the conductivity of ionic crystals increases with increasing temperature. At elevated temperatures, the ions themselves become mobile and ionic conductivity results.

(iv) Ionic solids crystallize in close- packed structures, of which the NaCl CsCI structures are the commonest.

4.13 Covalent Bond

In the covalent bond, atoms are held together by the sharing of electrons. Each atom participating in a covalent bond contributes an electron to the bond. These electrons are shared by both atoms rather than being the virtually exclusive property of one of them as in an ionic bond. Diamond is an example of a crystal whose atoms are linked by covalent bonds. Fig 4.13(a) shows the structure of a diamond crystal. The tetrahedral arrangements is a consequence of the ability of each carbon atom to from covalent bonds with four other atoms.

The binding of molecular hydrogen (H_2) is a simple example of covalent bond. In the H_2 molecule, two electrons are shared by the two atoms (Fig. 4.13(b)). As these electrons circulate, they spend more time between the atoms (in fact between the protons) than elsewhere and this produces an attractive force. Thus covalent crystals are composed of neutral atoms having slightly overlapping electron clouds. Diamond, germanium, silicon, and silicon carbide, (SiC) are examples of covalent crystals. In SiC each atom is surrounded by four atoms of the other kind in the same tetrahedral structure as that of diamond.

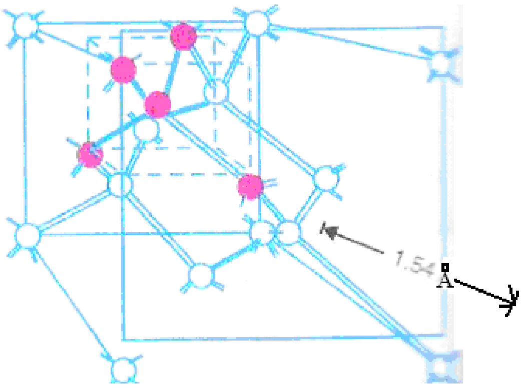


Fig. 4.13(a)

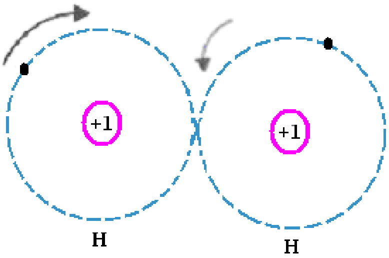
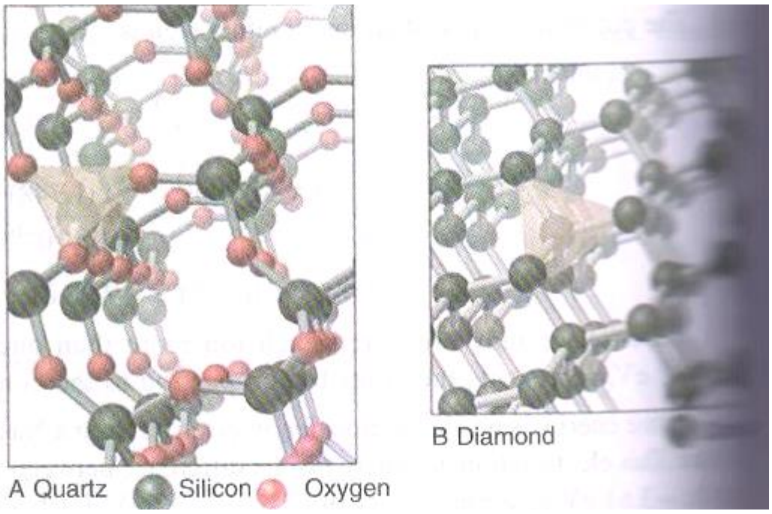


Fig. 4.13(b)

Characteristics of Covalent Crystals:

(1) Covalent bond is a strong bond. Cohesive energies of 6 to 12 eV/atom are typical of covalent crystals, which is more then the usual cohesive energies in ionic crystals. All covalent crystals are hard, have high melting points, and are insoluble in all ordinary liquids.



Covalent Bond Strongest Bond Formed by Sharing Electron

2) Covalent bonds are strongly directional The other important characteristic property of the covalent bond is its saturability. Saturability means that each atoms can form covalent bonds only with a limited number of itsneghbours.

3) The conductivity of covalent crystals varies over a wide range. Some crystals are insulators (diamond) and some are semiconductors (Ge). The conductivity increase with the increase of temperature.

4) The optical properties of the covalent crystals are characterized by high refractive index and high dielectric constant. Covalent crystals are transparent to long-wavelength radiation but opaque to shorter wavelengths.

4.14 Metallic Bond

In metallic crystals, the metallic bond arises when all of the atoms share all of the valence electrons. The valence electrons of the atoms comprising a metal are common to the entire aggregate, so that a kind of “gas” of free electrons pervades it. The crystal is held together by the electrostatic attraction between the negative electron gas and the positive metal ions. The best example for a metallic crystal is sodium.

The cohesion (i.e., the ability to remain a solid) of the metallic crystal results from a combination of forces:

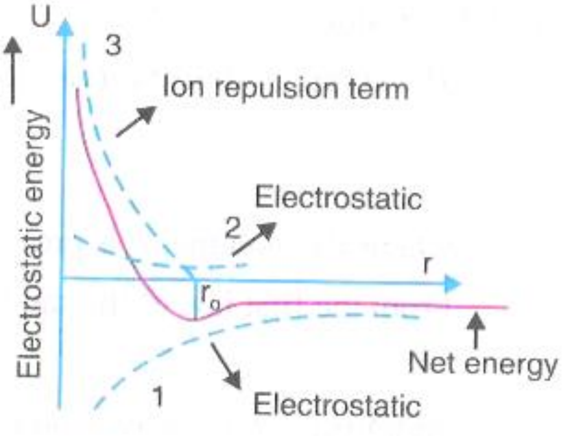


Fig. 4.14

i. The attraction of the electron cloud for the ion cores,

ii. The mutual repulsion of the electrons and

iii. The mutual repulsion of the cores.

Fig. 4.14 Shows a sketch of a reasonable form for the potential (free) energy contribution, as a function of nuclear spacing, from each of these charge interactions.

The following are the characteristics of metal crystals.

1. The presence of free electrons accounts for the high electrical and thermal conductivities of metals. The high electrical conductivity of metals is in turn responsible for their high optical reflection and absorption coefficients.
2. the other characteristic properties of metals are their ductility and metallic lustre. Since the metallic bonds are not localized between adjacent atoms, the atoms of a metal can be rearranged in position without rupturing the crystal. This explains ductility of metals. When light shines on a metal, the free electrons oscillate under the electromagnetic field of the incident light and become sources of light. Thus gives the metal its surface lustre.
3. Metallic bonds are weaker than ionic and covalent bonds.

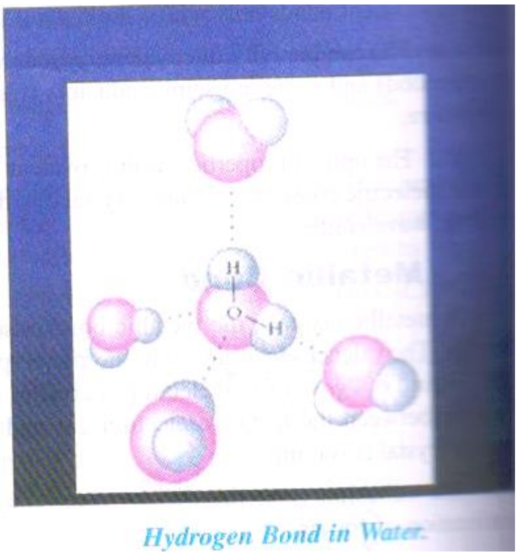
4.15 Molecular Bond

Neutral atoms with closed electron shells are bound together weakly by the Van der Waals forces. The van derWaals' attraction was first explained for electrically neutral gas molecules by Debye. He assumed that negihbouring molecules induced dipoles in each other because of their own changing electric fields. This interaction produces an attractive force that is inversely proportional to the seventh power of the separation. The molecules are located at the crystal lattice points and the bonds between them are developed by Van der Waals' forces. Solid argon and solid methane (CH_4) are examples of molecular crystals.

Properties: Van der Waals forces are much weaker than those found in ionic and covalent bonds. As a result, molecular crystals generally have low melting and boiling points and little mechanical strength. Molecular crystals have low cohesive energies. Molecular crystals are good insulators due to non-availability of free electrons.

4.16 Hydrogen Bond

Hydrogen bond is formed under certain conditions when a single hydrogen atom appears to be bonded to two distinct electronegative atoms. This configuration is represented as x-H…Y where X is called 'donor' while Y is called acceptor'. The weaker of the two bonds while the other dotted line) is the hydrogen bond while the other (shown by a full line) is a strong covalent bond. H_2O (ice), NH_3 and HF are examples of hydrogen-bonded crystals. These bonds are stronger than van der Waals bonds but weaker than ionic or covalent bonds.



Hydrogen Bond in Water.

Calculation of Repulsive exponent n

Born determined the repulsive exponent (n) from measurements of the compressibility of the crystal as follows:

The compressibility C (i.e. reciprocal of bulk modulus K) is defined as

$$\frac{1}{C} = K = -V\left(\frac{dP}{dV}\right) \qquad \text{….(1)}$$

Where V is volume, P is pressure

From the first law of thermodynamics

$$dQ = dU + P\,dV$$

At absolute zero there is no absorption of heat energy by the ions

$$\therefore \qquad dQ = 0$$

$\therefore$ change in internal energy, $dU = -P\,dV$

$$\frac{dU}{dV} = -P$$

(or) $$\frac{d^2U}{dV^2} = -\frac{dP}{dV} \qquad \ldots(2)$$

$\therefore$ $$\frac{1}{C} = V\frac{d^2U}{dV^2} \qquad \ldots(3)$$

Also $$\frac{dU}{dV} = \frac{dU}{dr}.\frac{dr}{dV}\, i.e., \frac{d}{dV} = \frac{dr}{dV}.\frac{d}{dr} \qquad \ldots(4)$$

and $$\frac{d^2U}{dV^2} = \frac{d}{dV}\left(\frac{dU}{dV}\right) = \frac{d}{dV}\left(\frac{dU}{dr}.\frac{dr}{dV}\right)$$

$$= \frac{dU}{dr}.\frac{d^2r}{dV^2} + \frac{dr}{dV}.\frac{d}{dV}\left(\frac{dU}{dr}\right)$$

$$= \frac{dU}{dr}.\frac{d^2r}{dV^2} + \left(\frac{dr}{dV}\right).\frac{dr}{dV}\frac{d}{dr}\left(\frac{dU}{dr}\right)$$

$$= \frac{dU}{dr}.\frac{d^2r}{dV^2} + \left(\frac{dr}{dV}\right)^2.\frac{d^2U}{dr^2} \qquad \ldots(5)$$

For NaCl crystal the volume of unit cell is

$$(2r)^3 = 8r^3$$

The unit cell has 4 sodium ions and 4 chloride ions. The volume $8r^3$ corresponds to 8 ions. So each ion corresponds to the volume r^3. If the crystal contains N-Na and N –Cl atoms, then $V=2Nr^3$

$\therefore$ $$\frac{dV}{dr} = 6Nr^2 \quad or\, \frac{dr}{dV} = \frac{1}{6Nr^2}$$

$$\frac{d^2r}{dV^2} = \frac{d}{dV}\left(\frac{dr}{dV}\right) = \frac{dr}{dV}.\frac{d}{dV}\left(\frac{dr}{dV}\right) = \frac{1}{6Nr^2}\, X\, \frac{d}{dr}\left(\frac{1}{6Nr^2}\right) = -\frac{1}{18N^2r^5}$$

Substituting these values in Eq. (5)

$$= \frac{d^2U}{dV^2} = -\frac{1}{18N^2r^5}\frac{dU}{dr} + \frac{1}{36N^2r^4}\frac{d^2U}{dr^2} \qquad \ldots(6)$$

At $r = r_0$, U is minimum and dU / dr=0

$$\left(\frac{d^2U}{dV^2}\right)_{r=r_0} = \frac{1}{36N^2r_0^4}\left(\frac{d^2U}{dr^2}\right)_{r=r_0} \qquad \ldots(7)$$

Substituting the above value in Eq. (3)

$$\frac{1}{C}=\left(V\frac{d^2U}{dV^2}\right)_{r=r_0}=2Nr_0^3 X\frac{1}{36N^2r_0^4}\left(\frac{d^2U}{dr^2}\right)_{r=r_0}$$

$$=\frac{1}{18N\,r_0}\left(\frac{d^2U}{dr^2}\right)_{r=r_0} \qquad \ldots(8)$$

But $$U=NU_i=N\left[\frac{B}{r^n}-\frac{\alpha e^2}{4\pi\varepsilon_0 r}\right] \qquad \ldots(9)$$

$$\therefore\ \frac{dU}{dr}=N\left(-\frac{Bn}{r^{n+1}}+\frac{\alpha}{4\pi\varepsilon_0}X\frac{e^2}{r^2}\right)$$

When $$r=r_0\,\frac{dU}{dr}=0$$

$$-\frac{Bn}{r_0^{\;n+1}}+\frac{\alpha}{4\pi\varepsilon_0}\frac{e^2}{r_0^{\;2}}=0$$

$$\therefore\qquad B=\frac{\alpha e^2-r_0^{\;n-1}}{4\pi\varepsilon_0 n} \qquad \ldots(10)$$

Also $$\frac{d^2U}{dr^2}=N\left(\frac{n(n+1)B}{r_0^{\;n+2}}-\frac{2\alpha e^2}{4\pi\varepsilon_0\,r_0^{\;3}}\right)$$

Substituting the value of B,

$$\left(\frac{d^2U}{dr^2}\right)=\frac{N\alpha e^2(n-1)}{4\pi\varepsilon_0\,r_0^{\;3}} \qquad \ldots(11)$$

Substituting this value in Eq. (8),

$$\frac{1}{C}=\frac{1}{18Nr_0}\frac{N\alpha e^2(n-1)}{4\pi\varepsilon_0\,r_0^{\;3}}$$

$$\therefore\qquad n=1+\frac{(4\pi\varepsilon_0)18r_0^{\;4}}{C\alpha e^2} \qquad \ldots(12)$$

Questions

1. What do you meant by nuclear models.
2. Explain liquid drop model. What are its merits and demerits.
3. Derive Weizacker semi-empirical mass formula.
4. Explain the Shell model of the nucleus.
5. What are the similarities between liquid drop and nucleus.
6. Explain the theory and principle of working of a synchrocyclotron.
7. What is the difference between a cyclotron and a synchrocyclotron?
8. Describe giving necessary theory the working of a betatron. Show that to keep the electron in a constant radial motion, the magnetic induction in the area enclosed by the path has to be maintained at any instant equal to double the induction over the circular orbits.
9. Give an account of the theory, construction and working of a modern synchrotron. Give its uses.
10. Describe the construction and working of an ionization chamber.
11. Describe a G.M. counter and explain its working as a particle detector.
12. Explain fully the working of a cloud chamber. How is it used to determine the energy of a particle passing through it?
13. Describe about Ionic Bond.
14. Describe the expression for the cohesive energy of an ionic crystal?
15. Short notes on Bonding in crystals.
16. Describe Covalent bond and its characteristics of covalent crystals.
17. Describe about Metallic Bond.
18. Explain briefly about Molecular Bond and its properties?
19. Write short notes on Hydrogen Bond.

