PERIYAR INSTITUTE OF DISTANCE EDUCATION

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PERIYAR UNIVERSITY

SALEM - 636 011.

BACHELOR OF BUSINESS ADMINISTRATION (B.B.A)

SECOND YEAR

PAPER - III : INTRODUCTION TO OPERATIONS RESEARCH

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PAPER - III : INTRODUCTION TO OPERATIONS RESEARCH

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UNIT - I

1.1 Introduction

Operation research is the study of optimization techniques. Rapid development and invention of new techniques occurred since the World War II essentially, because of the necessary to win the war with the limited resources available.

1.2 Scope or uses of the operation research

This operation research is useful for solving

- ★ Available resource allocation problems
- ★ Inventory Control Problems
- ★ Maintenance and replacement problems
- ★ Sequencing problem
- ★ Scheduling problem
- ★ Queuing Problem
- ★ Assignment of jobs to applicants to maximize profit Etc…

1.3 Models in OR

Models are representation of real objects or situations and can be presented in various forms.

In modeling terminology physical replicas are referred to as iconic models.

A second classification includes models that are physical in form but do not have the same physical appearance as the object being modeled. such models are referred to as analog models

A third classification of models-the type we will be primarily be studying-includes representation of a problem by a system of symbols and mathematical relationships or expressions. Such models are referred to as mathematical models.

Also as

- ★ Static models
- ★ Dynamic models
- ★ Deterministic models
- ★ Stochastic Models

Models can further be subdivided as

- ★ Descriptive Models
- ★ Prescriptive Models

★ Predictive Models

★ Analytic Models

★ Simulation Models

Now let we see about further explanations from static models to simulation models

Static models

This model which does not take time in to account. It assumes that the values of the variable do not change with time during a certain period of time horizon.

Example: A linear programming problem, as assignment problem, transportation problems etc.

Dynamic model

This model which considers time as one of the important variables.

Example: A replacement problem

Deterministic model

Is a model which considers time as one of the important variables.

Example: An assignment problem

Stochastic model: is a model which considers uncertainty as an important aspect of the problem

Example: Stochastic inventory models

Descriptive Models: is one which just describes a situation or system

Example: Any survey or opinion poll

Prescriptive Models: is one which prescribes or suggests a course of action for a problem.

Examples: Any programming problem

Predictive Models: is one which predicts something based on some data.

Example: Exit poll

Analytic Models: is a model in which exact solution is obtained by mathematical methods in closed form.

Simulation Models: is a virtual reality or artificially creating real situation

Example: queuing problem, inventory problems

1.4 General methods for solving OR models

(1) Analytical procedure: solving models by classical mathematical techniques like differential calculus, finite differences…etc. to get analytical solution

(2) **Iterative Procedure:** starts with a trial solution and a set of rules for improving it by repeating the procedure until further improvement is not possible.

(3) **Monte carlo technique :** Taking sample observations, computing probability distributions for the variable using random numbers and constructing some functions to determine values of the decision variables.

1.5 Limitations:

Operation research models involves with mathematical calculations. So, it is not about characteristic or emotional or qualitative which are quite real and influence the decision making. This is the major limitation of the operation research.

Exercise

1. What are the models available in operation research?
2. What is an analytic model?
3. What is simulation Model?
4. Narrate about dynamic Model?
5. Describe about Monte carlo technique?

NOTES

UNIT - II

LINEAR PROGRAMMING PROBLEM

Identify the models to be used for problem solving

1. Outline the conditions for a LPP
2. Identify the variables in a LPP
3. Identify the constraints in a LPP
4. Formulate the LPP

Formulation of LPP

Example-1

An Industry manufactures two items X & Y. To manufacture the item X, a certain machine has to be worked for 2.5 hours and in addition a workman has to work for 3 hrs. To manufacture the item Y, the machine has to be worked for 3.5 hrs and in addition the workmen has to work for 2.5 hrs. In a week factory can avail of 90 hours of machine time and 85 hours of craftsmen's time. The profit on each item X is Rs.130 and that on each item Y Rs.175. If the entire article produced canbe sold away, find how many of each item should be produced to earn the maximum profit per week. Formulate the problem as LP model.

Solution:

Step 1: Defining the variables.

$X1$ = Total number of the items produced in type X

$X2$ = Total number of the items produced in type Y

Step 2: Objective function.

Maximize (total profit) $Z = 130X1 + 175X2$

Step 3: Constraints.

$2.5X1 + 3.5 X2 <= 90$

$3X1 + 2.5X2 <= 85$

Step 4: Non - Negativity constraints.

$X1 >= 0$; $X2 >= 0$

Step 5: Complete Solution to the problem.

Max $Z = 130X1 + 175X2$

Subject to the Constraints

$2.5X1 + 3.5 X2 <= 90$

$3X1 + 2.5X2 <= 85$

and $X1, X2 >= 0$

Example2: The Village Butcher Shop traditionally makes its meat loaf from a combination of lean ground beef and ground pork. The ground beef contains 80 percent meat and 20 percent fat, and costs the shop Rs.80 per Kg; the ground pork contains 68 percent meat and 32 percent fat, and costs Rs.60 per Kg. How much of each kind of meat should the shop use in each pound of meat loaf if it wants to minimize its cost and to keep the fat content of the meat loaf to no more than 25 percent?

Solution:

Step 1: Defining the variables.

X1 = Total number of ground beef produced

X2 = Total number of ground pork produced

Step 2: Objective function.

Minimize (total cost) Z = 80X1 + 60X2

Step 3: Constraints.

0.20X1 + 0.32 X2 <= 25

X1 + X2 = 1

Step 4: Non - Negativity constraints.

X1 >= 0; X2 >= 0

Step 5: Complete Solution to the problem.

Min Z = 80X1 + 60X2

Subject to the Constraints

0.20X1 + 0.32 X2 <= 25

X1 + X2 = 1 and

X1, X2 >= 0

Example-3

The Futura Company produces two types of farm fertilizers, Futura Regular and Futura's Best. Futura Regular is composed of 25% active ingredients and 75% inert ingredients, while Futura's Best contains 40% active ingredients and 60% inert ingredients. Warehouse facilities limit inventories to 500 tons of active ingredients and 1200 tons of inert ingredients, and they are completely replenished once a week.Futura regular is similar to other fertilizers on the market and is competitively priced at $250 per ton. At this price, the company has had no difficulty in selling all the Futura Regular it produces. Futura's Best, however, has no competition, and so there are no constraints on its price. Of course, demand does depend on price, and through past experience the company has determined that price P (in dollars) and demand (D) are related by P = 600 - D. How many tons of each type of fertilizer should Futura produces weekly in order to maximize revenue?

Solution:

Step 1: Defining the variables.

X1 = Total number of the quantity produced in futura regular

X2 = Total number of the quantity produced in futura best

Revenue = P X D

P = 600 - D

Step 2: Objective function.

Maximize (Revenue) Z = 250X1 + (600 - X2) X2

Step 3: Constraints.

0.25X1 +0.4X2 <= 500

0.75X1 + 0.6X2 <= 200

Step 4: Non - Negativity constraints.

X1 >= 0 ; X2 >= 0

Step 5: Complete Solution to the problem.

Max Z = 250X1 + (600 - X2) X2

Subject to the Constraints

0.25X1 +0.4X2 <= 500

0.75X1 + 0.6X2 <= 200 and

X1, X2 >=

Example-4

A manufacturing company is engaged in producing three types of products: A, B and C. The Production department produces, each day, components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The following additional information is available.

Type of Product	Profit Contribution per Unit of Product (Rs)	Assembly Time per Product (Hrs)
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of product A and a total of 15 units of products B and C. Formulate this problem as an LP model so as to maximize the total profit.

Solution:

Step 1: Defining the variables.

X1 = Total number of the products produced in type A

X2 = Total number of the products produced in type B

X3 = Total number of the products produced in type C

Step 2: Objective function.

Maximize (total profit) Z = 12X1 + 20X2 + 45X2

Step 3: Constraints.

0.8X1 + 1.7 X2 + 2.5 X3 <= 100

X1 <= 50; X2 <= 25; X3 <= 30

X1 >= 20

X2 + X3 >= 15

Step 4: Non - Negativity constraints.

X1 >= 0 ; X2 >= 0; X3 >= 0

Step 5: Complete Solution to the problem.

Max Z = 12X1 + 20X2 + 45X2

Subject to the Constraints

0.8X1 + 1.7 X2 + 2.5 X3 <= 100

X1 <= 50

X2 <= 25

X3 <= 30

X1 >= 20

X2 + X3 >= 15 and

X1, X2, X3>= 0

GRAPHICAL METHOD

Learning Objectives

After reading this chapter the student must be able to

a. Plot the constraints of the LPP on a graph paper

b. Identify the feasible region in the graph

c. Identify the corner points of the region

d. Plot the objective function

e. Deduce the optimum solution of the problem

Basic terms

i. Solution

Values of the decision variable x;(i = 1,2,3, in) satisfying the constraints of a general linear programming model is known as the solution to that linear programming model.

ii. Feasible Solution

Out of the total available solution a solution that also satisfies the non-negativity restrictions of the linear programming problem is called a feasible solution.

iii. Basic Solution

For a set of simultaneous equations in Q unknowns (p Q) a solution obtained by setting (P - Q) of the variables equal to zero & solving the remaining P equation in P unknowns is known as a basic solution. The variables which take zero values at any solution are detained as non-basic variables & remaining are known as-basic variables, often called basic.

iv. Basic Feasible Solution

A feasible solution to a general linear programming problem which is also basic solution is called a basic feasible solution.

v. Optimal Feasible Solution

Any basic feasible solution which optimizes (i.e.; maximize or minimizes) the objective function of a linear programming modes known as the optimal feasible solution to that linear programming model

vi. Degenerate Solution

A basic solution to the system of equations is termed as degenerate if one or more of the basic variables become equal to zero.

I hope the concepts that we have so far discussed have been fully understood by all of you.

Friends, it is now the time to supplement our understanding with the help of examples.

Example-1 solve the following LPP graphically

$$Max\ Z = 3x_1 + 2x_2 \quad s.t$$
$$-2x_1 + x_3 \leq 1$$
$$x_1 \leq 2$$
$$x_1 + x_2 \leq 3$$
$$and\ x_1, x_2 \geq 0$$

Solution:

First consider the inequality constraints are to be equal

$$-2x_1 + x_3 = 1$$
$$x_1 = 2$$
$$x_1 + x_2 = 3$$

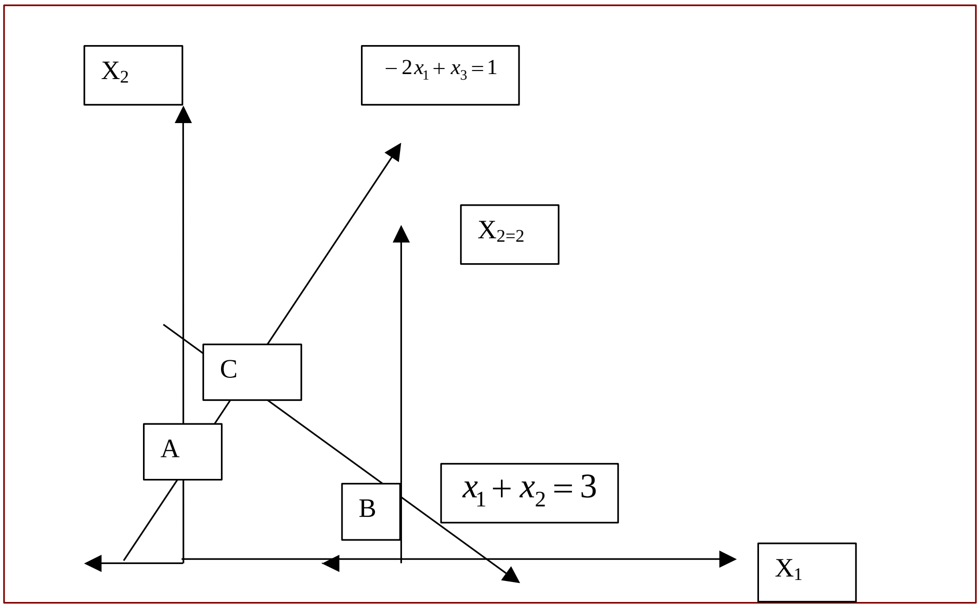
Let we assume like following

$$when\quad x_1 = 0\quad x_2 = 1\quad therefore\ thepoinsare\ (0,1)$$
$$when\ x_2 = 0\ x_1 = -0.5\ therefore\ thepoinsare\ (-0.5,0)$$

In the same way

X1=2

And for third equation we can get (0,3) and (3,0) this can be plotted in the graph



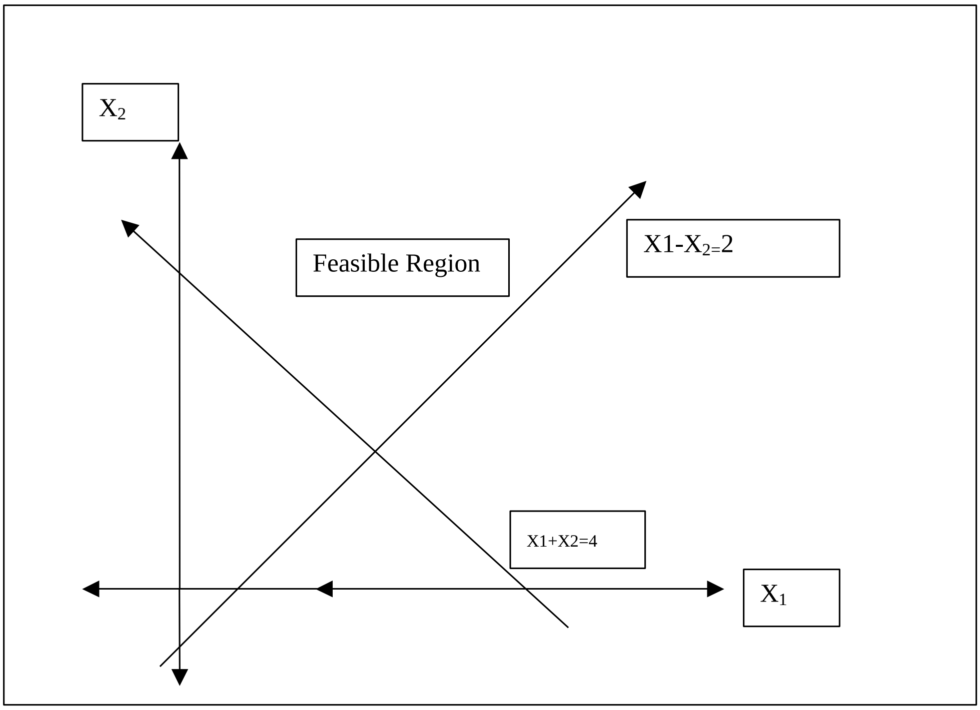
In the above graph common area is between O,A,B and C points, these area is called Feasible area and there fore

At O(0,0),A (2,0),B(2,1) and C(2/3,7/3)

Example-2 Solve graphically following LPP

$$Max\,Z = 2x_1 + 3x_2\quad s.t$$
$$x_1 - x_2 \leq 2$$
$$x_1 + x_2 \geq 4$$
$$and\ x_1, x_2 \geq 0$$

Solution :



in the above Graph it is noted that the feasible region is unbounded, there fore **solution is unbounded**

LINEAR PROGRAMMING -SIMPLEX METHOD

Objectives

a. Formulate the standardized LPP

b. Identify the use of slack, surplus and artificial variables

c. Identify the solving method

d. Formulate the initial simplex table

e. Calculate the tables for each iteration

f. Calculate the shadow costs/profits

Example-1

The LPP is maximize Z = 6x1 + 4x2

Subject to the constraints

2x1 + x2 ??390

3x1 + 3x2 ??810

X2 ??20

And x1, x2 ??0

Solution:

Introducing slack variables, the above problem becomes,

Maximize z = 4x1 + 3x2 + 0S1 + 0S2 + 0S3

Subject to

2x1 + 3x2 + S1 = 150

3x1 + 2x2 + S2 = 150

x1 + x2 + S2 = 100

x1, x2, S1, S3 ??0

		C_j	4	3	0	0	0	
Q	CV	C	x_1	x_2	S_1	S_2	S_3	Ratio
150	S_1	0	2	3	1	0	0	75
150	S_2	0	$\boxed{3}$	2	0	1	0	50 →
100	S_3	0	1	1	0	0	1	100
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-4↑	-3	0	0	0	
50	1	0	0	$-\boxed{\frac{5}{3}}$	1	$-\frac{2}{3}$	0	30 →
50	x_1	4	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	75
50	S_3	0	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	1	150
		Z_j	4	$\frac{8}{3}$	0	$\frac{4}{3}$	0	
		$Z_j - C_j$	0	$-\frac{1}{3}$↑	0	$\frac{4}{3}$	0	
30	x_2	3	0	1	$\frac{3}{5}$	$-\frac{2}{5}$	0	
30	x_1	4	1	0	$-\frac{2}{5}$	$\frac{3}{5}$	0	
40	S_3	0	0	0	$-\frac{1}{5}$	$-\frac{1}{5}$	1	
		Z_j	4	3	$\frac{1}{5}$	$\frac{6}{5}$	0	
	$Z_j - C_j$	0	0	$\frac{1}{5}$	$\frac{6}{5}$	0		

Since zj – cj ≥ 0, the optimal solution is reached. The optimal solution is $x_1 = 30$, $x_2 = 30$.

Maximum profit = Rs.210.

Example-2

Maximize Z = 6x1 + 4x2

Subject to the constraints

2x1 + x2 ??390

3x1 + 3x2 ??810

X2 ??20

And x1, x2 ??0

Solution:

Convert inequality constraints into equality constraints by introducing slack Variables.