NOTES

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UNIT - V

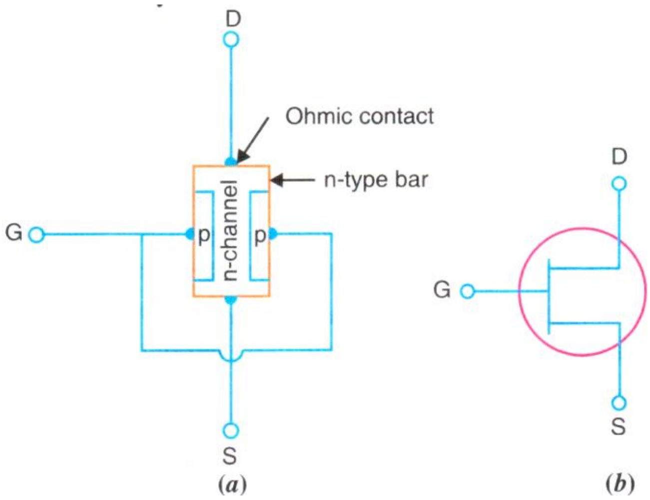
ELECTRONICS

5.1 Field-Effect Transistors

Introduction

The field effect transistor (FET) is a three terminal semiconductor device in which the output current is controlled by an applied electric field. Further the current in an FET is entirely due to the majority carriers whereas in a junction transistor both majority and minority carriers contribute to the current. Thus while the junction transistor is bipolar an FET is unipolar. There are two types of field effect transistors: (i) junction field effect transistor (JFET or simply FET). (ii) metal oxide semiconductor field effect transistor (MOSFET). A JFET can be either of the n-channel type or of the p-channel type. We shall here describe an n-channel JFET.

n-channel JFET. It consists of a channel (n-type) into which two p-regions are diffused. One end of this symmetric structure is called source (S) and the other end is called drain (D). The two o-regions are connected together to a third terminal called gate (G) (Fig. 5.1).



(*a*) (*b*)

Fig. 5.1

The p-regions are heavily doped compared to the n-region. During operation, majority carriers (electrons in this case) enter the channel through the source S and leave it through the drain D. The current is controlled by the gate which is always reverse-biased.

Drain Characteristics of an n Channel JFET

A curve plotted between drain current I_D and drain-to-source voltage V_{DS}, at a fixed gate-to-source voltage V_{GS} is called the drain characteristics. Fig. 5.1(a) shows the circuit diagram for determining the output characteristics. V_{GG} is the gate bias supply and V_{DD} is the drain voltage source.

Keeping V_{GS} fixed at some value, the drain source voltage (V_{DS}) is changed in steps and the corresponding drain current I_D is noted. A group of such drain characteristics curves are drawn by setting V_{GS} at different fixed values. Fig. 5.1(b) shows a family of drain characteristics. There are three distinct regions in the characteristic thus obtained.

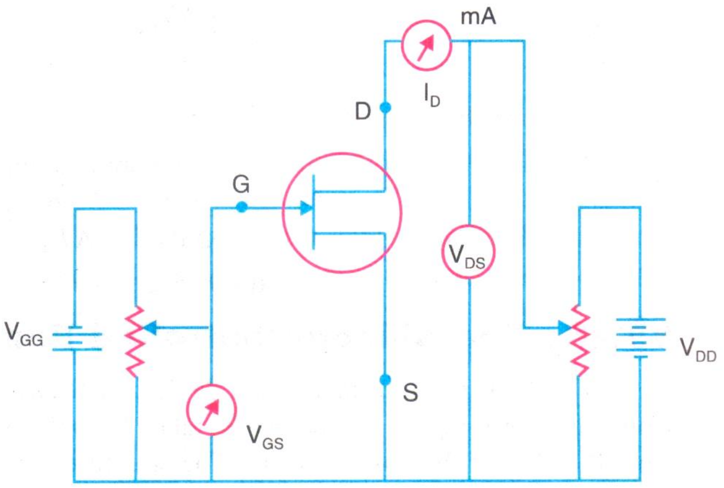


Fig. 5.1(a)

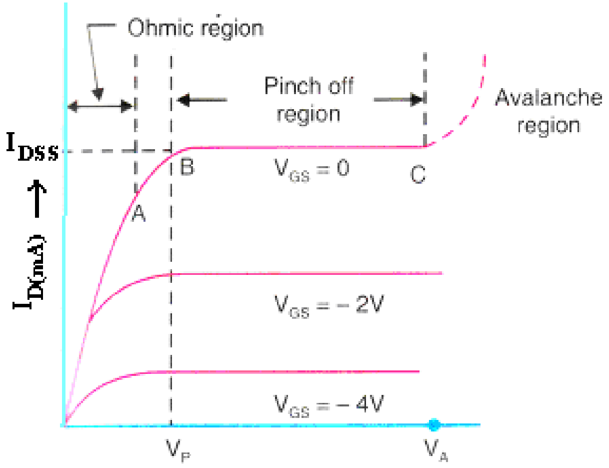


Fig. 5.1(b)

(i) When V_{DS} is small, the channel acts as a resistor. The current increases linearly with the voltage V_{DS} till point A is reached. This region of the characteristics is called the ohmic region.

(ii) When $V_{DS} = V_P$, the current I_D reaches its maximum value, I_{DSS}. If V_{DS} IS increased beyond V_P, the current does not increase further. The region BC is called saturation region or pinch off region.

(iii) At a certain voltage V_A, corresponding to point C, current increases suddenly due to avalanche breakdown. The covalent bonds in the depletion region break up and the current rises. This region is called the avalanche region. In actual practice this region is to be avoided.

Characteristic parameters of FET.

There are three main characteristic parameters of a FET which describe its performance in an electronic circuit.

(i) **Drain resistance r_d.** It is defined as the ratio of the change in drain-to-source voltage to the corresponding change in drain current at a constant gate-to-source voltage.

$$r_d = \left(\frac{\Delta V_{DS}}{\Delta I_D}\right)_{V_{GS}} \qquad \ldots(\mathrm{i})$$

It is given by the reciprocal of the slope of the drain characteristic.

(ii) **Transconductance g_m.** It is defined as the ratio of the change in drain current to the corresponding change in gate-to-source voltage at a constant drain-to-source voltage.

$$g_m = \left(\frac{\Delta I_D}{\Delta V_{GS}}\right)_{V_{DS}} \qquad \ldots(\mathrm{ii})$$

The transductance measures the control that the gate voltage has over the drain current.

(iii) **Amplification factor μ.** It is defined as the ratio of the change in drain-to-source voltage to the corresponding change in gate-to-source voltage at a constant drain current.

$$\mu = \left(\frac{\Delta V_{DS}}{\Delta V_{GS}}\right)_{I_D} \qquad \ldots(\mathrm{iii})$$

Relation between the three parameters.

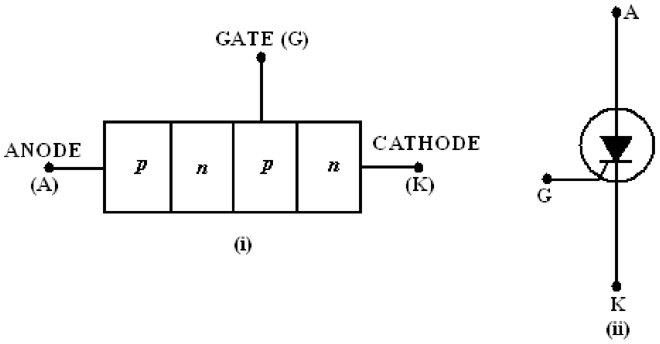
We have from the definition of the amplification factor

$$\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}} = \frac{\Delta V_{DS}}{\Delta I_D} \times \frac{\Delta I_D}{\Delta V_{GS}} = r_d \times g_m$$

$$\mu = r_d \times g_m.$$

5.2 Silicon Controlled Rectifier (SCR)

A silicon controlled rectifier is a semiconductor device that acts as a true electronic switch. It can change alternating current and at the same time can control the amount of power fed to the load. Thus SCR combines the features of a rectifier and a transistor.



(i) (ii)

Fig. 5.2

Constructional details. When a pn junction is added to a junction transistor, the resulting three pn junction device is called a silicon controlled rectifier. Fig. 5.2(i) shows the construction. It is clear that is essentially an ordinary rectifier (pn) and a junction transistor (npn) combined in one unit to form pnpn device. Three terminals are taken; one from the outer p-type material called anode A, second from the outer n-type material called cathode K and the third from the base of transistor section and is called gate G. In the normal operating conditions of SCR, anode is held at high positive potential w.r.t. cathode and gate at small positive potential w.r.t. cathode. Fig. 5.2 (ii) shows the symbol of SCR.

The silicon controlled rectifier is a solid state equivalent of thyratron. The gate, anode and cathode of SCR correspond to the grid, plate and cathode of thyratron. For this reason, SCR is sometimes called thyristor.

V-I Characteristics of SCR

It is the curve between anode-cathode voltage (V) and anode current (I) of an SCR at constant gate current. Fig. 5.2.1 shows the V-I characteristics of a typical SCR.

(i) **Forward characteristics.** When anode is positive w.r.t. cathode, the curve between V and I is called the forward characteristic. In Fig. 5.2.1, OABC is the forward characteristic of SCR at $I_G = 0$. If the supply voltage is increased from zero, a point is reached (point A) when the SCR starts conducting. Under this condition, the voltage across SCR suddenly drops as shown by dotted curve AB and most of supply voltage appears across the load resistance R_L. If proper gate current is made to flow, SCR can close at much smaller supply voltage.

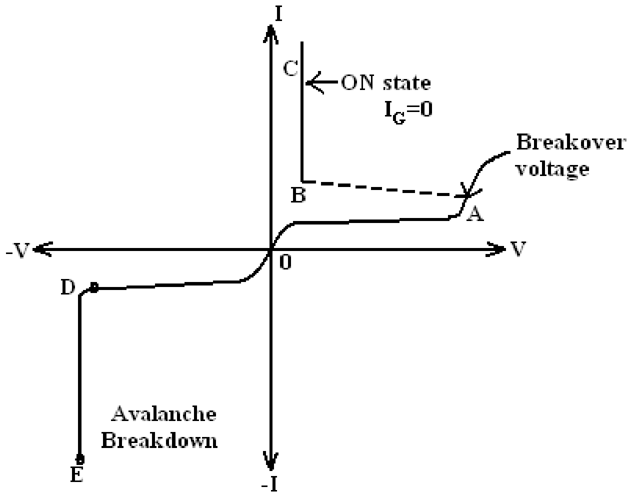


Fig. 5.2.1

(ii) **Reverse characteristics.** When anode is negative w.r.t. cathode, the curve between V and I is known as reverse characteristic. The reverse voltage does come across SCR when it is operated with a.c. supply. If the reverse voltage is gradually increased, at first the anode current remains small (i.e. leakage current) and at some reverse voltage, avalanche breakdown occurs and the SCR starts conducting heavily in the reverse direction as shown by the curve DE. This maximum reverse voltage at which SCR starts conducting heavily is known as reverse breakdown voltage.

SCR in Normal Operation

In order to operate the SCR in normal operation, the following points are kept in view:

(ii) The supply voltage is generally much less than breakover voltage.

(iii) The SCR is turned on by passing an appropriate amount of gate current (a few mA) and not by breakover voltage.

(iv) When SCR is operated from a.c. supply, the peak reverse voltage which comes during negative half-cycle should not exceed the reverse breakdown voltage.

(v) When SCR is to be turned OFF from the ON state, anode current should be reduced to holding current.

(vi) If gate current is increased above the required value, the SCR will close at much reduced supply voltage.

SCR as a Switch

The SCR has only two states, namely; ON state and OFF state and no state inbetween. When appropriate gate current is passed, the SCR starts conducting heavily and remains in this position indefinitely even if gate voltage is removed. This corresponds to the ON condition. However, when the anode current is reduced to the holding current, the SCR is turned OFF. It is clear that behaviour of SCR is similar to a mechanical switch. As SCR is an electronic device, therefore, it is more appropriate to call it an electronic switch.

Advantages of SCR as a switch. As SCR has the following advantages over a mechanical or electromechanical switch (relay):

(i) It has no moving parts. Consequently, it gives noiseless operation at high efficiency.

(ii) The switching speed is very high upto 10^9 operations per second.

(iii) It permits control over large current (30-100A) in the load by means of a small gate current (a few mA).

(iv) It has small size and gives trouble free service.

5.3 Unijunction Transistor (UJT)

Unijunction (abbreviated as UJT) is a three terminal semiconductor switching device. This device has a unique characteristic that when it is triggered, the emitter current increases regeneratively until it is limited by emitter power supply. Due to this characteristic, the unijunction transistor can be employed in a variety of applications e.g., switching, pulse generator, saw-tooth generator etc.

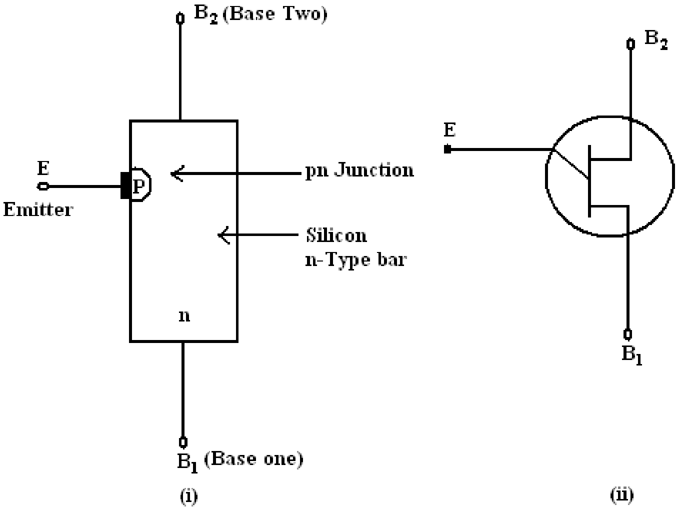


Fig. 5.3

Construction. Fig. 5.3(i) shows the basic structure of a unijunction transistor. It consists of an n-type silicon bar with an electrical connection on each end. The leads to these connections are called base leads base-one B_1 and base two B_2. Part way along the between the two bases, nearer to B_2 than B_1, a pn junction is formed between a p-type emitter and the bar. The lead to this junction is called the emitter lead E. Fig. 5.3(ii) shows the symbol of unijunction transistor. Note that emitter is shown closer to B_2 than B_1.

The following points are worth noting:

(i) Since the device has one pn junction and three leads, it is commonly called a unijunction transistor (uni means single).

(ii) With only one pn-junction, the device is really a form of diode. Because the two base terminals are taken from one section of the diode, this device is also called double-based diode.

(iii) The emitter is heavily doped having many holes. The n region, however, is lightly doped. For this reason, the resistance between the base terminals is very high (5 to 10 kΩ) when emitter lead is open.

Operation.

Fig. 5.3.1 shows the basic circuit operation of a unifunction transistor. The device has normally B_2 positive w.r.t B_1.