The LPP is

Max. Z = 6x1 + 4x2 + 0.S1 + 0.S2 + 0.S3

subject to

2x1 + x2 + S1 = 390

3x1 + 3x2 + S2 = 810

X2 + S3 = 200

x1, x2, S1, S2, S3 ??0

		C_j	6	4	0	0	0	
Q	CV	C	x_1	x_2	S_1	S_2	S_3	Ratio
390	S_1	0	2	1	1	0	0	195 →
810	S_2	0	3	3	0	1	0	270
200	S_3	0	0	1	0	0	1	∞
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-6↑	4	0	0	0	
195	S_1	6	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	390
225	S_2	0	0	$\frac{3}{2}$	$-\frac{3}{2}$	1	0	150 → $R_2 - 3R_1$
200	S_3	0	0	1	0	0	1	200 R_3
		Z_j	6	3	-3	0	0	R1
		$Z_j - C_j$	0	-1↑	3	0	0	$R_1 - \frac{1}{2} R_2$
120	x_2	6	1	0	1	$-\frac{1}{3}$	0	$R_1 - \frac{1}{2} R_2$
150	x_1	4	0	1	-1	$\frac{2}{3}$	0	
50	S_3	0	0	0	1	$-\frac{2}{3}$	1	$R_3 - R_2$
		Z_j	6	4	2	$\frac{2}{3}$	0	
		$Z_j - C_j$	0	0	2	$\frac{2}{3}$		

since all $Z_j - C_j \geq 0$, the optimal solution is reached

		C_j	6	4	0	0	0	
Q	CV	C	x_1	x_2	S_1	S_2	S_3	Ratio
390	S_1	0	[2]	1	1	0	0	195 →
810	S_2	0	3	3	0	1	0	270
200	S_3	0	0	1	0	0	1	∞
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-6↑	4	0	0	0	
195	S_1	6	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	390
225	S_2	0	0	[$\frac{3}{2}$]	$-\frac{3}{2}$	1	0	150→ $R_2 - 3R_1$
200	S_3	0	0	1	0	0	1	200 R_3
		Z_j	6	3	-3	0	0	R1
		$Z_j - C_j$	0	-1↑	3	0	0	$R_1 - \frac{1}{2} R_2$
120	x_2	6	1	0	1	$-\frac{1}{3}$	0	$R_1 - \frac{1}{2} R_2$
150	x_1	4	0	1	-1	$\frac{2}{3}$	0	
50	S_3	0	0	0	1	$-\frac{2}{3}$	1	$R_3 - R_2$
		Z_j	6	4	2	$\frac{2}{3}$	0	
		$Z_j - C_j$	0	0	2	$\frac{2}{3}$		

Since all Zj - Cj ??0, the optimal solution is reached

The optimal solution is

x1 = 120

x2 = 150

Maximum profit = 120 x 6 + 150 x 4

= Rs.1,320

Example:3

Solve the following linear programming problem by simplex method.

Minimize Z = 16x1 + 16x2

Subject to

2x1 + 4x2 ??3

3x1 + 2x2 ??4

x1, x2 ??0

Solution:

Convert inequality constraints into equality constraints by introducing surplus and artificial variables.

The LPP is minimize Z = 16x1 + 16x2 + 0.S1 + 0.S2 + MA1 + MA2

Subject to

2x1 + 4x2 - S1 + A1 = 3

3x1 + 2x2 - S2 + A2 = 4

x1, x2, S1, S2, A1, A2 ??0

from the following table we can conclude that all the Zj - Cj ??0.

With Min Z =22

x1 =5/4,;

x2 =1/8

		C_j	16	16	0	0	M	M	
Q	CV	C	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
3	A_1	M	2	$\underline{4}$	-1	0	1	0	$\frac{3}{4}$ →
4	A_1	M	3	2	0	-1	0	1	2
		Z_j	5M	6M	-M	-M	M	M	
		$Z_j - C_j$	5M – 16	6M – 16↑	-M	-M	0	0	
$\frac{3}{4}$	x_2	16	$\frac{1}{2}$	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{3}{2}$
$\frac{5}{2}$	A_2	M	2	0	$\frac{1}{2}$	-1	$-\frac{1}{2}$	1	$\frac{5}{4}$ → $R_1 - 2R_2$
		Z_j	2M+8	8	$\frac{M}{2} - 4$	-M	$\frac{-M}{2} + 4$	M	
		$Z_j - C_j$	2M+8↑	8	$\frac{M}{2} - 4$	-M	$\frac{-3M}{2} + 4$	0	
$\frac{1}{8}$	x_2	16	0	1	$-\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$-\frac{1}{4}$	$R_1 - \frac{1}{2}R_2$
$\frac{5}{4}$	x_1	16	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	
		Z_j	16	16	-2	-4	2	4	
		$Z_j - C_j$	0	0	-2	-4	2 – M	4 -M	

Exercise

1) Define a feasible region.

2) Define a feasible solution

3) What is a redundant constraint

4) Define optimal solution

5) What is the difference between feasible solution and basic feasible

solution

6) Define the following:

(a) Basic solution

(b) non-degenerate solution

(c) degenerate solution.

(a) Basic solution

7) Define unbounded solution

8) What are the two forms of a LPP?

9) When does the simplex method indicate that the LPP has unbounded solution?

10) What is meant by optimality?

11) How will you find whether a LPP has got an alternative optimal solution or not, from the optimal simplex table?

12) What are the methods used to solve an LPP involving artificial variables?

13) Define artificial variable.

14) When does an LPP possess a pseudo-optimal solution?

15) What is degeneracy?

16) Solve Graphically Following LPP

$$MaxZ = x_1 - 2x_2$$
$$s.t$$
$$-x_1 + x_2 \leq 1$$
$$6x_1 + 4x_2 \leq 24$$
$$0 \leq x_1 \leq 5$$
$$2 \leq x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

17) Solve Graphically the following LPP

$$MaxZ = 5x_1 + 3x_2$$
$$s.t$$
$$4x_1 + 5x_2 \leq 1000$$
$$5x_1 + 2x_2 \leq 1000$$
$$3x_1 + 5x_2 \leq 1200$$
$$x_1, x_2 \geq 0$$

24. Solve graphically the following LPP

$$MinZ = 6000x_1 + 4000x_2$$
$$s.t$$
$$3x_1 + x_2 \geq 40$$
$$x_1 + 2.5x_2 \geq 22$$
$$3x_1 + 5x_2 \geq 40$$
$$x_1, x_2 \geq 0$$

25. Solve using simplex method

$$MaxZ = x_1 + 2x_2 + x_3$$
$$s.t$$
$$2x_1 + x_2 - x_3 \leq 2$$
$$-2x_1 + x_2 - 5x_3 \geq -6$$
$$4x_1 + x_2 + x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

26. Solve using simplex method

$$MaxZ = 10x_1 + x_2 + 2x_3$$
$$s.t$$
$$x_1 + x_2 - 2x_3 \leq 10$$
$$4x_1 + x_2 + x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

27. Solve using simplex method

$$MaxZ = 12x_1 + 20x_2$$
$$s.t$$
$$6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1, x_2 \geq 0$$

NOTES

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UNIT – III

TRANSPORTATION PROBLEM

Learning Objectives

After reading this chapter the student must be able to

a. Formulate a transportation problem

b. Generate the initial basic solution for the problem and resolve degeneracy

c. Optimize the initial basic solution using MODI method

Transportation models

The **basic transportation problem** was developed in 1941 by F.I. Hitchaxic. However it could be solved for optimally as an answer to complex business problem only in 1951, when Geroge B. Dantzig applied the concept of Linear Programming in solving the Transportation models. **Transportation models or problems** are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses (called demand destinations).

The **objective** in a transportation problem is:-

To fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacture to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase the profit on sales. Transportation problems arise in all such cases. It aims at providing assistance to the top management in ascertaining how many units of a particular product should be transported from each supply origin to each demand destinations to that the total prevailing demand for the company's product is satisfied, while at the same time the total transportation costs are minimized.

Initial Feasible Solutions.

In a transportation problem, the initial feasible solution can be generated by a number of methods. Three of the most commonly

methods are discussed as under:

1. The North-West Corner Rule (NWC Rule)

As the name itself suggests, under this rule, first of all the upper-left or the north-west corner of each cell is selected. It is one of the simplest methods which provides an initial feasible solution to a transportation problem. The steps involved are as under:

Step I

Upper-left corner cell of the transportation table is selected. Allocate as many units to this cell as possible. This will be the least value of demand and supply. This process is repeated for all the respective rows and columns.

Step II

After exhausting the supply for the first row, move down vertically to the Ist cell in the 2nd row and Ist column. Then repeated the above Step I.

Step III

After exhausting the demand for the first column, move along i.e.; horizontally to the next cell in the IInd column & Ist row.Then repeated the above Step I.

Step IV

Where demand = supply for a cell, then further allocation is made in either the next row or the next column cell. Thisprocedure is continued till the total quantity that is available is fully allocated to the various cells, as required.