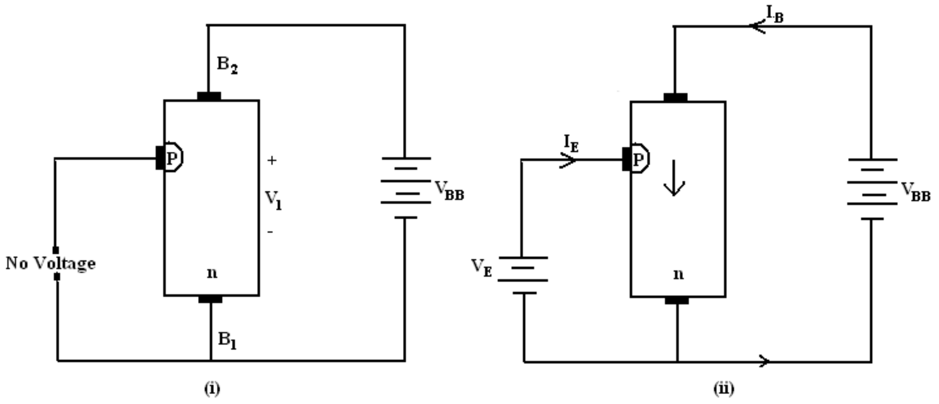


Fig. 5.3.1

(i) If voltage V_{BB} is applied between B_2 and B_1 with emitter open [see fig. 5.3.1(i)], a voltage gradient is established along the n-type bar. Since the emitter is located nearer to B_2, more than half of V_{BB} appears between the emitter and B_1. The voltage V_1 between emitter and B_1 establishes a reverse bias on the pn junction and the emitter current is cut off. Of course, a small leakage current flows from B_2 to emitter due to minority carriers.

(ii) If a positive voltage is applied at the emitter [see Fig. 5.3.1 (ii)], the pn junction will remain reverse biased so long as the input voltage is less than V_1. If the input voltage to the emitter exceeds V_1, the pn junction becomes forward biased. Under these conditions, holes are injected from p-type material into the n-type bar. These holes are repelled by positive B_2 terminal and they are attracted towards B_1 terminal of the bar. This accumulation of holes in the emitter to B_1 region results in the decrease of resistance in this section of the bar. The result is that internal voltage drop from emitter to B_1 is decreased and hence the emitter current I_E increases. As more holes are injected, a condition of saturation will eventually be reached. At this point, the emitter current is limited by emitter power supply only. The device is now in the ON state.

(iii) If a negative pulse is applied to the emitter, the pn junction is reverse biased and the emitter current is cut off. The device is then said to be in the OFF state.

Characteristics of UJT

Fig. 5.3.2a shows the curve between emitter voltage (V_E) and emitter current (I_E) of a UJT at a given voltage V_{BB} between the bases. This is known as the emitter characteristic of UJT. The following points may be noted from the characteristics:

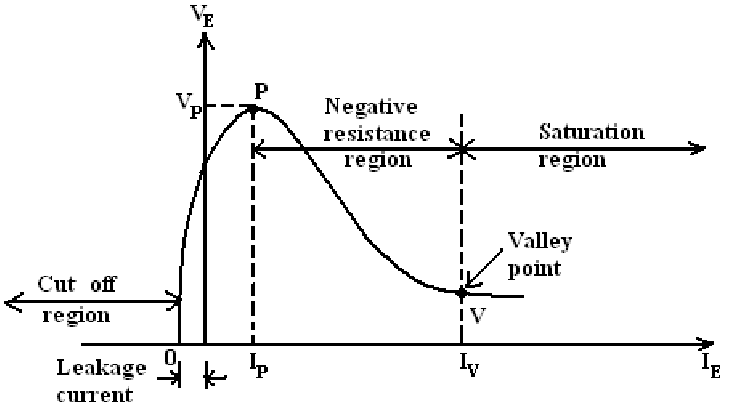


Fig. 5.3.2a

(i) Initially, in the cut-off region, as V_E increases from zero, slightly leakage current flows from terminal B_2 to the emitter. This current is due to the minority carriers in the reverse biased diode.

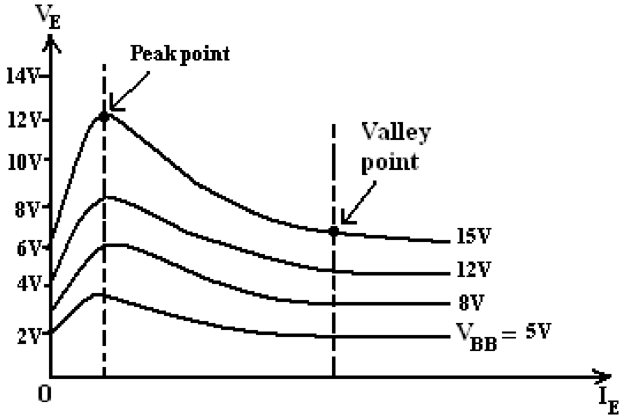


Fig. 5.3.2b

(ii) Above a certain value of V_E, forward I_E begins to flow, increasing until the peak voltage V_P and current I_P are reached at point P.

(iii) After the peak point P, an attempt to increase V_E is followed by a sudden increase in emitter current I_E with a corresponding decrease in V_E. This is a negative resistance portion of the curve because with increase in I_E, V_E decreases. The device, therefore, has a negative resistance region which is stable enough to be used with a great deal of reliability in many areas e.g., trigger circuits, saw-tooth generators, timing circuits.

(iv) The negative portion of the curve lasts until the valley point V is reached wit valley-point voltage V_V and valley-point current I_V. After the valley point, the device is driven to saturation.

Fig. 5.3.2b shows the typical family of V_E/I_E characteristics of a UJT at different voltages between the bases. It is clear that peak-point voltage (= $\eta V_{BB} + V_D$) falls steadily with reducing V_{BB} and so does the valley point voltage V_V. The difference V_P-V_V is a measure of the switching efficiency of UJT and can be seen to fall off as V_{BB} decreases. For a general purpose UJT, the peak-point current is of the order of 1 μA at V_{BB} = 20 V with a valley-point voltage of about 2.5 V at 6 mA.

Example 1. The intrinsic stand-off ratio for a UJT is determined to be 0.6. If the inter-base resistance is 10kΩ, what are the values of R_{B1} and R_{B2}?

Sol. $R_{BB} = 10K\Omega, \eta = 0.6$

Now $R_{BB} = R_{B1} + R_{B2}$

or $10 = R_{B1} + R_{B2}$

Also $$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}}$$

or $$0.6 = \frac{R_{B1}}{10} \qquad (\because R_{B1} + R_{B2} = 10K\Omega)$$

$\because$ $R_{B1} = 10 \times 0.6 = 6K\Omega$

and $R_{B2} = 10 - 6 = 4K\Omega$

Example 2 A unijunction transistor has 10V between the bases. If the intrinsic stand off ratio is 0.65, find the value of stand off voltage. What will be the peak-point voltage if the forward voltage drop in the pn junction is 0.7V?

Sol. $V_{BB} = 10V; \eta = 0.65; V_D = 0.7V$

Stand off voltage = $\eta V_{BB} = 0.65 \times 10 = 6.5V$

Peak-point voltage, $V_P = \eta V_{BB} + V_D = 6.5 + 0.7 = 7.2V$

Advantages of UJT

The UJT was introduced in 1948 but did not become commercially available until 1952. Since then, the device has achieved great popularity due to the following reasons:

(i) It is a low cost device.

(ii) It has excellent characteristics.

(iii) It is a low-power absorbing device under normal operating conditions.

Due to above reasons, this device is being used in a variety of applications. As few include oscillators, trigger circuits, saw-tooth generators, bistable network etc.

Applications of UJT

Unijunction transistors are used extensively in oscillator, pulse and voltage sensing circuits. Some of the important applications of UJT are discussed below:

(i) **UJT relaxation oscillator.** Fig. 5.3.2c shows UJT relaxation where the discharging of a capacitor through UJT can develop a saw-tooth output as shown.

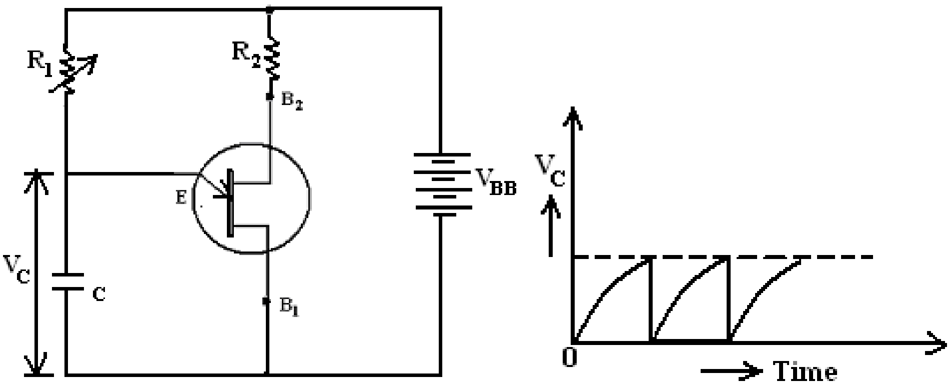


Fig. 5.3.2c

When battery V_{BB} is turned on, the capacitor C charges through resistor R_1. During the charging period, the voltage across the capacitor rises in an exponential manner until it reaches the peak-point voltage. At this instant of time, the UJT switches to its low resistance conducting mode and the capacitor is discharged between E and B_1. As the capacitor voltage flys back to zero, the emitter ceases to conduct and the UJT is switched off. The next cycle then begins, allowing the capacitor C to charge again. The frequency of the output saw-tooth wave can be varied by changing the value of R_1 since the controls the time constant R_1C of the capacitor charging circuit.

The time period and hence the frequency of the saw-tooth wave can be calculated as follows. Assuming that the capacitor is initially uncharged, the voltage V_C across the capacitor prior to breakdown is given by:

$$V_C = V_{BB}\left(1 - e^{-t/R_1C}\right)$$

where $R_1C =$ charging time constant of resistor-capacitor circuit

$t =$ time from the commencement of waveform.

The discharge of the capacitor occurs when V_C us equal to the peak-point voltage ηV_{BB} i.e.

$$\eta V_{BB} = V_{BB}\left(1 - e^{-t/R_1C}\right)$$

or $\eta = 1 - e^{-t/R_1C}$

or $e^{-t/R_1C} = 1 - \eta$

or $t = R_1C \log_e \frac{1}{1-\eta}$

$$\therefore \quad \text{Time period, } t = 2.3R_1C\log_{10}\frac{1}{1-\eta}$$

Frequency of saw-tooth wave, $f = \frac{1}{t \; in \; \text{seconds}} Hz$

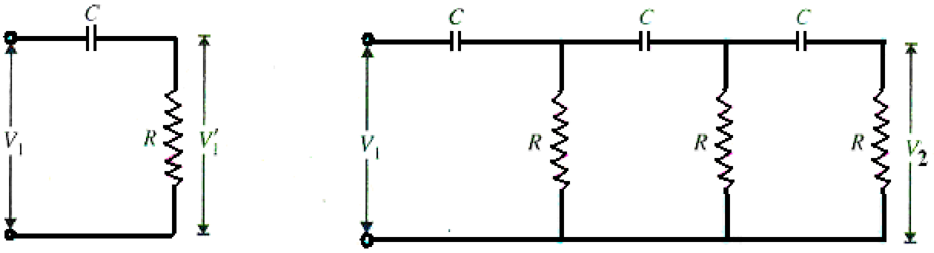
(i) **Overvoltage detector.** A warning pilot-lamp L is connected between the emitter and B_1 circuit. So long as the input voltage is less than the peak-point voltage (V_P) of the UJT, the device remains switched off. However, when the input voltage exceeds V_P, the UJT is switched on and the capacitor discharges through the low resistance path between terminals E and B_1. The current flowing in the pilot lamp L light it, thereby indicating the overvoltage in the circuit.

5.4 Principle of Phase Shift Oscillators

One desirable feature of an oscillator is that it should feedback energy of correct phase to the tank circuit to overcome the losses occurring in it. In the oscillator circuits discussed so far, the tank circuit employed inductive (L) and capacitive (c) elements. In such circuits, a phase shift of 180° was obtained due to inductive or capacitive coupling and a further phase shift of 180° was obtained due to transistor properties. In this way, energy supplied to the tank circuit was in phase with the generated oscillations. The oscillator circuits employing L-C elements have two general drawbacks. Firstly, they suffer from frequency instability and poor waveform. Secondly, they cannot be used for very low frequencies because they become too much bulky and expensive.

Good frequency stability and waveform can be obtained from oscillator employing resistive and capacitive elements. Such amplifiers are called R-C or phase shift oscillators and have the additional advantage that they can be used for very low frequencies. In a phase shift oscillator, a phase shift of 180^o is obtained with a phase shift circuit instead of inductive or capacitive coupling. A further phase shift of 180^o is introduced due to the transistor properties. Thus, energy supplied back to the tank circuit is assured of correct phase.

Phase shift circuit. A phase-shift circuit essentially consists of an R-C network. Fig. 5.4 (i) shows a single section of RC network. From the elementary theory of electrical engineering, it can be shown that alternating voltage V'_1 across R leads the applied voltage V'_1 by $\phi^{o.}$ The value of ϕ depends upon the values of R and C. If resistance R is varied, the value of ϕ also changes. If R were reduced to zero, V'_1 will lead V_1 by 90^o i.e., $\phi = 90^o$. However, adjusting R to zero would be impracticable because it would lead to no voltage across R. Therefore, in practice, R is varied to such a value that makes V'_1 to lead V_1 by 60^o.



(*i*) (*ii*)

Fig. 5.4

Fig. 5.4 (ii) shows the three sections of RC network. Each section produces a phase shift of 60^o. Consequently, a total phase shift of 180^o is produced i.e. voltage V_2 leads the voltage V_1 by 180^o.

Phase shift Oscillator

Fig. 5.4.1 shows the circuit of a phase shift oscillator. It consists of a conventional single transistor amplifier and a RC phase shift network. The phase shift network consists of three sections R_1C_1, R_2C_2 and R_3C_3. At some particular frequency f_0, the phase shift in each RC section is 60^o so that the total phase-shift produced by the RC network is 180^o. The frequency of oscillations is given by:

$$f_0 = \frac{1}{2\pi RC\sqrt{6}} \qquad \ldots(i)$$

where $R_1 = R_2 = R_3 = R$

$$C_1 = C_2 = C_3 = C$$

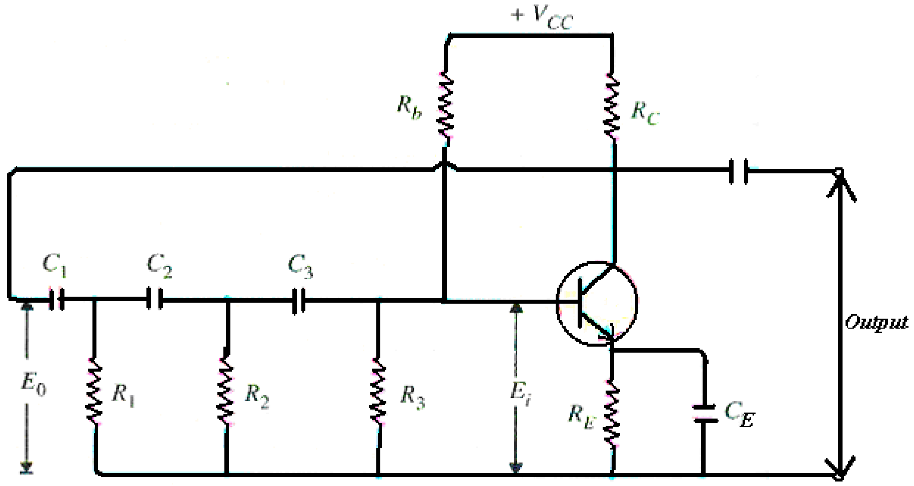


Fig. 5.4.1

Circuit operation. When the circuit is switched on, it produces oscillations of frequency determined by exp. (i). The output E_0 of the amplifier is feedback network. This network produces a phase shift of 180^o and a voltage E_i appears at its output which is applied to the transistor amplifier.

Obviously, the feedback fraction $m = E_i/E_0$. The feedback phase is correct. A phase shift of 180^o is produced by the transistor amplifier. A further phase shift of 180^o is produced by the RC network. As a result, the phase shift around the entire look is 360^o.

Advantages

(i) It does not require transformers or inductors.

(ii) It can be used to produce very low frequencies.

(iii) The circuit provides good frequency stability.

Disadvantages

(ii) It is difficult for the circuit to start oscillations as the feedback is generally small.

(iii) The circuit gives small output.

5.5 Multivibrators

An electronic circuit that generates square waves (or other non-sinusoidals such as rectangular, saw-tooth waves) is known as a multivibrator.

A multivibrator is a switching circuit which depends for operation on positive feedback. It is basically a two-stage amplifier with output of one feedback to the input of the other as shown in Fig. 5.5.

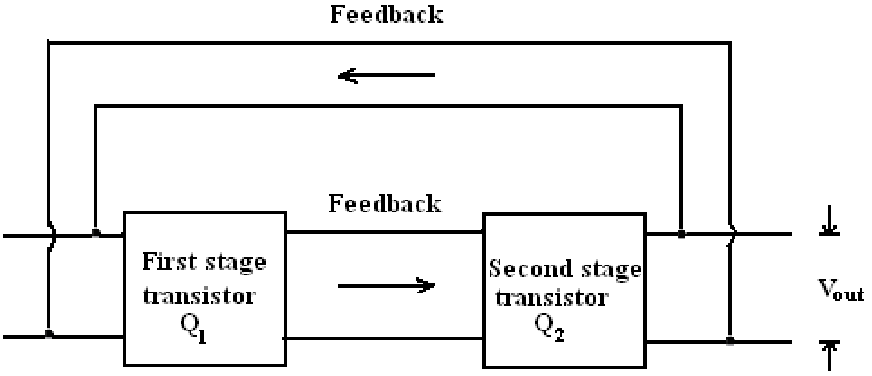


Fig. 5.5

The circuit operates in two states (viz ON and OFF) controlled by circuit conditions. Each amplifier stage supplies feedback to the other in such a manner that will drive the transistor of one stage of saturation (ON state) and the other to cut off (OFF state).

After a certain time controlled by circuit conditions, the action is reversed i.e. saturated stage is driven to cut off and the cut off stage is driven to saturation. The output can be taken across either stage and may be rectangular or square wave depending upon the circuit conditions.

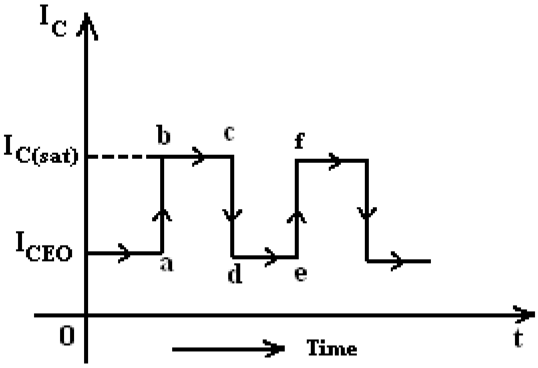


Fig. 5.5.a

Fig. 5.5 shows the block diagram of a multivibrator. It is a two-stage amplifier with 100% positive feedback. Suppose output is taken across the transistor Q_2. At any particular instant, one transistor is ON and conducts $I_{C(sat)}$ while the other is OFF. Suppose Q_2 is ON and Q_1 is OFF. The collector current in Q_2 will be $I_{C(sat)}$ as shown in Fig, 5.5.a. This condition will prevail for a time (bc is this case) determined by circuit conditions. After this time, transistor Q_2 is cut off and Q_1 is turned ON. The collector current in Q_2 is now I_{CEO} as shown. The circuit will stay in this condition for a time de. Again Q_2 is turned ON and Q_1 is driven to cut off. In this way, the output will be a square wave.

Types of Multivibrators

A multivibrator is basically a two-stage amplifier with output of one fedback to the input of the other. At any particular instant, one transistor is ON and the other is OFF. After a certain time depending upon the circuit components, the stages reverse their conditions- the conducting stage suddenly cuts off and the non-conducting stage suddenly starts to conduct. The two possible states of a multivibrator are:

	ON	**OFF**
First State	Q_1	Q_2
Second state	Q_2	Q_1

Depending upon the manner in which the two stages interchange their states, the multivibrators are classified as:

(i) Astable or free running multivibrator

(ii) Monostable or one-shot multivibrator

(iii) Bi-stable or flip-flop multivibrator.

Fig. 5.5.b shows the input/output relations for the three types of multivibrators.

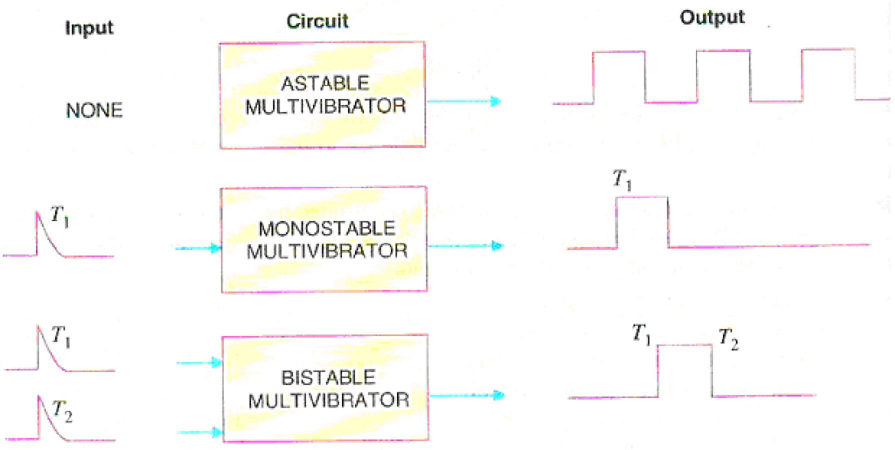


Fig. 5.5.b

(i) The astable or free running multivibrator alternates automatically between the two states and remains in each for a time dependent upon the circuit constants. Thus it is just an oscillator since it requires no external pulse for its operation. Of course, it does require a source of d.c. power. Because it continuously produces the square-wave output, it is often referred to as a **free running multivibrator.**

(ii) The monostable or one-shot multivibrator has one state stable and one quasi-stable (i.e. half-stable) state. The application of input pulse triggers the circuit into its quasi-stable state, in which it remains for a period determined by circuit constants. After this period of time, the circuit returns to its initial stable state, the process is repeated upon the application of each trigger pulse. Since the monostable multivibrator produces a single output pulse for each input trigger pulse, it is generally called **one-shot multivibrator.**

(iii) The bistable multivibrator has both the tow states stable. It requires the application of an external triggering pulse to change the operation from either one state to the other. Thus one pulse is used to generate half-cycle of square wave and another pulse to generate the next half-cycle of square wave. It is also known as a **flip-flop multivibrator** because of the two possible states it can assume.

5.5.1 Transistor Astable Multivibrator

A multivibrator which generates square waves of its own (i.e. without any external triggering pulse) is known as an **astable or free running multivibrator**.

The astable multivibrator has no stable state. It switches back and for the from one state to the other, remaining in each state for a time determined by circuit constants. In other words, at first one transistor conducts (i.e. ON state) and the other stays in the OFF state for some time. After this period of time, the second transistor is automatically turned ON and the first transistor is turned OFF. Thus the multivibrator will generate a square wave output of its own. The width of the square wave and its frequency will depend upon the circuit constants.

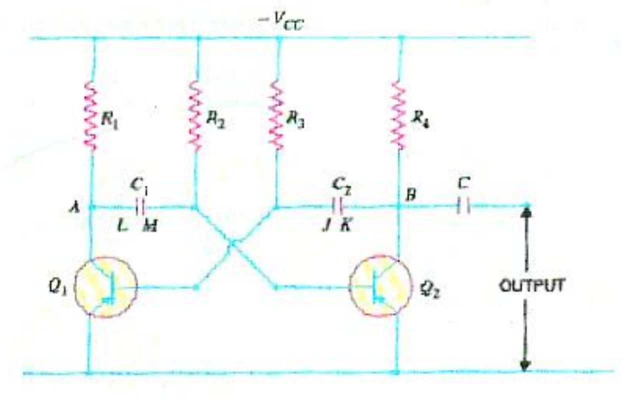


Fig. 5.5.1

Circuit details. Fig. 5.5.1 shows the circuit of a typical transistor astable multivibrator using two identical transistors Q_1 and Q_2. The circuit essentially consists of two symmetrical CE amplifier stages, each providing a feedback to the other. Thus collector loads of the two stages are equal i.e. $R_1 = R_4$ and the biasing resistors are also equal ie. $R_2 = R_3$. The output of transistor Q_1 is coupled to the input of Q_2 through C_1 while the output of Q_2 is fed to the input of Q_1 through C_2. The square wave output can be taken from Q_1 or Q_2.

Operation.

When V_{CC} is applied, collector currents start flowing in Q_1 and Q_2. In addition, the coupling capacitors C_1 and C_2 also start charging up. As the characteristics of no two transistors $(i.e. \beta, V_{BE})$ are exactly alike, therefore, one transistor, say Q_1, will conduct more rapidly than the other. The rising collector current in Q_1 drives its collector more and more positive. The increasing positive output at point A is applied to the base of transistor Q_2 through C_1. This establishes a reverse bias on Q_2 and its collector current starts decreasing. As the collector of Q_2 is connected to the base of Q_1 through C_2, therefore, base of Q_1 becomes more negative i.e. Q_1 is more forward biased. This further increases the collector current in Q_1 and causes a further decrease of collector current in Q_2. This series of actions is repeated until the circuit drives Q_1 to saturation and Q_2 to cut off. These actions occur very rapidly and may be considered practically instantaneous. The output of Q_1 (ON state) is approximately zero and that of Q_2 (OFF state) is approximately V_{CC}. This is shown by ab in Fig. 5.5.1a

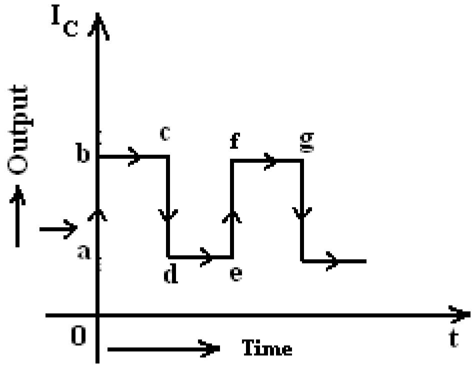


Fig. 5.5.1a

When Q_1 is at saturation and Q_2 is cut off, the full voltage V_{CC} appears across R_1 and voltage across R_4 will be zero. The charges developed across C_1 and C_2 are sufficient to maintain the saturation and cut off conditions at Q_1 and Q_2 respectively. This condition is represented by time interval bc in Fig. 5.5.1a. However, the capacitors will not retain the charges indefinitely but will discharge through their respective circuits. The discharge path for C_1, with plate L negative and Q_1 conducting, is $LAQ_1V_{CC}R_2M$ as shown in Fig. 5.5.1a(i).

The discharge path for C_2, with plate K negative Q_2 cut off, KBR_4R_3J as shown in fig. 5.5.1a(ii). As the resistance of the discharge path for C_1 is lower that that of C_2, therefore, C_1 will discharge more rapidly.

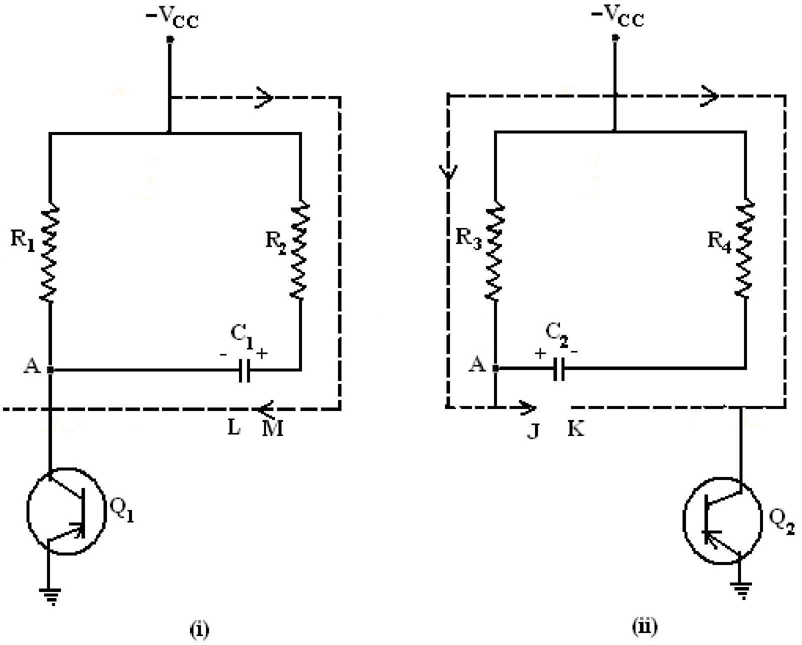


Fig. 5.5.1.a

As C_1 discharges, the base bias at Q_2 becomes less positive and at a time determined by R_2 and C_1, forward bias is re-established at Q_2. This causes the collector current to start in Q_2. The increasing positive potential at collector of Q_2 is applied to the base of through the capacitor C_2. Hence the base of Q_1 will become more positive i.e. Q_1 is reverse biased. The decrease in collector current in Q_1 sends a negative voltage to the base of Q_2 through C_1, thereby causing further increase in the collector current of Q_2. With this set of actions taking place, Q_2 is quickly driven to saturation and Q_1 to cut off. This condition is represented by cd in fig. 5.5.1a. The period of time during which Q_2 remains at saturation and Q_1 at cut off is determined by C_2 and R_3.

ON or OFF time: The time for which either transistor remains ON or OFF is given by

ON time for Q_1 (or OFF time for Q_2) is

$$T_1 = 0.694 R_2 C_1$$

OFF time for Q_1 (or ON time for Q_2) is

$$T_2 = 0.694 R_3 C_2$$

Total time period of the square wave is

$$T = T_1 + T_2 = 0.694\left(R_2 C_1 + R_3 C_2\right)$$

As $R_2 = R_3 = R$ and $C_1 = C_2 = C$,

$\therefore \qquad T = 0.694\left(RC + RC\right) \simeq 1.4RC \sec onds$

Frequency of the square wave is

$$f = \frac{1}{T} \simeq \frac{0.7}{RC} Hz$$

It may be noted that in these expressions, R is in ohms and C in farad.

Example 2. In the astable multivibrator shown in Fig. 21.12, $R_2 = R_3 = 10K\Omega$ and $C_1 = C_2 = 0.01\mu F$. Determine the time period and frequency of the square wave.