2. Least Cost Method (LCM)

This is a time-saving method since it drastically reduces the numerous calculation required to be done under the northwest corner rule.

The following steps are involved in this method:

Step I

Among all the rows and columns in the transportation table, select that cell which has the lowest (minimum) transportation cost.

Step II

Where the smallest cost is not unique i.e.; there are other cells having the same smallest cost, select any cell which has this smallest cost.

Step III

To the cell chosen in the above step, allocated the maximum possible units. Eliminate that row or the column where either the demand is satisfied or the supply is exhausted.

Step IV

For the reduced table so obtained, repeat the above steps till total demand and supply are exhausted.

3. Vogel's Approximation Method (VAM)

The Steps involved in this method are give below :-

Step I

Calculate the penalty for all rows and columns. (Penalty is the difference between the smallest and the next smallest cost)

Step II

Select the row or column having maximum penalty. In this row or column, select the cell having the least cost. Allocate maximum possible units (quantity) to this lowest cost cell.

Remarks

In case of a tie, select the row/column having minimum cost. It, still, a tie persists, select the row/column having the maximum possible assignments or you may simply select any row or column in case of a tie without considering minimum costs etc.

Step III

Reduce the demand or supply by the amount assigned to the cell.

Step IV

If row supply is zero-eliminate it If column demand is zero-eliminate it If both are zero-eliminate both.

Step V

Again calculate penalty and repeat the same steps.

Remarks

At the end, check that the total number of filled cells = M + N- 1. only then the initial solution would be feasible. If filled cells are <M + N - 1, an empty cell is to be filled in a particular manner.

Modified Distribution (MODI) Method

The following are the steps involved in the Modified Distribution (MODI) method of testing the optimality of a feasible solution in a transportation problem:-

Step I

By using any one of the three methods discussed above, obtain an initial feasible solution, having M + N - 1 allocation in independent position.

Step II

Assign an arbitrary value (zero) to one of the variables without violating the equations. (Since there are M + N - 1 occupied cells, there will be M + N - 1 equations).

Step III

For every empty cell, calculate the improvement index i.e.; its opportunity cost. This has to be calculated by adding the corresponding row and column number and then substracting the actual cost of this cell from it.

The solution is optimal, if the opportunity cost of all the empty cells>0.

Step IV

Where the solution is not optimal i.e.; we have empty cells with negative improvement index (opportunity cost), select the empty cell having the largest value of negative opportunity cost.

Step V

For the empty cell selected in Step IV, draw a closed path - and assign alternate positive (+) and negative (-) signs at the empty cell - lying on the corner points of the path. The cell being evaluated i.e.; as selected in Step IV will have a positive (+) sign.

Step VI

Repeat this procedure till an optimal solution is achieved.

Example-1

Find initial basic feasible solution for the following transportation problem

Destination

		A	B	C	D	E	supply
	P	2	11	10	3	7	4
Origin	Q	1	4	7	2	1	8
	R	3	9	4	8	12	9
	Demand	3	3	4	5	6	21

Solution: since here Demand =Supply =21, the given problem is balanced. following north west corner rule ,the first allocation is made in the cell(1,1)

Here corresponding demand is only 3 but available capacity is 4 ,there fore we can allocate all required 3 to cell(1,1) .Now plant P remaining capacity is 1.destination A is completely satisfied. so we can leave corresponding column.

Destination

		B	C	D	E	supply
	P	11	10	3	7	1
Oorigin	Q	4	7	2	1	8
	R	9	4	8	12	9
	Demand	3	4	5	6	21

Again here north west corner is cell (1,2) corresponding demand is 3 but Plant P capacity is 1, there fore only 1 is allocated to cell (1,2).now plant P capacity is exhausted there fore we can leave that particular row.

		Destination				
		B	C	D	E	sup ply
Oorigin	Q	4	7	2	1	8
	R	9	4	8	12	9
	$Demand$	2	4	5	6	21

In the same way now north west corner is cell (2,2) corresponding demand is 2 but Plant Q capacity is 8, there fore all required 2 is allocated to cell (2,2).Now at B all the requirement is satisfied so column B can be deleted. There fore remaining at Q is 6.

		Destination			
		C	D	E	sup ply
Oorigin	Q	7	2	1	6
	R	4	8	12	9
	$Demand$	4	5	6	21

At present north west corner is cell (2,3) corresponding demand is 4 but Plant Q capacity is 6, there fore all required 4 is allocated to cell (2,3).Now at C all the requirement is satisfied so column C can be deleted. There fore remaining at Q are 2.

		Destination			
		C	D	E	sup ply
Oorigin	Q		2	1	2
	R		8	12	9
	$Demand$		5	6	21

Now the north west corner is cell (2,4) corresponding demand is 5 but Plant Q capacity is 2, there fore only two will be allocated to the cell (2,4).Now at D the demand is 3,there fore 2nd row can be deleted. There fore remaining at D are 3.

		Destination		
		D	E	sup ply
Oorigin				
	R	8	12	9
	$Demand$	3	6	21

the cell (3,4) is in north west corner position, here the corresponding demand is 3 but the plant capacity is 9,there fore all three can be allocated to the cell(2,4).

		E	sup *ply*
Ori			
R		12	6
Demand		6	21

Now the corresponding demand is 6 and requirement also 6.there fore the entire requirement will be satisfied.

IBFS is = $2\times3+11\times1+4\times2+7\times4+2\times2+8\times3+12\times6$ =Rs153

Example-2

Find initial basic feasible solution for the following transportation problem by least cost method

					sup *ply*
To	1	2	1	4	30
From	3	3	2	1	50
	4	2	5	9	20
Demand	20	40	30	10	100

Solution: since here Demand =Supply =100, the given problem is balanced transportation problem. In the above table minimum cost cells are (1,1),(1,3) and (2,4).here more than one minimum cost cells are available, therefore we can take arbitrarily any one of the minimum cost cell for allocation. Here we can choose cell (1,1). Corresponding demand is 20 but available is 30. therefore complete 20 can be allocated to the cell (1,1). Now demand is satisfied therefore we can delete column.

				sup *ply*
To	2	1	4	10
From	3	2	1	50
	2	5	9	20
Demand	40	30	10	

In the above table there are two minimum cost cells available namely (1, 3) and (2, 4). Let we choose arbitrarily (1, 3). Corresponding demand is 30 but available is 10. Therefore available 10 can be allocated to the cell (1, 3) complete 20 can be allocated to the cell (1, 3). Now demand is not satisfied therefore we can delete exhausted row.

				sup *ply*
To				
From	3	2	1	50
	2	5	9	20
Demand	30	30	10	

From the above table minimum cost cell is (2, 4).corresponding demand is 10 but availability is 50, therefore all 10 can be allocated to the cell (2, 4)

To				sup ply
From		3	2	40
		2	5	20
	Demand	30	30	

In the above table there are two minimum cost cells available namely (2, 3) and (3, 2). Let we choose arbitrarily (2, 3). Corresponding demand is 30 but available is 40. Therefore all the 30 can be allocated to the cell (2, 3). Now demand is satisfied therefore we can delete exhausted column.

To			sup ply
From		3	10
		2	20
	Demand	30	

Now the cell (3, 2) is having minimum cost, here demand is 30 but available is 20.let we allocate all 20 to that cell. There fore the exhausted row can be deleted.

To			sup ply
From		3	10
	Demand	10	

Here demand and supply is equal there fore all 10 can be allocated.

Now IBFS is= =Rs 180

Example :3

Find initial basic feasible solution for the following transportation problem by vogles method (VAM)

		Destination				
		A	B	C	D	sup ply
Origin	P	11	13	17	14	250
	Q	16	18	14	10	300
	R	21	24	13	10	400
	Demand	200	225	275	250	

Solution: here Demand=supply=950, there fore balanced transportation problem

From the above table, let we calculate penalty value by subtracting minimum value with just greater than of the same minimum value for each row and column. Among the penalty value biggest can be opted, here 5 is the maximum penalty value. But here there are two 5 is available. Let we choose any one arbitrarily; say first column can be chosen. in the first column minimum cost cell is (1,1)Here the corresponding demand is 200 but available is 250.there fore all 200 can be allocated to the minimum cost cell (1,1).exhausted column must be deleted. And again penalty will be calculated as follows

	B	C	D	SUPPLY	
	13	17	14	50	1
	18	14	10	300	4
	24	13	10	400	3
DEMAND	225	275	250		
	5	1	0		

Here maximum penalty value is 5and corresponding minimum cost cell is (2,1).demand is 225 and available is 50,all 50 can be allocated. and exhausted row will be deleted

	B	C	D	SUPPLY	
	18	14	10	300	4
	24	13	10	400	3
DEMAND	175	275	250		
	6	1	0		

Here maximum penalty value is 6and corresponding minimum cost cell is (2,2).demand is 175 and available is 300,all175 can be allocated. and exhausted column will be deleted

	C	D	SUPPLY	
	14	10	125	4
	13	10	400	3
DEMAND	275	250		
	1	0		

If we repeat same process the following tables will be obtained

	C	D	SUPPLY
	13	10	400
DEMAND	275	125	

Followed to the above table we can get the following table

	C	D	SUPPLY
	13		275
DEMAND	275		

Final allocation will be made to the cell(3,3),

there fore IBFS is $=11\times 200+13\times 50+18\times 175+10\times 125+13\times 275+10\times 125$ =Rs 12075

Finding optimal solution:

Modified Distribution (MODI) Method

The following are the steps involved in the Modified Distribution (MODI) method of testing the optimality of a feasible solution in a transportation problem:-

Step I

By using any one of the three methods discussed above, obtain an initial feasible solution, having M + N - 1 allocation in independent position.