Sol

Here $R = 10K\Omega = 10^4\Omega; C = 0.01\mu F = 10^{-8} F$

Time period of the square wave is

$$T = 1.4RC = 1.4 \times 10^4 \times 10^{-8} \sec ond$$

$$= 1.4 \times 10^{-4} \sec ond = 1.4 \times 10^{-4} \times 10^3 m \sec$$

$$= 0.14m \sec$$

Frequency of the square wave is

$$f = \frac{1}{T \text{ in second}} Hz = \frac{1}{1.4 \times 10^{-4}} Hz$$

$$= 7 \times 10^3 Hz = 7kHz$$

5.5.2 Transistor Monostable Multivibrator

A multivibrator in which one transistor is always conducting (i.e. in the ON state) and the other is non-conducting (i.e. in the OFF state) is called a **monostable multivibrator.**

A monostable multivibrator has only one state stable. In other words, if one transistor is conducting and the other is non-conducting, the circuit will remain in this position. It is only with the application of external pulse that the circuit will interchange the states. However, after a certain time, the circuit will automatically switch back to the original stable state and remains there until another pulse is applied. Thus a monostable multivibrator cannot generate square waves of its own like an astable multivibrator. Only external pulse will cause it to generate the square wave.

Circuit details:

Fig, 5.5.2 shows the circuit of a transistor monostable multivibrator. It consists of two similar transistors Q_1 and Q_2 with equal collector loads i.e. $R_1 = R_4$. The values of V_{BB} and R_5 are such as to reverse bias Q_1 and keep it at cut off. The collector supply V_{CC} and R_2 forward bias Q_2 and keep it at saturation. The input pulse is given through C_2 to obtain the square wave. Again output can be taken from Q_1 or Q_2.

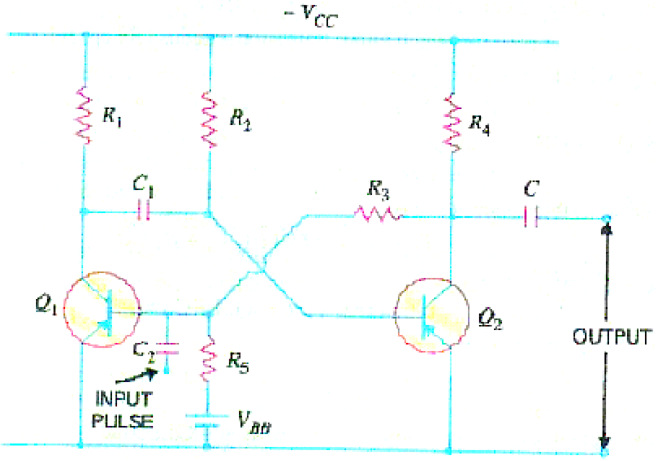
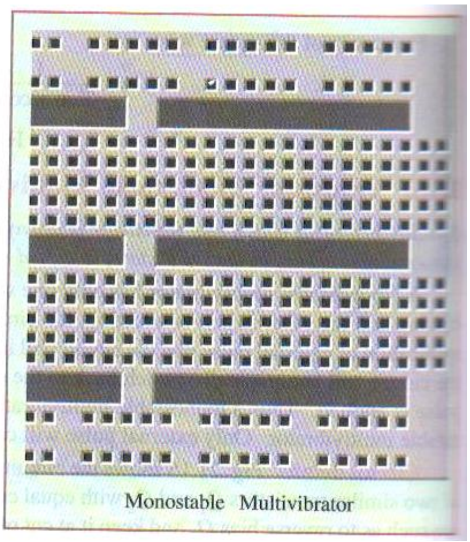


Fig. 5.5.2

Operation. With the circuit arrangement shown, Q_1 is at cut off and Q_2 is at saturation. This is the stable state for the circuit and it will continue to stray in this state until a triggering pulse is applied at C_2. When a negative pulse of short duration and sufficient magnitude is applied to the base of Q_1 through C_2, the transistor Q_1 starts conducting and positive potential is established at its collector. The positive potential at the collector of Q_1 is coupled to the base of Q_2 through capacitor C_1. This decreases the forward bias on Q_2 and its collector current decreases. The increasing negative potential on the collector of Q_2 is applied to the base of Q_1 through R_3. This further increases the forward bias on Q_1 and hence its collector current. With this set of actions taking place, Q_1 is quickly driven to saturation and Q_2 to cut off.



With Q_1 at saturation and Q_2 at cut off, the circuit will come back to the original stage, (i.e. Q_2 at saturation and Q_1 at cut off) after some time as explained in the following discussion. The capacitor C_1 (charged to approximately V_{CC}) discharges through the path $R_2V_{CC}Q_1$. As C_1 discharges, it sends a voltage to the base of Q_2 to make it less positive. This goes on until a point is reached when forward bias is re-established on Q_2 and collector current starts to flow in Q_2. The step by step events already explained occur and Q_2 is quickly driven to saturation and Q_1 to cut off. This is the stable state for the circuit and it remains in this condition until another pulse causes the circuit to switch over the states.

5.5.3 Transistor Bistable Multivibrator

A multivibrator which has both the states stable is called a **bistable multivibrator.**

The bistable multivibrator has both the states stable. For instance, suppose at any particular instant, transistor Q_1 is conducting and transistor Q_2 is at cut off. It left to itself, the bistable multivibrator will stay in this position forever. However, if an external pulse is applied to the circuit in such a way that Q_1 is cut off and Q_2 is tuned on, the circuit will stay in the new position. Another trigger pulse is then required to switch the circuit back to its original state.

Circuit details.

Fig. 5.5.3. shows the circuit of a typical transistor bistable multivibrator. It consists of two identical CE amplifier stages with output of one fed to the input of the other. The feedback is coupled through resistors (R_2, R_3) shunted by capacitors C_1 and C_2. The main purpose of capacitors C_1 and C_2 is to improve the switching characteristics of the circuit by passing the high frequency components of the square wave. this allows fast rise and fall times and hence distortionless square wave output. The output can be taken across either transistor.

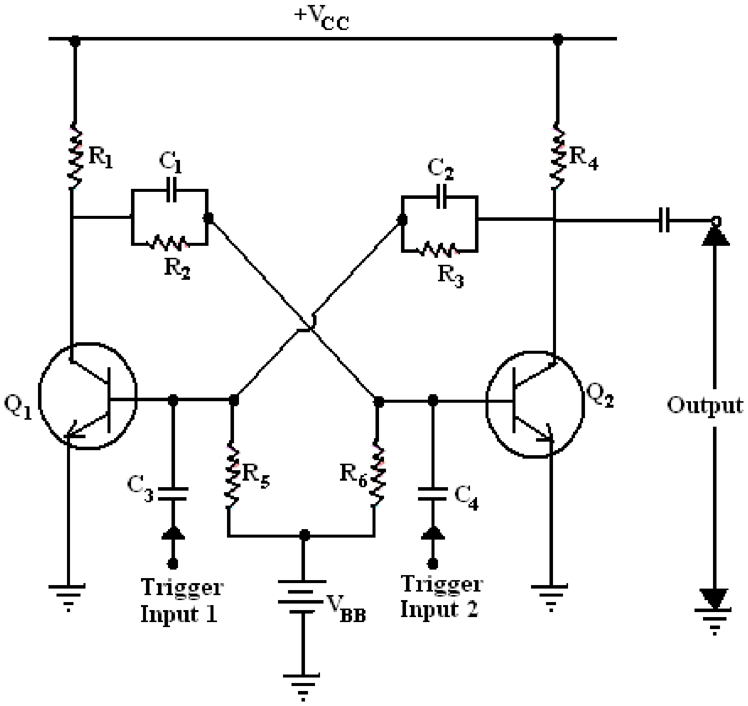


Fig. 5.5.3

Operation.

When V_{CC} is applied, one transistor will start conducting slightly ahead of the other due to some difference in the characteristics of the transistors. This will drive one transistor to saturation and the other to cut off in a manner described for the astable multivibrator. Assume that Q_1 is turned ON and Q_2 is cut OFF. If left to itself, the circuit will stay in this condition. In order to the base of Q_1 through C_3 will cut it off or a positive pulse applied to the base of Q_2 through C_4 will cause it to conduct.

Suppose a negative pulse of sufficient magnitude is applied to the base of Q_1 through C_3. This will reduce the forward bias on Q_1 and cause a decrease in its collector current and an increase in collector voltage. The rising collector voltage is coupled to the base of Q_2 where it forward biases the base-emitter junction of Q_2. This will cause an increase in its collector current and decrease in collector voltage. The decreasing collector voltage is applied to the base of Q_1 where it further reverse biases the base-emitter junction Q_1 to decrease its collector current. With this set of actions taking place, Q_2 is quickly driven to saturation and Q_1 to cut off. The circuit will now remain stable in this state until a negative trigger pulse at Q_2 (or a positive trigger pulse at Q_1) changes this state.

Number Systems

5.6 Decimal Number System

This is the frequently used number system in our daily life. It uses ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The base or the radix of the decimal system is 10. A number in decimal system is expressed in terms of the position or place values.

For example, the number 10523 is represented as

$$10523 = 1\times10^4+0\times10^3+5\times10^2+2\times10^1+3\times10^0.$$

The digit (3) has the position value 10^0 and is the least significant digit (LSD).

The digit (1) has the place value 10^4 and is the most significant digit (MSD).

Similarly, the number 2564.397 can be expressed as

$$2564.397 = 2\times10^3+5\times10^2+6\times10^1+4\times10^0+3\times10^{-1}+9\times10^{-2}+7\times10^{-3}.$$

That is the powers to the base 10 are numbered to the left of the decimal point starting with 0 and to the right of the decimal point starting with -1.

Binary Number System

In a binary system of representation the base (or radix) is 2. It uses only two numerals 0 and 1. In a digital system there are only two possible state or conditions. For example, a situation may be True or False, a switch close or open, a voltage signal High or Low etc. These sates or conditions are designated as 1 and 0 respectively. The binary digits 0 and 1 are termed as bits. Like the decimal system, the binary system also has a place or position value representation.

For example, the number 15 of decimal system is written in the binary system as 1111, since,

$$1111 = 1\times2^3+1\times2^2+1\times2^1+1\times2^0 = 15.$$

This can also be written as $1111_2 = 15_{10}$, the subscript indicating the number system. In the binary 1111, the bit 1 at the extreme left is the MSB (most significant bit) and the bit 1 at the extreme right is the LSB.

Similarly, the binary 1011.011 is written in the decimal system as 11.375, since

$$\begin{aligned}1011.011 &= 1\times2^3+0\times2^2+1\times2^1+1\times2^0+0\times2^{-1}+1\times2^{-2}+1\times2^{-3}\\ &= 8+0+2+1+0+\frac{1}{4}+\frac{1}{8} = 11.375\end{aligned}$$

or $\quad 1011.011_2 = 11.375_{10}$

Thus, the place values of the bits in a binary number are given by ascending powers of 2 to the left of binary point starting from 0 and to the right of binary point in the descending powers of 2 starting from -1.

Conversion of Binary Number into Decimal Number

(i) Conversion of integral binary numbers

Example 1. Convert $(1010)_2$ into its decimal equivalent.

$$1010 = 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 = 8+0+2+0 = 10$$

$$\Rightarrow (1010)_2 = (10)_{10}$$

(ii) Conversion of fractional binary numbers

As an example, let us find the decimal equivalent of 0.1101.

$$0.1101 = 1\times 2^{-1} + 1\times 2^{-2} + 0\times 2^{-3} + 1\times 2^{-4}$$

$$= \frac{1}{2} + \frac{1}{4} + 0 + \frac{1}{16} = 0.8125$$

$$\therefore \quad \underset{\substack{\downarrow \\ \text{Binary} \\ \text{point}}}{0.1101_2} = \underset{\substack{\downarrow \\ \text{Decimal} \\ \text{point}}}{0.8125_{10}}$$

(iii) Mixed Number

For mixed numbers, that is, having integer and fractional parts, each part is handled separately according to the following rules.

(i) The integer numbers are multiplied by their corresponding powers of base 2, i.e., $2^0, 2^1, 2^2, \ldots$ from left of the binary point and then added.

(ii) The fractional numbers are multiplied by their corresponding powers of base 2, i.e., $2^{-1}, 2^{-2}, 2^{-3}, \ldots$ from right of the binary point and then added.

Consider the binary number 1101. 101. Its decimal equivalent is

$$1101.101 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3}$$

$$= 8+4+0+1+\frac{1}{2}+0+\frac{1}{8} = 13.625$$

Example 2. Convert $(1011.0101)_2$ into its decimal equivalent.

$$\begin{aligned}1011.0101 &= 1\times2^{0}+1\times2^{1}+0\times2^{2}+1\times2^{3}+0\times2^{-1}+1\times2^{-2}+0\times2^{-3}+1\times2^{-4}\\ &=1+2+0+8+0+\frac{1}{4}+0+\frac{1}{16}\\ &=11.3125\end{aligned}$$

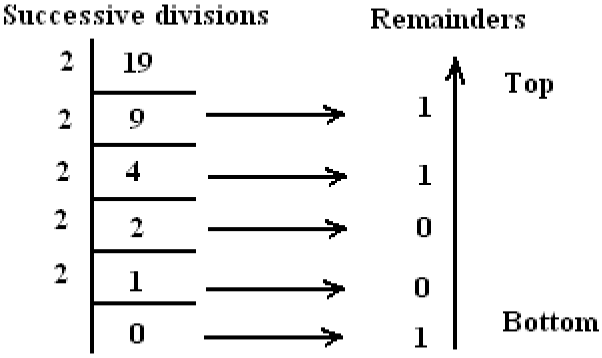
$$(1011.0101)_2 = \left(11.3125\right)_{10}$$

Conversion of Decimal number into Binary number

(i) Conversions of integral decimal numbers

The given decimal number is divided progressively by 2, until we get zero. The remainders, taken in the reverse order, give the binary number.

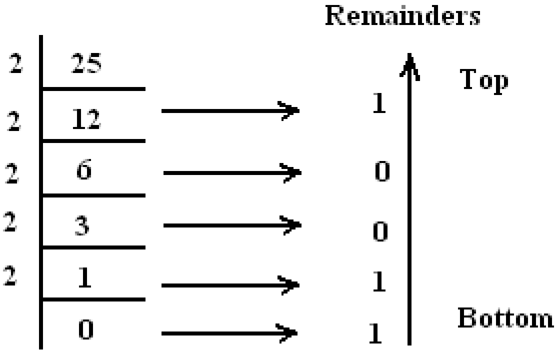
As an example let us covalent the decimal 19 into its binary equivalent.



Reading the remainders from the bottom to the top, the binary equivalent of 19 is found to be 10011, or

$19_{10} = 10011_2$.

Example 3. Convert $(25)_{10}$ into binary number.



$(25)_{10} = (11001)_2$

(ii) Conversion of fractional – decimal numbers

The given decimal number is multiplied by 2 progressively. For each step that results in a 1 in the unit place, transfer the 1 to the binary record and repeat the process with the fractional number. For each multiplication by 2 that results in a product less than unity, record a 0 in the binary number and carry on the process. The last step is reached if the fractional part is zero or it is terminated when the desired accuracy is attained. The carries are taken in the forward (top to bottom) direction to give the equivalent binary.

Let us convert 0.9125 into its binary equivalent.

$$\begin{array}{l}
0.9125\times 2 = 1.8250 = 0.8250 \text{ with a carry } 1 \\
0.8250\times 2 = 1.6500 = 0.6500 \text{ with a carry } 1 \\
0.6500\times 2 = 1.3000 = 0.3000 \text{ with a carry } 1 \\
0.3000\times 2 = 0.6000 = 0.6000 \text{ with a carry } 0 \\
0.6000\times 2 = 1.2000 = 0.2000 \text{ with a carry } 1 \\
0.2000\times 2 = 0.4000 = 0.4000 \text{ with a carry } 0
\end{array} \Bigg\downarrow$$

The process is terminated here to get an approximate result, namely, representation of 0.9125 by six binary digits.

$\therefore \quad 0.9125_{10} = 0.111010_2$

The point in front of the binary is referred to as the binary point.

To find the binary equivalent of a decimal number like 35.625, we split the number into an integer of 35 and a fraction of 0.625. Then the binary equivalent of each part is obtained separately by methods discussed above.

First let us find out the binary equivalent of the integer part 35 by divide-by-two method.

$$\begin{array}{lll}
35\div 2 = 17+1 & \text{remainder } 1 & \text{Top} \\
17\div 2 = 8+1 & \text{remainder } 1 & \uparrow \\
8\div 2 = 4+0 & \text{remainder } 0 & \\
4\div 2 = 2+0 & \text{remainder } 0 & \\
2\div 2 = 1+0 & \text{remainder } 0 & \\
1\div 2 = 0+1 & \text{remainder } 1 & \\
35_{10} = 100011_2 & &
\end{array}$$

The binary equivalent of 0.625 is found by the multiple-by-two method,

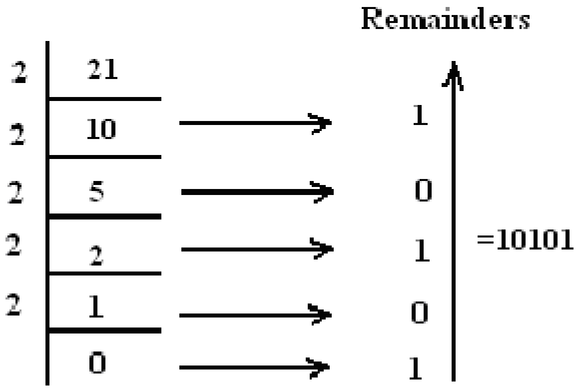
$$\begin{array}{lllll}
0.625\times 2 = 1.25 & = 1+0.25 & \text{carry}1 & \text{Top} \\
0.25\times 2 = 0.50 & = 0+0.50 & \text{carry}0 & \downarrow \\
0.50\times 2 = 1.0 & = 0+1 & \text{carry}1 & \text{Bottom}
\end{array}$$

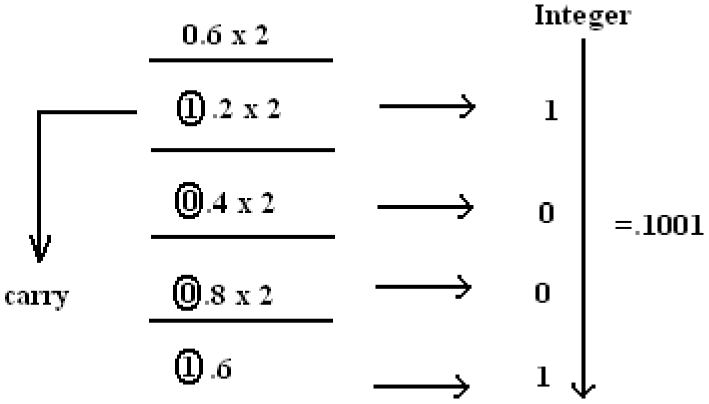
Taking the carrys from top to bottom

$0.625_{10} = 101$

$\therefore \qquad 35.625_{10} = 100011.101_2$

Example 1. Convert $(21.6)_{10}$ into binary number.





$$(21.6)_{10} = (10101.1001)_2$$

Binary Multiplication

The following are the rules for binary multiplication

(*i*) $0 \times 0 = 0$
(*ii*) $0 \times 1 = 0$
(*iii*) $1 \times 0 = 0$
(*iv*) $1 \times 1 = 1$

Binary multiplication is carried out as in decimal system.

Example 1. Multiply 10110 by 110.

10110×110	
00000	*Verification*
10110	22×6
10110	132
10000100	

$\therefore$ The result is 10000100.

Binary Division

Binary division is done as in decimal system.

Example 2. Divide 1111 by 110.

Verification

$15 \div 6$

$$\begin{array}{r|l} & \quad 10 \\ \hline 110 & 1111 \\ & 110 \\ \hline & \quad 11 \end{array} \qquad\qquad \begin{array}{r|l} & 2 \\ \hline 6 & 15 \\ \hline & 12 \\ \hline & 3 \end{array}$$

Quotient: 10	Quotient: 2
Remainder: 11	Remainder: 3

Octal Number System

The radix or base of this system is 8. It uses the eight numerals 9, 1,2,3,4,5,6,7. For counting beyond 7, 2-digit combinations are formed taking the second digit followed by the first, then the second digit followed by the second and so on. For example, after 7, the next number in the octal system is 10 (second digit followed by the first), then 11 (second digit followed by the second). The position value of each digit is in ascending powers of 8 for integers and descending powers of 8 for fractions as shown below:

$$\leftarrow 8^3 \quad 8^2 \quad 8^1 \quad 8^0 \quad \underset{\substack{\uparrow \\ \text{Octal point}}}{.} \quad 8^{-1} \quad 8^{-2} \quad 8^{-3} \rightarrow$$

For example, 425.350 in the octal system can be expressed in decimal system in the following way.

$$425.350 = 4\times 8^2 + 2\times 8^1 + 5\times 8^0 + 3\times 8^{-1} + 5\times 8^{-2} + 0\times 8^{-3}$$

$$= 4\times 64 + 2\times 8 + 5\times 1 + 3\times\frac{1}{8} + 5\times\frac{1}{64}$$

$$= 256 + 16 + 5 + 0.375 + 0.078 = 277.453_{10}$$

Hexadecima Number system

In the hexadecimal system the radix or baste is 16. It uses the sixteen numerals

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

Thus, A represents 10, B represents 11, … F represents 15. After reaching F, two digit combinations are formed taking the second digit followed by the first, then the second followed by the second and so on. For example, in the hexadecimal system 10 (second digit followed by the first represents 16 in the decimal system. The position value for each digit is in ascending powers of 26 for integers and descending powers of 16 for fractions.

Example of Hexadecimal to Decimal conversion

(i) $3C8^{16} = 3\times16^2 + 12\times16^1 + 8\times16^0$

$= 768 + 192 + 8 = 968_{10}$

(ii) $E5F8_{16} = 14\times16^3 + 5\times16^2 + 15\times16^1 + 8\times16^0$

$= 57344 + 1270 + 240 + 8 = 58.872_{10}$

5.7 Boolean Algebra

Digital circuit perform the binary arithmetic operations with binary digits 1 and 0. These operations are called logic functions or logical operations. The algebra used to symbolically describe logic functions is called Boolean algebra. Boolean algebra is a set of rules and theorems by which logical operation can be expressed symbolically in equation form and be manipulated mathematically. As with the ordinary algebra, the letters of alphabet (e.g. A, B, C etc.) can be used to represent the variables. Boolean algebra differs from ordinary algebra in that Boolean constant and variables can have only two values; 0 and 1. There are four connecting symbols used in Boolean algebra viz.

(i) equals sign (=) (ii) Plus sign (+)
(iii) multiply sign (.) (iv) Bar (-)

(i) **Equals sign (=).** The equals sign in Boolean algebra refers to the standard mathematical equality. In other words, the logical value on one side of the sign is identical to the logical value on the other side of the sign. Suppose we are given two logical variables such that A = B. Then if A = 1, then B =1 and if A =0, then B =0.

(ii) **Plus sign (+).** The plus sign is Boolean algebra refers to the logical OR operation. Thus, when the statement A+B=1 appears in Boolean algebra, it means A ORed with B equals 1. Consequently, either A =1or B =1or both equal 1.

(iii) **Multiply sign (.).** The multiply sign in Boolean algebra refers to AND operation. Thus, when the statement A.B =1 appears in Boolean algebra, it means A ANDed with B equals 1. Consequently, A =1 and B =1. The function A.B is often written as AB, omitting the dot for convenience.

(iv) **Bar sign (-).** The bar sign in Boolean algebra refers to NOT operation. The NOT has the effect of inverting (complementing) the logical value. Thus, if A =1, then $\overline{A}=0$.

Boolean Algebra

The algebra of logic prominently used in the operation of computer devices is the algebra developed by George Boole. It is a binary or two valued logic, i.e., it permits only two values or states for its variables. These two states are ‘true’ and ‘false’ in logic but are represented by ‘on’ and ‘off’ states of electronic circuits.

The two variables of the Boolean algebra are usually represented by 0 and 1. Hence, every variable is either a 0 or a 1. There are no negative or fractional numbers. Logically, we may write:

If $X=0$ then $X\neq 1$

And If $X=1$ then $X\neq 0$.

Boolean algebra uses only three operations on its variables. These operations are:

(i) The OR addition represented by a + (plus) sign.

(ii) The AND multiplication represented by a × (cross) or a.(dot) sign. We will normally write the adjacent letters without the dot such as AB.

(iii) The NOT operation represented by a bar over a variable.

The rules for OR, AND and NOT operations of Boolean algebra are summarized in Table 10.1.

Table 10.1 Rules of Boolean algebra OR, AND and NOT operations

OR operation	AND operation	NOT operation
0+0=0 0+1=1 1+0=1 1+1=1	0.0=0 0.1=0 1.0=0 1.1=1	$\overline{0}=1$ $\overline{1}=0$

Postulates and theorems of Boolean Algebra

1. **OR Laws:** OR laws in general form may be expressed as

$(i) A+0=A$

$(ii) A+1=1$

$(iii) A+A=A$

$(iv) A+\overline{A}=1$

2. **AND Laws:**

$(v) A.0 = 0$

$(vi) A.1 = A$

$(vii) A.A = A$

$(viii) A.\overline{A} = 0$

2. Complementation (or NOT) Laws:

$(ix) \overline{0} = 0$

$(x) \overline{1} = 0$

$(xi) If$ A=0, then $\overline{A} = 1$

$(xii) If$ A=1, then $\overline{A} = 0$

$(xiii) \overline{\overline{A}} = A$

Here $\overline{\overline{A}}$ refers to double complement operation.

Rule (xiii) states that if a variable is complemented twice, the result is the variable itself.

3. Commutative Laws:

$(xiv) A + B = B + A$

$(xv) A.B = B.A$

4. Associative Laws:

$(xvi) A + (B + C) = (A + B) + C$

$(xvii) (A + B) + (C + D) = A + B + C + D$

$(xviii) A.(B.C) = (A.B).C$

Thus removal of brackets from logical expressions and regrouping of variables is allowed.

5. Distributive Laws:

$(xix) A.(B + C) = A.B + A.C$

$(xx) A + B.C = (A + B).(A + C)$

$(xxi) A + \overline{A}.B = A + B$

Thus factoring or multiplying out of expressions is permitted.

6. Absorptive Laws:

$(xxii) A + A.B = A$

$(xxiii) A.(A + B) = A$

$(xxiv) A.(\overline{A} + B) = AB$

Each of these identities can be proved by substituting the two possible values of the variables i.e., 0 and 1, on each side of the identity. In each case, the left hand side will come out to be equal to the right hand side.

Example 1. Prove that (A+B)(A+C)=A+BC

Sol L.H.S.

$$(A+B)(A+C)=AA+AC+BA+BC \qquad \ldots\text{Distributive Law}$$

$$=A+AC+AB+BC \qquad (\because \text{A.A=A})$$

$$=A(1+B)+AC+BC$$

$$=A+AC+BC \qquad (\because 1+\text{B}=1)$$

$$=\text{A}(1+\text{C})+\text{BC}$$

$$=\text{A}+\text{BC}=\text{R.H.S.}$$

Example 2. Using Boolean Algebra show that

$$\text{AC}+\text{ABC}=\text{AC}$$

Sol. L.H.S.

$$\text{AC}+\text{ABC}=\text{AC}(1+\text{B})$$

$$=\text{AC}.1 \qquad (\because 1+\text{B}=1)$$

$$\text{AC}=\text{R.H.S} \qquad (\because C.1=\text{C})$$

Example 3. Using Boolean Algebra show that

$$\text{A}+\overline{A}.B=A+B$$

Sol. L.H.S.

$$\text{A}+\overline{A}.B=A.1+\overline{A}.B \qquad (\because A.1=A)$$

$$=A(1+\text{B})+\overline{A}.B$$

$$=A.1+A.B+\overline{A}.B \qquad \text{(Distributive law)}$$

$$=A+B(A+\overline{A})$$

(Commutative law and distributive law)

$$=A+B.1$$

$$=A+B$$

Example 4. Reduce $AB+ABC+\overline{A}B+A\overline{B}C$ using laws of Boolean algebra.

Sol. L.H.S. $AB+ABC+\overline{A}B+A\overline{B}C$

$$=AB+ABC+\overline{A}B+A\overline{B}C \qquad \text{(Commutative law)}$$

$$=B(A+\overline{A})+AC(B+\overline{B})$$

(Distributive law and commutative law)

$$= B.1 + AC.1 \qquad (\because A+\overline{A}=B+\overline{B}=1)$$

$$=B+AC$$

$$\therefore AB+ABC+\overline{A}B + A\overline{B}C = B + AC$$

Example 5. Prove that Boolean expression $Y = ABC + A\overline{B}C + AB\overline{C}$ is equal to the Boolean expression Y=(B+C).

Sol. L.H.S.

$$ABC + A\overline{B}C + AB\overline{C} = AC(B+\overline{B}) + AB\overline{C}$$

$$= AC + AB\overline{C} \qquad (\because B+\overline{B}=1)$$

$$= A(C + B\overline{C})$$

$$= A(B+C) \qquad (\because C+B\overline{C}=C+B)$$

Example 6. Show that

$(i) AC + ABC = AC$

$(ii) A + AB = A$

Sol.

$(i) AC + ABC = AC(1+B) = AC \qquad (\because 1+B=1)$

$(ii) A + AB = A(1+B) = A$

Example 7. Show that $A+\overline{A}=1$.

Sol. This identity can be proved by substituting on both sides, the two possible values (0 and 1) of the variable, i.e., A. A can have only two values, 0 and 1.

When $A=0, A+\overline{A}=0+\overline{0}=0+1=1.$

When $A=1, A+\overline{A}=1+\overline{1}=1+0=1.$

Hence $A+\overline{A}=1$ for all possible values of A.

Example 8. Prove the following Boolean identity

$$(A+B)(A+\overline{B})(\overline{A}+C) = AC.$$

Sol. L.H.S.

$$(A+B)(A+\overline{B})(\overline{A}+C)$$

$$= (AA + A\overline{B} + BA + B\overline{B})(\overline{A}+C)$$

$$= (A + A\overline{B} + AB)(\overline{A}+C) \qquad (\because B\overline{B}=0)$$

$$=[A(1+B)+A\overline{B}(\overline{A}+C)$$

$$= (A + A\overline{B})(\overline{A}+C) \qquad (\because 1+B=1)$$

$=\text{A}(1+\overline{B})(\overline{A}+C)$

$=(A.1)(\overline{A}+C)$ $\qquad (\because 1+\overline{B}=1)$

$\text{A}(\overline{A}+C)$

$\text{A}\overline{A}+AC$

$AC=R.H.S$ $\qquad (\because \text{A}\overline{A}=0)$

5.8 De Morgan's Theorem

Theorem 1

Statement: The complement of the sum of two or more variables is equal to the product of the complements of the variables.