Step II

Assign an arbitrary value to one of the variables without violating the equations. (Since there are M + N - 1 occupied cells, there will be M + N - 1 equations).

Step III

For every empty cell, calculate the improvement index i.e.; its opportunity cost. This has to be calculated by adding the corresponding row and column number and then subtracting the actual cost of this cell from it. The solution is optimal, if the opportunity cost of all the empty cells>0.

Step IV

Where the solution is not optimal i.e.; we have empty cells with negative improvement index (opportunity cost), select the empty cell having the largest value of negative opportunity cost.

Step V

For the empty cell selected in Step IV, draw a closed path - and assign alternate positive (+) and negative (-) signs at the empty cell - lying on the corner points of the path. The cell being evaluated i.e.; as selected in Step IV will have a positive (+) sign.

Step VI

Repeat this procedure till an optimal solution is achieved.

Remarks

Both the stepping stone method and MODI differ in approach but provide the same optimal solution. i.e.; both give the solution (to the transportation problem) having the lowest shipping costs

Example4: Find initial basic feasible solution for the following transportation problem by vogles method (VAM)

		Destination			
		A	B	C	$SUPPLY$
Origin	P	7	3	2	2
	Q	2	1	3	3
	R	3	4	6	5
	$Demand$	4	1	5	

Solution: here Demand=supply=10, there fore balanced transportation problem

By using VAM if we find IBFS we can get the following table

7	3	2 (2)
2	1 (1)	3 (2)
3 (4)	4	6 (1)

There fore initial basic feasible solution is= =29

For optimality

First let we check M+N-1= number of basic cells=5

(Here M= number of row and N = number of column)

There fore the problem is non-degeneracy

(i) Find $C_{ij} = u_i + v_j$ for all basic cells(the cell where allocations are made is called basic cell)

$$2 = u_1 + v_3$$
$$1 = u_2 + v_2$$
$$3 = u_2 + v_3$$
$$3 = u_3 + v_1$$
$$6 = u_3 + v_3$$

let $v_3 = 0$ there fore we can get the following values

$$u_1 = 2$$
$$u_2 = 3$$
$$u_3 = 6$$
$$v_1 = -3$$
$$v_2 = -2$$
$$v_3 = 0$$

(ii) Let we find $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$$\Delta_{11} = 7 - (u_1 + v_1) = 7 - (2 - 3) = 8$$
$$\Delta_{12} = 3 - (u_1 + v_2) = 3 - (2 - 2) = 3$$
$$\Delta_{21} = 2 - (u_2 + v_1) = 2 - (3 - 3) = 2$$
$$\Delta_{32} = 4 - (u_3 + v_2) = 4 - (6 - 2) = 0$$
$$here \quad all$$
$$\Delta_{ij} \geq 0$$

There fore this current table is optimum so optimum solution is =Rs 29

Example5:

Find initial basic feasible solution for the following transportation problem by vogles method (VAM) and find optimal solution

Destination

Origin		A	B	C	D	E	SUPPLY
	P	4	1	2	6	9	100
	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
	Demand	40	50	70	90	90	

Solution: Here Demand = supply = 340, there fore balanced transportation problem By using VAM if we find IBFS we can get the following table

4	1 (50)	2 (50)	6	9
6 (10)	4	3 (20)	5	7 (90)
5 (30)	2	6	4 (90)	8

There fore initial basic feasible solution is=

$1\times50+2\times50+6\times10+3\times20+7\times90+5\times30+4\times90$ = Rs 1410

For optimality

First let we check M+N-1= number of basic cells=7

(Here M= number of row and N = number of column)

There fore the problem is non-degeneracy

(i)Find $C_{ij} = u_i + v_j$ for all basic cells(the cell where allocations are made is called basic cell)

$$1 = u_1 + v_2$$
$$2 = u_1 + v_3$$
$$6 = u_2 + v_1$$
$$3 = u_2 + v_3$$
$$7 = u_2 + v_5$$
$$5 = u_3 + v_1$$
$$4 = u_3 + v_4$$

Let $u_2 = 0$ there fore we can get the following values

$$u_1 = -1$$
$$u_2 = 0$$
$$u_3 = -1$$
$$v_1 = 6$$
$$v_2 = 2$$
$$v_3 = 3$$
$$v_4 = 5$$
$$v_5 = 7$$

(ii) Let we find $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$$\Delta_{11} = 4 - (u_1 + v_1) = 4 - (-1 + 6) = -1 \leftarrow$$
$$\Delta_{14} = 6 - (u_1 + v_4) = 6 - (-1 + 5) = 2$$
$$\Delta_{15} = 9 - (u_1 + v_5) = 9 - (-1 + 7) = 3$$
$$\Delta_{22} = 4 - (u_2 + v_2) = 4 - (0 - 2) = 2$$
$$\Delta_{24} = 5 - (u_2 + v_4) = 5 - (0 + 5) = 0$$
$$\Delta_{32} = 2 - (u_3 + v_2) = 2 - (-1 + 2) = 1$$
$$\Delta_{33} = 6 - (u_3 + v_3) = 6 - (-1 + 3) = 4$$
$$\Delta_{35} = 8 - (u_3 + v_5) = 8 - (-1 + 7) = 2$$

here all

Δ_{ij} *not* ≥ 0

There fore this current table is not optimum, here cell (1, 1) is giving Δ_{ij} -ve .let we choose that corresponding cell for table reconstruction. Let we draw loop from cell (1,1)through basic cell corners and in all corner of the loop alternative sign must be marked. among negative corner we must take minimum allocation to add and subtract in all +ve corners and -ve corners of the loop.

4 (+)	1 **(50)**	2 **(50) (-)**	6	9
6 **(10)** **(-)**	4	3 **(20) (+)**	5	7 **(90)**
5 **(30)**	2	6	4 **(90)**	8

Here among negative corners 10 is the minimum value, this 10 to be added with all +ve corners and subtracted with all the negative corners. there fore the new table is

4 (10)	1 (50)	2 (40)	6	9
6	4	3 (30)	5	7 (90)
5 (30)	2	6	4 (90)	8

Again we have to check

(i) Find $C_{ij} = u_i + v_j$ for all basic cells

$$4 = u_1 + v_1$$
$$1 = u_1 + v_2$$
$$2 = u_1 + v_3$$
$$3 = u_2 + v_3$$
$$7 = u_2 + v_5$$
$$5 = u_3 + v_1$$
$$4 = u_3 + v_4$$

let $u_1 = 0$ there fore we can get the following values

$$u_1 = 0$$
$$u_2 = 1$$
$$u_3 = 1$$
$$v_1 = 4$$
$$v_2 = 1$$
$$v_3 = 2$$
$$v_4 = 3$$
$$v_5 = 6$$

(ii) Let we find $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$$\Delta_{14} = 6 - (u_1 + v_4) = 6 - (0+3) = 3$$
$$\Delta_{15} = 9 - (u_1 + v_5) = 9 - (0+6) = 3$$
$$\Delta_{21} = 6 - (u_2 + v_2) = 6 - (1+4) = 1$$
$$\Delta_{22} = 4 - (u_2 + v_2) = 4 - (1+1) = 2$$
$$\Delta_{24} = 5 - (u_2 + v_4) = 5 - (1+3) = 1$$
$$\Delta_{32} = 2 - (u_3 + v_2) = 2 - (1+1) = 0$$
$$\Delta_{33} = 6 - (u_3 + v_3) = 6 - (1+2) = 3$$
$$\Delta_{35} = 8 - (u_3 + v_5) = 8 - (1+6) = 1$$
$$\textit{here all}$$
$$\Delta_{ij} \geq 0$$

There fore above table is optimum

The optimum solution is=

$4\times10+1\times50+2\times40+6\times10+3\times30+7\times90+5\times30+4\times90$ =Rs1400

Degeneracy in Transportation

In transportation problem if we get M+N-1 < number of basic cell, then the transportation problem is in degenerate. It may occur either at the initial stage or at intermediate stage of the iteration. To solve this we can introduce extremely small amount to one of the empty cells of the table.

Example6:

Find initial basic feasible solution for the following transportation problem by vogles method (VAM) also find optimal solution

Destination		1	2	3	4	*SUPPLY*
	P	14	56	48	27	70
Origin	*Q*	82	35	21	81	47
	R	99	31	71	63	93
	Demand	70	35	45	60	210

Solution: Here Demand=supply=210, there fore balanced transportation problem By using VAM if we find IBFS we can get the following table

14 **(70)**	56	48	27
82	35	21 **(45)**	81 **(2)**
99	31 **(35)**	71	63 **(58)**

Here initial basic feasible solution is

$= 14\times70+21\times45+81\times2+31\times35+63\times58 = 6826$

For optimality

First let we check M+N-1= 6

But number of basic cells=5

Here M+N-1 $\neq$ number of basic cells, there fore the problem is in degeneracy

Let we introduce $\in (\in > 0)$ to the cell (1,4). So the new table becomes

14 **(70)**	56	48	27 ($\in$)
82	35	21 **(45)**	81 **(2)**
99	31 **(35)**	71	63 **(58)**

Now, M+N-1 = number of basic cells=6, there fore now it is non-degeneracy

(i) Find $C_{ij} = u_i + v_j$ for all basic cells

$$14 = u_1 + v_1$$
$$27 = u_1 + v_4$$
$$21 = u_2 + v_3$$
$$81 = u_2 + v_4$$
$$31 = u_3 + v_2$$
$$63 = u_3 + v_4$$

let $v_4 = 0$ there fore we can get the following values

$$u_1 = 27$$
$$u_2 = 81$$
$$u_3 = 63$$
$$v_1 = -13$$
$$v_2 = -32$$
$$v_3 = -60$$
$$v_4 = 0$$

(ii) Let we find $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$$\Delta_{12} = 56 - (u_1 + v_2) = 56 - (27 - 32) = 61$$
$$\Delta_{13} = 48 - (u_1 + v_3) = 48 - (27 - 60) = 81$$
$$\Delta_{21} = 82 - (u_2 + v_1) = 82 - (81 - 13) = 14$$
$$\Delta_{22} = 35 - (u_2 + v_2) = 35 - (81 - 32) = -14 \leftarrow$$
$$\Delta_{31} = 99 - (u_3 + v_1) = 99 - (63 - 13) = 49$$
$$\Delta_{33} = 71 - (u_3 + v_3) = 71 - (63 - 60) = 68$$
$$here \quad all$$
$$\Delta_{ij} not \geq 0$$

There fore this current table is not optimum, here cell (2, 2) is giving Δ_{ij} -ve. let we choose that corresponding cell for table reconstruction. Let we draw loop from cell (2, 2) through basic cell corners and in all corner of the loop alternative sign must be marked. among negative corner we must take minimum allocation to add and subtract in all +ve corners and -ve corners of the loop.

14 **(70)**	56	48	27 ∈)
82	35 +ve	21 **(45)**	81(-ve) **(2)**
99	31(-ve) **(35)**	71	63(+ve) **(58)**

Among the negative corner smallest negative corner value is 2.There fore the new table is

14 **(70)**	56	48	27 (∈)
82	35 **(2)**	21 **(45)**	81
99	31 **(33)**	71	63 **(60)**

Again (i) Find $C_{ij} = u_i + v_j$ for all basic cells

$$14 = u_1 + v_1$$
$$27 = u_1 + v_4$$
$$35 = u_2 + v_2$$
$$21 = u_2 + v_3$$
$$31 = u_3 + v_2$$
$$63 = u_3 + v_4$$

let $u_2 = 0$ there fore we can get the following values

$$u_1 = -40$$
$$u_2 = 0$$
$$u_3 = -4$$
$$v_1 = 54$$
$$v_2 = 35$$
$$v_3 = 21$$
$$v_4 = 67$$

(iii) Let we find $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$$\Delta_{12} = 56 - (u_1 + v_2) = 56 - (-40 + 35) = 61$$
$$\Delta_{13} = 48 - (u_1 + v_3) = 48 - (-40 + 21) = 67$$
$$\Delta_{21} = 82 - (u_2 + v_1) = 82 - (0 + 54) = 28$$
$$\Delta_{24} = 81 - (u_2 + v_4) = 81 - (0 + 67) = 14$$
$$\Delta_{31} = 99 - (u_3 + v_1) = 99 - (-4 + 54) = 49$$
$$\Delta_{33} = 71 - (u_3 + v_3) = 71 - (-4 + 21) = 54$$

here all

$$\Delta_{ij} \geq 0$$

There fore the optimum solution =

$$14\times 70+27\times \epsilon+35\times 2+21\times 45+31\times 33+63\times 60=6798$$

$$(27\times \epsilon=0)$$

Unbalanced Transportation Problem:

A transportation problem is said to be unbalanced if the supply and demand are not equal.

Two situations are possible:-

i. If Supply < demand, a dummy supply variable is introduced in the equation to make it equal to demand.

ii. Likewise, if demand < supply, a dummy demand variable is introduced in the equation to make it equal to supply.

Example 7: Find initial basic feasible solution for the following transportation problem by vogles method (VAM) also find optimal solution

Destination

		1	2	3	4	*SUPPLY*
	P	11	20	7	8	50
Origin	*Q*	21	16	20	12	40
	R	8	12	18	9	70
	Demand	30	25	35	40	

Solution: Here Demand $\neq$ supply there fore it is unbalanced transportation problem

In this problem Demand=215 and supply=195, that is supply is getting short of 20, there fore let we introduce dummy row with zero cost. The new table is

6	1	9	3
11	5	2	8
10	12	4	7
0	0	0	0

By using VAM if we find IBFS we can get the following table

6 **(65)**	1 **(5)**	9	3
11	5 **(30)**	2 **(25)**	8
10	12	4 **(25)**	7 **(45)**
0 **(20)**	0	0	0

IBFS is = $6 \times 65 + 1 \times 5 + 5 \times 30 + 2 \times 25 + 4 \times 25 + 7 \times 45 + 0 \times 20 = Rs1010$

ASSIGNMENT PROBLEMS

Learning Objectives

After reading this chapter the student must be able to

a. Identify the use of assignment problem

b. Solve a standard assignment problem

c. Solve an unbalanced assignment problem

d. Formulate flight scheduling problem

e. Solve special assignment problems- traveling salesman problem, impossible Assignments

Example:1: A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allows 30 minutes for lunch. The processing time in minutes and the profit for each of the product are given below:

Operators ↓	Project			
	A	B	C	D
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
Profit (Rs. Per unit)	8	6	5	4

Find the optimal assignment of product to operators.

Solution: The net working time is 450 minutes per day. The number of items that could be produced by the four operators is given in the adjoining table.

Operators ↓	Project			
	A	B	C	D
1	30	50	45	75
2	45	75	50	75
3	18	30	30	50
4	30	50	45	45

Multiplying with the corresponding profit, we obtain the adjoining matrix for finding Maximizing profit.

Operators ↓	Project			
	A	B	C	D
1	240	300	225	300
2	360	450	250	300
3	144	180	150	200
4	240	300	225	180

The given maximization problem can be converted into a minimization problem by subtracting form the largest element (i.e. 450) all the elements of the given table. The new cost table so obtained is given in Table. Apply step 1 of the Hungarian method to get the opportunity cost table in the usual manner as shown in table.

Operators ↓	Project			
	A	B	C	D
1	210	150	225	150
2	90	0	200	150
3	306	270	300	250
4	210	150	225	270

Operators ↓	Project			
	A	B	C	D
1	4	0	25	0
2	34	0	150	150
3	0	20	0	0
4	4	0	25	120

Make assignments in the above table by applying the usual method as shown in table

Operators ↓	Project			
	A	B	C	D
1	4	0	25	0
2	34	0	150	150
3	0	20	0	0
4	4	0	25	120

Operators ↓	Project			
	A	B	C	D
1	0	0	21	0
2	30	0	146	150
3	0	24	0	4
4	0	0	21	120

Solution shown in the above Table (first) is not optimal since only three assignments are made. Cover the zeros with the minimum number of lines (=3) as shown in the above table (seconed) by the usual method discussed earlier. Develop the revised cost matrix by selecting the minimum element (=4) among all uncovered elements by the lines. Subtract 4 from each uncovered element including itself and add it to the element at the intersection of the lines. A revised cost table so obtained is shown in the following table.

Operators ↓	Project			
	A	B	C	D
1	0	0	21	0
2	30	0	146	150
3	0	24	0	4
4	0	0	21	120

The optimal assignment is :

Operator		Product		Profit
1	→	D	-	300
2	→	B	-	450
3	→	C	-	150
4	→	A	-	240
			Rs.	1,140

Example:3 Solve the adjoining unbalanced assignment problem minimizing total time for doing all the jobs.

Operators ↓	Job				
	1	2	3	4	5
1	6	2	5	3	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

Step 1. Since the number of operators is not equal to the number of jobs, a dummy job 6 is created. The time consumed by any operator for the dummy job is 0.

Operators ↓	Job					
	1	2	3	4	5	6
1	6	2	5	2	6	0
2	2	5	8	7	7	0
3	7	8	6	9	8	0
4	6	2	3	4	5	0
5	9	3	8	9	7	0
6	4	7	4	6	8	0

Subtract smallest number in each column from all the numbers of that column and cover zeros by minimum number of lines (row subtraction will give the same numbers and therefore is not necessary).