For two variables A and B,

$$\overline{A+B}=\overline{A}.\overline{B}$$

Proof: This theorem can be proved by substituting the two permitted values of the variables i.e., 0 and 1 on both sides of the identity.

In each case, the left hand side will equal the right hand side, thus proving the theorem.

Case 1. When $A=0, B=0$

$$L.H.S=\overline{A+B}=\overline{0+0}=\overline{0}=1.$$

$$R.H.S=\overline{A}.\overline{B}=\overline{0}.\overline{0}=1.1=1.$$

Case 2. When $A=0, B=1$

$$L.H.S=\overline{A+B}=\overline{0+1}=\overline{1}=0.$$

$$R.H.S=\overline{A}.\overline{B}=\overline{0}.\overline{1}=1.0=0.$$

Case 3. When $A=1, B=0$

$$L.H.S=\overline{A+B}=\overline{1+0}=\overline{1}=0.$$

$$R.H.S=\overline{A}.\overline{B}=\overline{1}.\overline{0}=0.1=0.$$

Case 4. When $A=1, B=1$

$$L.H.S=\overline{A+B}=\overline{1+1}=\overline{1}=0.$$

$$R.H.S=\overline{A}.\overline{B}=\overline{1}.\overline{1}=0.0=0.$$

In every case, left hand side (L.H.S.) is equal to right hand side (R.H.S).

Therefore, the theorem is proved.

Theorem 2

Statement: The complement of the product of two or more variables is equal to the sum of the complements of the variables.

For two variables A and B.

$$\overline{A.B} = \overline{A} + \overline{B}$$

Proof: This theorem can be proved by substituting the two permitted values of the variables i.e., 0 and 1 on both sides of the identity.

In each case, the left hand side will equal the right hand side, thus proving this theorem.

Case 1. When $A = 0, B = 0$

$$L.H.S = \overline{A.B} = \overline{0.0} = \overline{0} = 1.$$

$$R.H.S = \overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 + 1 = 1.$$

Case 2. When $A = 0, B = 1$

$$L.H.S = \overline{A.B} = \overline{0.1} = \overline{0} = 1.$$

$$R.H.S = \overline{A} + \overline{B} = \overline{0} + \overline{1} = 1 + 0 = 1.$$

Case 3. When $A = 1, B = 0$

$$L.H.S = \overline{A.B} = \overline{1.0} = \overline{0} = 1.$$

$$R.H.S = \overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1.$$

Case 4. When $A = 1, B = 1$

$$L.H.S = \overline{A.B} = \overline{1.1} = \overline{1} = 0.$$

$$R.H.S = \overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0.$$

In every case, left hand side of the expression is equal to right hand side of the expression. Therefore, the theorem is proved.

5.9 NAND Gates is a Universal gate

A NAND gate is known as a universal gate because it can be used to realize all the three basic logic functions of an OR gate, ANDN gate and Not gate. Thus, in digital circuits, NAND gate serves as a building block.

1. As OR Gate: OR gate can be made out of the three NAND gates (Fig. 1).

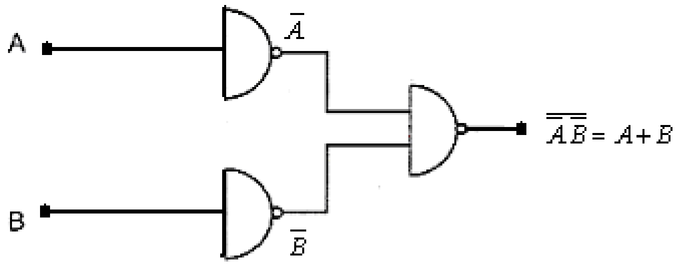


Fig. 1

However, the OR function may not be very clear from the figure because DE Morgan's theorem is needed to prove that $\overline{\overline{A}.\overline{B}} = A + B$

2. **As AND Gate:**

An AND gate can be produced by using two NAND gates (Fig. 2).

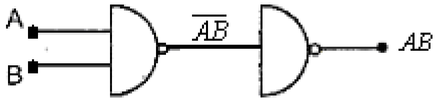


Fig. 2

3. As NOT Gate:

(i) If the two inputs of a NAND gate are connected together, then we get a NOT gate [Fig. 3].

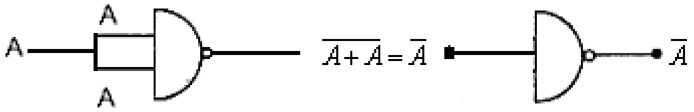


Fig. 3.1

(ii) When only single input is used, the circuit symbol of Fig. 3.1 is used.

5.8.1 NOR gate is a universal gate

A NOR gate can be used to realize the basic logic functions: OR, AND and NOT. So NOR gate is also called universal gate. Thus, in digital circuits, NOR gate serves as a building block.

1. As OR gate: An OR gate can be realized by connecting the ouput of a NOR gate to an inverter (Fig. 1).

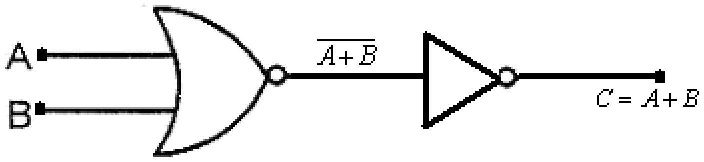


Fig. 1

The output of NOR gate is $\overline{A+B}$ which is inverted by inverter to give C = A + B. This is the logic function of a normal OR gate.

2. As AND gate: Twp inverters and a NOR gate are used to get an AND gate (Fig. 2).

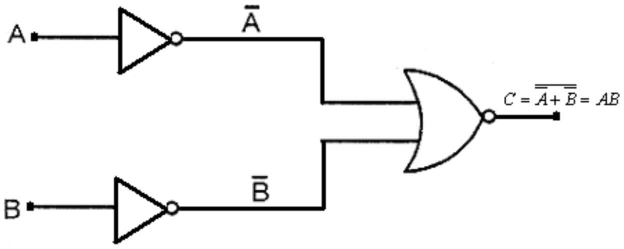


Fig. 2

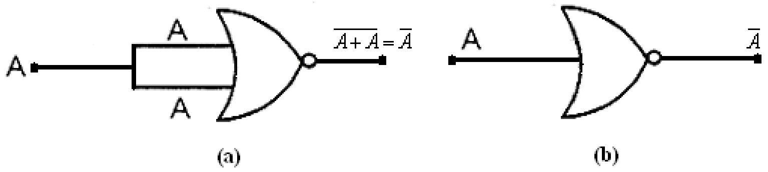
Here, two inverters are used, one for each input. The inverted inputs $\overline{A}$ and $\overline{B}$ are applied to the NOR gate.

The output of NOR is

$$\overline{\overline{A}+\overline{B}} = AB \text{ (By De Morgan's Theorem)}$$

3. As NOT gate:

(ii) The two inputs of NOR gate are tied together, as shown in Fig. 10.46 (a)



(a) (b)

Fig. 3

The output is $\overline{A+A}$.

By De Morgan's Theorem,

$$\overline{A+A}=\overline{A}$$

(ii) When only single input is used, the circuit symbol of Fig. 10.46 (b) is widely used.

5.10 Karnaugh Map

Though the method of factorization could be done by algebraic method, still a better method is being followed which is known as Karnaugh Map method. Karnaugh map is a table in which the fundamental products corresponding to a truth table is displayed in rows and columns.

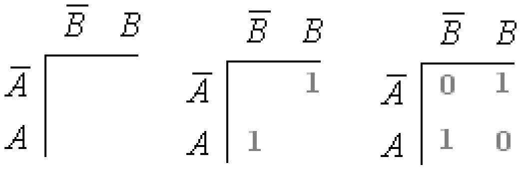
Construction of Karnaugh Map

(i) For two variables.

Consider the truth table shown here in which, the output 1 occurs for AB = 0 1 and AB = 1 0. This indicates that the fundamental products $A\overline{B}=1$ and $A\overline{B}=1$ and the Boolean equation is, $Y=\overline{A}B+A\overline{B}$

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

To draw a Karnaugh map for the above given truth table, a diagram as shown in the figure IV(a) is drawn in which $\overline{A}$ and A are entered in one column outside the figure and B and $\overline{B}$ are centered in one row above the figure. From the truth table, select the row which have the fundamental product value 1 and enter them in the figure. In the present case, $\overline{A}B=1$ and $A\overline{B}=1$ and these are shown in the figure IV(b). The remaining columns are filled with 0s. Figure IV(c) is the final form of the required Karnaugh map.



(a) (b) (c)

Fig. IV

(ii) For three variables.

Let us assume a truth table as shown here. The fundamental product value 1 occurs for the combinations ABC = 1 0 0 and ABC = 1 1 0. Thus, $A\overline{B}\,\overline{C}=1$ and $AB\overline{C}=1$. The sum of product is given by

$$Y = A\overline{B}\,\overline{C} + AB\overline{C}$$

This is the required Boolean equation.

Input			Output
A	B	C	y
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	1
0	1	1	0
1	1	0	1
1	0	1	0
1	1	1	0

This could be simplified as

$$Y = A\overline{C}\left(\overline{B}+B\right)$$

$$= A\overline{C} \qquad \text{Since } \left(\overline{B}+B\right)=1$$

The Karnaugh map is drawn for this problem in the following way:

$\overline{A}\overline{B}$, $\overline{A}B$, AB and $A\overline{B}$ are written in a vertical column outside the figure and $\overline{C}$ and C are written in a horizontal row above the figure. The four combinations of A and B or C and D are chosen in this order only as there is only one variable changes form complemented to uncomplemented form or vice versa. This condition is necessary.

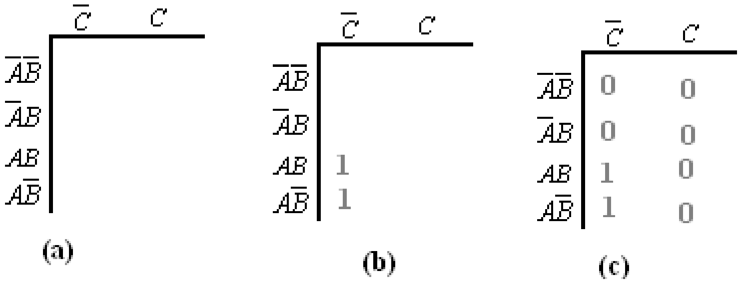


Fig V

The fundamental product value 1 is filled in the figure corresponding to $AB\overline{C}$ and $A\overline{B}\overline{C}$. The remaining spaces are filled with 0s. Figure (c) is the final form of the required Karnaugh map.

By the same procedure Karnaugh map could be drawn for four variables also. Karnaugh map is used to obtain the Boolean equation without actually simplifying the algebraic equation. So, many complicated Boolean equations could be simplified by Karnaugh map method.

5.10.1 Pairs, Quads and Octets

In the Karnaugh map shown above, two 1s occur one below the other corresponding of ABC and ABC. In case two combinations, B varies form uncomplemented to complemented form. Whenever such pair occurs it should be eliminated.

Now, $Y = AB\overline{C} + A\overline{B}\overline{C}$

$= A\overline{C}\left(B + \overline{B}\right)$

$= A\overline{C}$

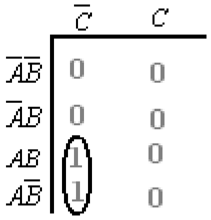


Fig. VI

This value Y $= A\bar{C}$ could easily be obtained by ignoring the variable (here, B) that changes from complemented to uncomplemented form of vice versa. Thus,

$$\mathrm{Y} = A\bar{C}$$

The same procedure may also be adopted for a pair occurring along a row. For two 1s occurring diagonally, this procedure is not permitted.

In another Karnaugh map for four variables (assumed), two pairs of 1s occur for $\bar{A}\bar{B}\bar{C}D$ & $\bar{A}\bar{B}CD$ and $AB\bar{C}\bar{D}$ & $A\bar{B}\bar{C}\bar{D}$. The Boolean equation for this is given by

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}.$$

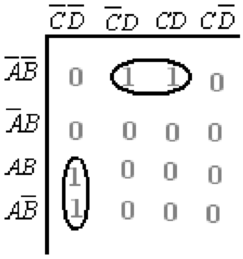


Fig. VII

In the pair occurring in the first row, C changes from complemented to uncomplemented form. Hence, it is ignored. The sum of product is reduced the form $\bar{A}\bar{B}D$. In the pair occurring in the first column, B changes from uncomplemented to complemented form. Ignoring B, we have the reduced form of the sum of products as $A\bar{C}\bar{D}$.

Thus, the Boolean equation in the simplified form is

$$Y = \bar{A}\bar{B}D + A\bar{C}\bar{D}$$

Here, the actual process of going into the algebraic simplification is not necessary.

Just like the elimination of pairs, a quad could also be eliminated. A quad is a group of four 1s, all occurring either along a column or like a 2 X 2 matrix (square form). Wherever such a quad occurs it could be eliminated, for the reasons explained earlier, and a simple form of Boolean equation could be obtained quickly.

Example : In the Karnaugh map shown here, first the pair of 1s in the first and the second rows, B changes from complemented to uncomplemented form. Ignoring B, we get the Boolean equation as $Y = \bar{A}CD$. Similarly, for the lower pair, ignoring B, the Boolean equation is $Y = ACD$. Hence, the combined Boolean equation is

$$\begin{aligned} Y &= \bar{A}CD + ACD \\ &= CD \end{aligned}$$

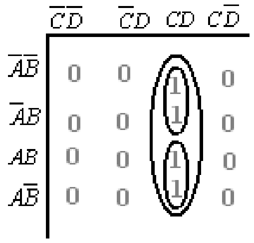


Fig. VIII

Thus, in the case of the occurance of a quad the two changing variable lie along the 1s are ignored and the remaining variables is the Boolean equation.

In the case of square quad also, two variables that change from complemented to uncomplemented or vice versa are removed and a simple form of Boolean equation is obtained.

In the figure IX shown here, along the row D changes from complemented to uncomplemented form and along the column B changes from uncomplemented to complemented form. Eliminating B and D, we have,

$$Y = A\overline{C}$$

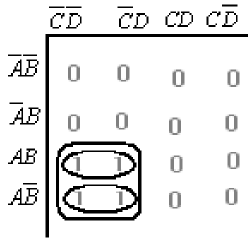


Fig. IX

An octet (group of 8 adjacent 1s) may also occur in a Kanaugh map. In such a case 3 pairs of variables are removed and only one variable is therein the final form of the Boolean equation.

Here in the figure X along the column, A changes from complemented to uncomplemented form. Along the row, C and D change from one to another form. Thus, eliminating A, C and D we have the Boolean equation,

$$Y = B$$

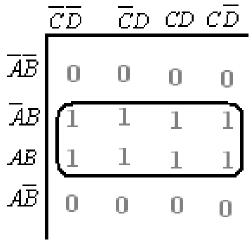


Fig. IV - X

It should be remembered that if the same problem is done by algebraic simplification it may be a laborious method.