Operators ↓	Job					
	1	2	3	4	5	6
1	4	0	2	0	1	0
2	0	3	5	5	2	0
3	5	6	3	7	3	0
4	4	0	0	2	0	0
5	7	1	5	7	2	0
6	2	5	1	4	3	0

Thus no more than four lines are needed to cover all zeros, which is not equal to the number of assignments. Also it may be noted that all rows contain at least one zero and 1 is the smallest value not covered by a line.

Operators ↓	Job					
	1	2	3	4	5	6
1	~~4~~	~~0~~	~~2~~	~~0~~	~~1~~	~~1~~
2	~~0~~	~~3~~	~~5~~	~~5~~	~~2~~	~~1~~
3	~~4~~	~~5~~	~~2~~	~~6~~	~~2~~	~~0~~
4	~~4~~	~~0~~	~~0~~	~~2~~	~~0~~	~~1~~
5	~~6~~	~~0~~	~~4~~	~~6~~	~~1~~	~~0~~
6	~~1~~	~~4~~	~~0~~	~~3~~	~~2~~	~~0~~

Now the minimum number of lines equals the number of assignments that can be made. An optimal assignment can now be made by selecting one zero in each row so that no two selected zeros are in the same column. Hence the optimal assignment is as follows

Operators ↓	Job					
	1	2	3	4	5	6
1	4	0 ✗	2	[0]	1	1
2	[0]	3	5	5	2	1
3	4	5	2	6	2	[0]
4	4	0 ✗	0 ✗	2	[0]	1
5	6	[0]	4	6	1	0 ✗
6	1	4	[0]	3	2	0 ✗

Operator 1 to job 4, operator 2 to job 1

Operator 3 to dummy 6, operator 4 to job 5

Operator 5 to job 2, operator 6 to job 3

Minimum time = 2 + 2 + 0 5 + 3 + 4 = 16

Example:3

A methods engineer wants to assign four new methods to three work centers. The assignment of the new methods will increase productions and they are given below. If only one method can be assigned to a work centre, determine the optimal assignment.

Methods ↓	Increase in production (unit) Work Centers			
	I	II	III	IV
1	38	29	33	22
2	26	27	28	28
3	34	26	32	29
4	33	21	26	28
5	31	26	31	26

Elements of the given matrix relate to units increase in production due to introduction of new methods, hence the given problem is of maximization type. We will, first of all, convert it into minimization problem by subtracting each element of the given matrix from the maximum element in the matrix (i.e. 38) and a dummy work centre V, has been added with zero costs for all assignments in its column, since the problem is an unbalanced one. Hence the given problem becomes:

Methods ↓	Work Centers				
	I	II	III	IV	V
A	0	9	5	16	0
B	12	11	10	10	0
C	4	12	6	9	0
D	5	17	12	10	0
E	7	12	7	12	0

The dummy column provides a zero for each row, making the row subtraction step redundant. Successive solution steps of column subtraction, line, covering, number modification, second, line-covering check, another round of number modifications, and a final line-covering check reveal the following optimal assignments.

Column subtraction and Values modified by subtracting 1 from

	I	II	III	IV	V_d
A	0	0	0	7	0
B	12	2	5	1	0
C	~~4~~	~~3~~	~~1~~	~~0~~	~~0~~
D	5	8	7	1	0
E	7	3	2	3	0

	I	II	III	IV	V_d
A	~~0~~	~~0~~	~~0~~	~~7~~	~~1~~
B	11	1	4	0	0
C	4	3	1	0	1
D	4	7	6	0	0
E	6	2	1	2	0

Column subtraction and Values modified by subtracting 1 from first zero covering

	I	II	III	IV	V_d	
A	[0]	0	0	8	2	→ First assignment – A to I
B	10	[0]	3	0	0	→ Second assignment – B to II
C	3	2	0	0	1	Matrix reduced to 3 x 3
D	3	6	5	0	0	
E	5	1	0	2	0	

Table 6.43 (a)

	III	IV	V_d
C	[0]	0	1
D	5	[0]	0
E	0	2	[0]

	III	IV	V_d
C	0	[0]	1
D	5	0	[0]
E	0	2	0

Assignment 1 :	A to I	B to II	C to III	D to IV	
Expected production	38	27	32	28	Total 125
Assignment 2 :	A to I	B to II	E to III	C to IV	
Expected production	38	27	31	29	Total 125

Example 4 (Traveling Salesman problem). A traveling salesman has to visit 5 cities. He wishes to start from a particular city, visit each city once and they return to his starting point. The traveling cost for each city from a particular city is given below:

	To city				
From City	A	B	C	D	E
A	x	4	7	3	4
B	4	x	6	3	4
C	7	6	x	7	5
D	3	3	7	x	7
E	4	4	5	7	x

What is the sequence of visit of the salesman, so that the cost is minimum?

Solution .

Step 1.

The given problem may be solved as an assignment problem with two more restrictions on our choice of assignment. Clearly, we cannot make an assignment along the diagonal and this can be avoided by filing the leading diagonal with infinitely large elements. The other restriction is that, having visited the city D, say, the salesman does not wish to visit it again until he has visited all other cities.

	To				
From	A	B	C	D	E
A	M	4	7	3	4
B	4	M	6	3	4
C	7	6	M	7	5
D	3	3	7	M	7
E	4	4	5	7	M

2. Solving the problem by assignment technique, we get Table 6.41 showing a solution indicated by encircled zeros. The zeros of this particular matrix provide a solution to the assignment problem. However, this is not a solution of the traveling salesman problem, as it gives A _ D, B _ A while B is not allowed to follow A unless C and E are processed.

	To				
From	A	B	C	D	E
A	M	~~0~~	2	[0]	~~0~~
B	[0]	M	1	~~0~~	~~0~~
C	2	1	M	3	0
D	~~0~~	[0]	3	M	4
E	~~0~~	~~0~~	[0]	3	M

We examine the matrix for some of the next best solutions to the problem and try to find one that satisfies additional restrictions. The smallest non-zero element is 1, so we try the effect of putting such an element in the solution.

	To				
From	A	B	C	D	E
A	M	0	2	0	0
B	0	M	1	0	0
C	2	1	M	3	0
D	0	0	3	M	4
E	0	0	0	3	M

We start by making an assignment from B to C using element 1 and delete row B and column C. In the remaining matrix, we observe that the assignments can be made using zeros. The set of assignments obtained is also a feasible solution to the traveling salesman problem.

4. Hence the best route for the salesman is : A _ D _ B _ C _ E _ A and total distance traveled = 3 + 3 + 6 + 5 + 4 = 21 hundred km.

Example 5: In the modification of a plant layout of a factory four new machines M1, M2, M3, and M4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M2 cannot be placed at C and M3 cannot be placed at A. The cost of locating a machine at a place (in hundred rupees) is as follows.

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	-	10	9
Machine M_3	-	11	14	11	7
M_4	14	8	12	7	8

Find the optimal assignment schedule.

Solution

As the cost matrix is not balanced, add one dummy row (machine) with a zero cost element in that row. Also assign a high cost, denoted by M, to the pair (M2, C) and (M3, A). The cost matrix so obtained is given in the Table.

Apply the usual Hungarian method of solving this problem.

	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	M	10	9
M_3	M	11	14	11	7
M_4	14	8	12	7	8
M_5	0	0	0	0	0

	A	B	C	D	E
M_1	[0]	2	6	1	2
M_2	3	[0]	M	1	~~0~~
M_3	M	4	7	4	[0]
M_4	7	1	5	[0]	1
M_5	~~0~~	~~0~~	[0]	~~0~~	~~0~~

The total minimum cost (Rs.) and optimal assignment made are as follows :

Machine	Location	Cost (in Rs.100)
M_1	A	9
M_2	B	9
M_3	E	7
Md	D	7
M5 (dummy)	C	0
	Total Rs.	32

Example:6 A traveling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The traveling cost (in Rs.'000) of each city from a particular city is given below:

	To city				
From City	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Solution:

Solving the given traveling salesman problem as an assignment problem by Hungarian method of assignment, an optimal solution is shown in Table 45. However, this solution is not the solution to the traveling salesman problem as it gives the sequence A - E - A. This violates the condition that salesmen can visit each city only once.

	To city				
From City	A	B	C	D	E
A	∞	1	3	6	[0]
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	[0]	1	∞	1
E	[0]	2	~~0~~	7	∞

The 'next best' solution to the problem which also satisfies this extra condition of unbroken sequence of visit to all cities, can be obtained by brining the next (non-zero) minimum element, i.e. 1 into the solution. In table 46, the cost 1 occurs at three different places. Therefore, consider all three different cases one by one until the acceptable solution is reached.

Case 1 :

Make the unit assignment in the cell (A, B) instead of zero assignment in the cell (A, E) and delete row A and column B so as to eliminate the possibility of any other assignment in row A and column B. Now make the assignments in the usual manner.

The resulting assignments are shown in the following table.