5.11 Half Adder

A logic circuit that adds two bits producing a sum and a carry to be used in the next higher positions is called a half-adder.

Fig. 5.11(a) shows the circuit of a half-adder. It consists of an EXCLUSSIVE OR gate and an AND gate. The output of the exclusive OR gate is called the SUM, while the output of the AND gate is called the CARRY. Fig. 5.11(b) shows the symbol of a half-adder.

The two inputs A and B represents the bits to be added.

S and C are the two outputs.

S represents the output of EXCLUSIVE OR gate.

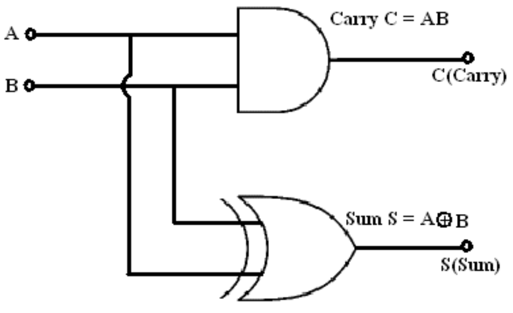
C represents the CARRY bit. It is the output of AND gate.

With two inputs A and B, there are four distinct cases.

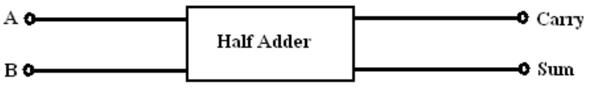
(I) When A =0 and B =0,

Carry C = AB =0

Sum S = $A \oplus B = 0 \oplus 0 = 0.$



(a) Half Adder



(b)

Fig. 5.11

(II) When A =0 and B =1,

Carry C = AB = 0.1 = 0

Sum S = $A \oplus B = 0 \oplus 1 = 1.$

(III) When A = 1 and B =0,

Carry C = AB = 1.0 = 0

Sum S = $A \oplus B = 1 \oplus 0 = 1.$

(IV) When A = 1 and B =1,

Carry C = AB =1.1 = 1

Sum S = $A \oplus B = 1 \oplus 1 = 0.$

The truth table for the Half-Adder operation is given in Table 1.

Truth Table 1

Input		Output	
A	B	Sum (S)	Carry (C)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

The sum column represents XOR operation.

The carry column represent the AND operation.

The Boolean expressions for half-adder operation may be written as,

Sum S = $A \oplus B$

Carry C = AB.

2. Full Adder

We define full-adder as a logic circuit that adds three bits-two bits to be added and a carry bit from previous addition, which results in a sum and a carry.

The half-adder accomplishes the first step in the addition of bits. Two half-adders are combined into a full adder, in order to add the carry bits to the bit sum. Let us illustrate this operation by an example.

$$\begin{array}{rrr} 1 & 1 & 1 \\ +1 & 0 & 1 \\ \hline 11 & 0 & 0 \end{array}$$

In the least significant column,

1+1=10, Sum = 0, Carry =1.

Thus for this operation, we need as half-adder.

In the next column, we must add 3 bits because of carry,

1+0+1 = 10, Sum =0, Carry =1.

By connecting two half-adders and an OR gate, we get a full adder (Fig. 5.11(2)).

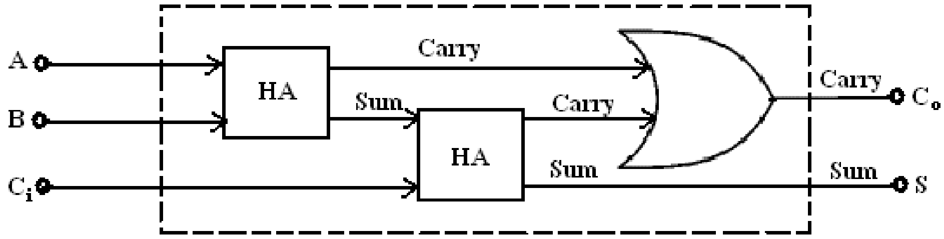


Fig. 5.11(2).

It can add three bits at a time. Fig. 3 shows the symbol of a full-adder. It has two inputs A and B, plus a third input C. Input C is also called the CARRY IN (C_i). In comes from a lower-order column. There are two outputs, Sum and Carry. The output carry is also called the CARRY OUT (C_o).

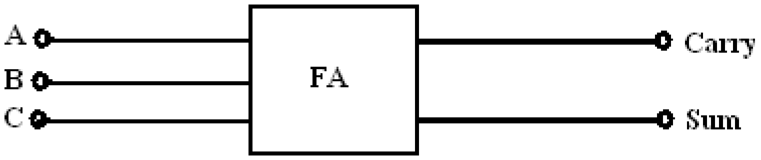


Fig. 5.11(3)

It goes to the next higher column.

The truth-table of a FA is given in Table 2.

It has three inputs and two outputs.

Truth Table 2

Input			Output	
A	B	C_i	Sum (S)	Carry (C_o)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Here C_i represents a carry from a preceding stage.

C_o represents the carry to be added to the next stage.

Working

To illustrate its operation, let us take two examples

7. **A =1, B =1, C_i=0**

Fig. 5.11(4) shows the full adder with these three inputs.

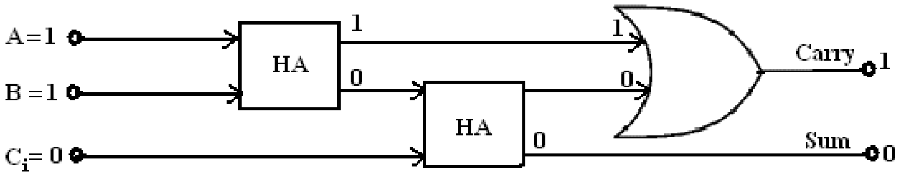


Fig. 5.11(4)

(i) The output of first half adder consists of a sum of 0 with a carry of 1.

(ii) The sum 0 of first half adder and carry 0 when fed to the second half adder, give a sum of 0 with a carry of 0.

(iii) The carry outputs of both the half adders is fed into the input of OR gate.

The final output is SUM 0, CARRY 1.

We get the same result from binary addition: 1+1+0 = 10_2.

8. **A =1, B=1, C_i=1.**

(i) The output of the first half adder is a sum of 0 with a carry 1 (Fig 5.11(5)).

(ii) The output of second half adder is a sum of 1 with a carry of 0.

(iii) The final output is SUM 1 with a CARRY 1.

We get the same result from binary addition: 1+1+1 = 11_2.

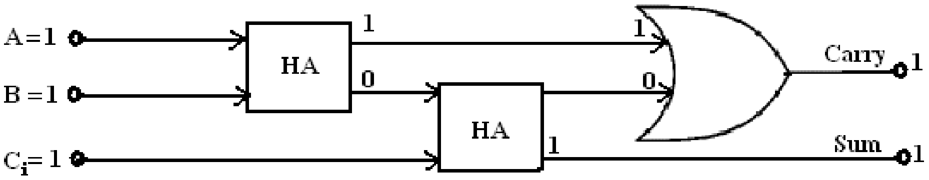


Fig. 5.11(5)

5.12 Half Subtractor

A logic circuit, which subtracts two binary bit is called a half subtractor. It contains two inputs and two outputs (Difference and Borrow). The truth table of half subtractor is shown in the fig. 5.12 (a)

A	B	Diff.	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

(a) truth table

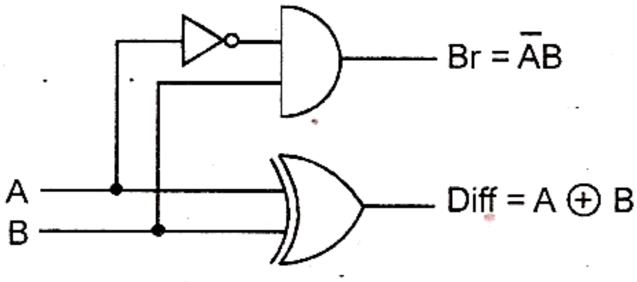


Fig. 5.12(a) Half subtractor

From the truth table, the difference and borrow will be written in the form of “sum of products” as.

Difference = $\overline{A}B + A\overline{B} = A \oplus B$

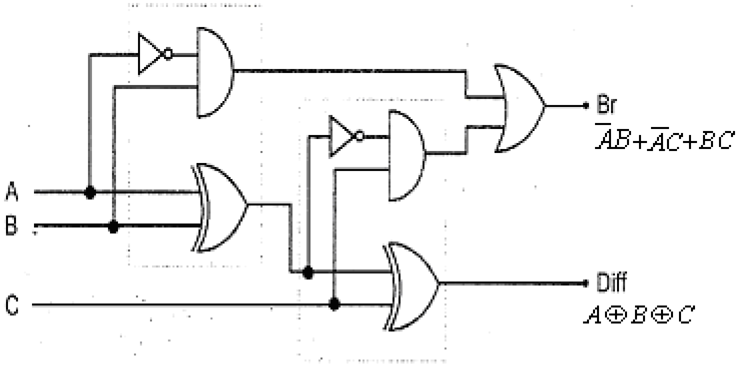
Borrow = $\overline{A}B$

The logic diagram for half subtractor is shown in fig. 5.12 (b)

Full Subtractor

A	B	C	Diff	Bo
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

(a) truth table



(b) Fig. 5.12 Logic Diagram

Fig. Full subtractor

A logic circuit, which subtracts three binary bits is called full subtractor. It contains three inputs and two outputs. The truth table of full subtractor is shown in the Fig 5.14 (a)

From the truth table, the Difference and Borrow will be written in the form of "sum of products" as follows.

$$\text{Difference} = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + ABC$$

$$= \left(A \oplus B\right) \oplus C.$$

$$\text{Borrow} = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + ABC$$

$$= \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + BC\left(A + \overline{A}\right) \qquad (\because A+\overline{A}=1)$$

$$= \overline{A}(\overline{B}C + B\overline{C}) + BC$$

$$= \overline{A}(B \oplus C) + BC$$

The logic diagram of full subtractor is shown in the fig. 5.14

The difference output is 0 whenever the inputs A and B are the same; the difference output is 1 whenever A and B are different. So, we can use an exclusive-OR gate to produce the difference output. Second, the borrow output is 1 only when A is 0 and B is 1. We can get this borrow output by ANDing $\overline{A}$ and B.

Figure 5-13 shows one way to build a half-subtractor, a circuit that subtracts one binary digit from anotherYou can see there will be a borrow only when A =0 and B =1. Further, the difference output will be correct for each of the four possible A – B combinations.

The half-subtractor handles only 2 bits at a time and can be used for the least significant column of a subtraction problem. To take care of a higher-order column, we need a full-subtractor. Fig. 5.14 shows a full-subtractor; it uses two half-subtractors and an OR gate.

Half-and full – subtractors are analogous to half- and full – adders; by cascading half- and full-subtractors as shown in Fig. 5.14, we have a system that directly subtracts $B_3B_2B_1B_0$ from $A_3A_2A_1A_0$.

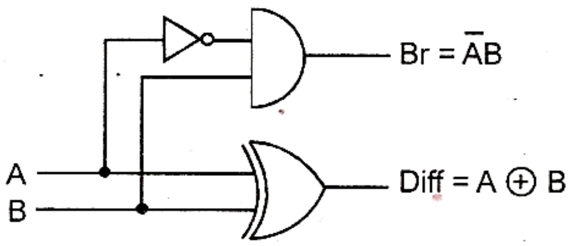


Fig. 5-13 Half-subtractor

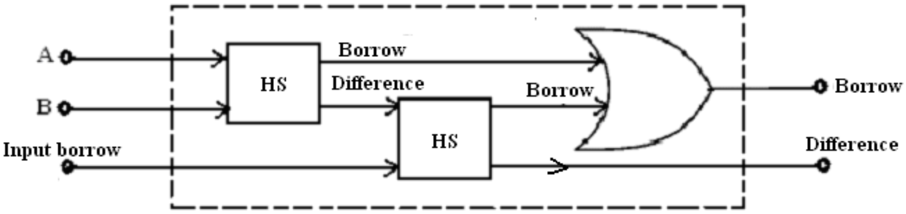


Fig. 5-14. Full subtractor.

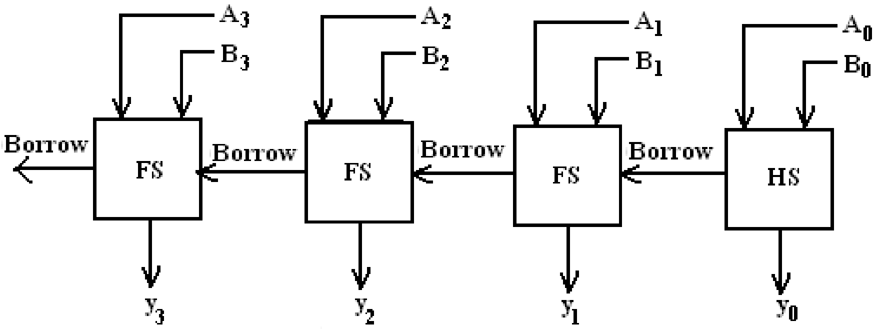


Fig. 5.15. Parallel 4-bit binary subtractor.

The adders and subtractors give us the basic circuits needed for binary arithmetic; multiplication and division can be done by repeated additions and subtractions (discussed in later chapters, after we have studied registers).

Questions

1. Explain the construction and working of a FET?
2. How will you determine the drain characteristics of FET? What do they indicate?
3. Briefly describe some practical applications of FET?
4. Write short notes on the advantages of FET?
5. Explain the construction and working of an SCR?
6. Draw the V-I characteristics of an SCR. What do you infer from them?
7. Explain the action of an SCR as a switch. What are the advantages of SCR switch over a mechanical or electro-mechanical switch?
8. Discuss some important applications of SCR?
9. Explain the construction and working of a UJT?
10. Discuss UJT working from the circuit.
11. Describe some important application of a UJT?
12. What is binary number system? How does it differ from decimal number system? Why is the binary number system used in computers?
13. What are binary numbers? Explain with illustrations the methods of conversion from decimal to binary and binary to decimal numbers.

14. What are octal and hexadecimal number system?
15. What is Boolean algebra? Mention its unique feature. Discuss the fundamental laws of Boolean algebra.
16. Write and explain the Boolean expressions for OR, AND and NOT circuits.
17. State and prove De Morgan's theorems.
18. State the rules for binary multiplication.
19. What is a multivibrator? Explain the principle on which it works.
20. With a neat sketch, explain the working of (i) astable multivibrator (ii) monostable multivibrator (iii) bistable multivibrator.
21. What is the basic difference among the three types of multivibrators?
22. What is Karnaugh map? Explain how is it constructed for (i) 2 variables (ii) 3 variables and 4 variables.
23. Explain the terms pairs, quads and octets. Explain their use in Karnaugh mapping.
24. Explain NAND, NOR gates as universal building blocks?

Books for Study and Reference.

1. Properties of Matter and acoustics, R. Murugesan S Chand &Co
2. Heat and Thermodynamics Brijlal and Subramaniam S Chand & Co
3. Optics Brijlal & Subramanian S Chand & Co
4. Electricity and Magnetism R. Murugesan S Chand & Co
5. Modern Physics R Murugesan S Chand & Co
6. Principles of Electronics V K Mehta S Chand & Co
7. Digital Principles and applications, Malvino & Leach TMH.

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