	To city				
From City	A	B	C	D	E
A	∞	[1]	3	6	[0]
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	~~0~~	1	∞	[1]
E	[0]	2	~~0~~	7	∞

The solution given in Table 46 gives the sequence : A ??B, B ??C, C ?D, D ??E, E ?A. The cost corresponding to this feasible solution is Rs.15,000.

Case 2

If we make the assignment in the cell (D, C) instead of (D, E), then no feasible solution is obtained in terms of zeros or which may give cost less than Rs.15,000. Hence, the best solution is : A - B - C - D - E - A, and the total cost associated with this solution is Rs.15,000.

Example 7: Solve the following assignment problem to minimize the total cost represented as elements in the matrix (cost in thousand rupees).

	Contractor			
Building	1	2	3	4
A	48	48	50	44
B	56	60	60	68
C	96	94	90	85
D	42	44	54	46

Solution

Step 1

Choose the least element in each row of the matrix and subtract the same from all the elements in each row so that each row contains at least one zero. Thus we have table

Building	Contractor 1	2	3	4
A	4	4	6	0
B	0	4	4	12
C	11	9	5	0
D	0	2	12	4

Step 2

Choose the least element in each column and subtract the same from all the elements in that column to ensure that there is at least one zero in each column. Thus we have table.

Building	Contractor 1	2	3	4
A	4	2	2	[0]
B	[0]	2	~~0~~	12
C	11	7	1	~~0~~
D	~~0~~	[0]	8	4

Step 3

We make the assignment in each row and column as explained previously. This results in table

Building	Contractor 1	2	3	4
A	4	2	2	[0]
B	[0]	2	~~0~~	12
C	11	7	1	~~0~~
D	~~0~~	[0]	8	4

Here we have only three assignment. But we must have four assignment. With this maximal assignment we have to draw the minimum number of lines to cover all the zeros. This is carried out as explained in steps 4 to 9.

Building	Contractor 1	2	3	4	
A	4	2	2	[0]	✓
B	[0]	2	~~0~~	12	
C	11	7	1	~~0~~	✓
D	~~0~~	[0]	8	4	

Step 4

Mark (_) the unassigned row (row C).

Step 5

Against the marked row C, look for any 0 element and mark that column (column4).

Step 6

Against the market column 4, look for any assignment and mark that row (row A).

Step 7

Repeat steps 6 and 7 until the chain of markings ends.

Step 8

Draw lines through all unmarked rows (row B and row D) and through all marked columns (column 4). (Check : There should be only three lines to cover all the zeros).

Step 9

Select the minimum from the elements that do not have a line through them. In this example we have I as the minimum element, subtract the same from all the elements that do not have a line through them and add this smallest element at the intersection of two lines. Thus we have the matrix shown in table 4.16. lines through all unmarked rows (row B and row D) and through all marked columns (column 4). (Check : There should be only three lines to cover all the zeros).

	Contractor			
Building	1	2	3	4
A	3	1	1	[0]
B	[0]	2	0	13
C	10	6	[0]	0
D	0	[0]	8	5

Step 10

Go ahead with the assignment with the usual procedure. Thus we have four assignments.

Building A is allotted to contractor 4

Building B is allotted to contractor 1

Building C is allotted to contractor 3

Building D is allotted to contractor 2

Total cost is 44 + 56 + 90 + 44 = Rs.234 thousands.

Exercise

1. What do you understand by transportation problem?
2. Define feasible, basic feasible, non-degenerate solution of a T.P.
3. Give reasons as to why the L.P.P solution techniques is not made use for solving aT.P
4. List any three approaches used with T.P for determining the starting solution.
5. Define the optimal solution to a T.P
6. State the necessary and sufficient condition for the existence of a feasible solution to a T.P.
7. What is the purpose of MODI method?
8. When does a T.P have a unique solution?
9. What do you mean by degeneracy in a T.P?
10. Explain how degeneracy in a T.P may be resolved?
11. What do you mean by an unbalanced T.P?
12. How do you convert the unbalanced T.P into a balanced one?
13. List the merits and limitations of using North West corner rule
14. Vogel's approximation method results in the most economical initial basic feasible solution. Why
15. How will you identify that a T.P has got an alternate optimal solution?
16. A solution that satisfies all conditions of supply and demand but it may or may not be
17. Optimal is called an initial feasible solution.
18. The transportation model is restricted to dealing with a single commodity only. Say true of false.
19. What is an assignment problem? Give two applications?
20. What do you mean by an unbalanced assignment problem?
21. Why can the transportation technique or the simplex method not be used to solve the assignment problem?
22. State the difference between the T.P and the A.P.
23. What is the objective of the traveling salesman problem?
24. How do you convert the maximization assignment problem into a minimization one?

PART-B

1. Solve the following transportation problem and find optimal solution

Destination

		A	B	C	D	E	supply
Origin	P	2	11	10	3	7	4
	Q	1	4	7	2	1	8
	R	3	9	4	8	12	9
	Demand	3	3	4	5	6	21

2. Solve the following transportation problem and find optimal solution (use least cost method)

To

						supply
From		1	2	1	4	30
		3	3	2	1	50
		4	2	5	9	20
	Demand	20	40	30	10	100

3. Find optimal solution for the following transportation problem by vogles method (VAM)

Destination

		A	B	C	D	supply
Origin	P	11	13	17	14	250
	Q	16	18	14	10	300
	R	21	24	13	10	400
	Demand	200	225	275	250	

4. Find optimal solution for the following transportation problem by vogles method (VAM)

Destination

		A	B	C	SUPPLY
Origin	P	7	3	2	2
	Q	2	1	3	3
	R	3	4	6	5
	Demand	4	1	5	

5. Find initial basic feasible solution for the following transportation problem by vogles method (VAM) and find optimal solution

Destination

		A	B	C	D	E	SUPPLY
Origin	P	4	1	2	6	9	100
	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
	Demand	40	50	70	90	90	

6. Find initial basic feasible solution for the following transportation problem by vogles method (VAM) also find optimal solution

		Destination				
		1	2	3	4	*SUPPLY*
Origin	*P*	14	56	48	27	70
	Q	82	35	21	81	47
	R	99	31	71	63	93
	Demand	70	35	45	60	210

7. Find initial basic feasible solution for the following transportation problem by vogles method (VAM) also find optimal solution

		Destination				
		1	2	3	4	*SUPPLY*
Origin	*P*	11	20	7	8	50
	Q	21	16	20	12	40
	R	8	12	18	9	70
	Demand	30	25	35	40	

8. A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allows 30 minutes for lunch. The processing time in minutes and the profit for each of the product are given below:

Operators ↓	Project			
	A	B	C	D
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
Profit (Rs. Per unit)	8	6	5	4

9. A methods engineer wants to assign four new methods to three work centers. The assignment of the new methods will increase productions and they are given below. If only one method can be assigned to a work centre, determine the optimal assignment.

Methods ↓	Increase in production (unit) Work Centers			
	I	II	III	IV
1	38	29	33	22
2	26	27	28	28
3	34	26	32	29
4	33	21	26	28
5	31	26	31	26

10. A traveling salesman has to visit 5 cities. He wishes to start from a particular city, visit each city once and they return to his starting point. The traveling cost for each city from a particular city is given below:

	To city				
From City	A	B	C	D	E
A	x	4	7	3	4
B	4	x	6	3	4
C	7	6	x	7	5
D	3	3	7	x	7
E	4	4	5	7	x

What is the sequence of visit of the salesman, so that the cost is minimum ?

NOTES

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UNIT - IV

GAME THEORY

Objective:

After reading this chapter the student must be able to

a. Formulate a game model problem

b. Recollect the assumptions game theory problem

c. Identify the usage of the model in appropriate places

Introduction

Game theory was developed for the purpose of analyzing competitive situations involving conflicting interests. In other words, game theory is used for decision making under conflicting situations where there are one or more opponents (i.e., players). For example, chess, poker, etc., are the games which have the characteristics of a competition and are played according to definite rules. Game theory provides solutions to such games, assuming that each of the players wants to maximize his profits and minimize his losses

The game theory models can be classified into several categories. Some important categories are listed below.

- **Two-person & N-person games:** If the number of players is two, it is known as two-person game. On the other hand, if the number of players is N, it is known as N-person game.

- **Zero sum & Non-zero sum game:** In a zero sum game, the sum of the points won equals the sum of the points lost, i.e., one player wins at the expense of the other. To the contrary, if the sum of gains or losses is not equal to zero, it is either positive or negative, then it is known as non-zero sum game. An example of non-zero sum game is the case of two competing firms each with a choice regarding its advertising campaign. In such a situation, both the firms may gain or loose, though their gain or loss may not be equal.

- **Games of Perfect and Imperfect information:** If the strategy of a player can be discovered by his competitor, then it is known as a perfect information game. In case of imperfect information games no player has complete information and tries to guess the real situation.

- **Pure & Mixed strategy games:** If the players select the same strategy each time, then it is referred to as pure strategy games. If a player decides to choose a course of action for each play in accordance with some particularly probability distribution, it is called mixed strategy game. There are finite number of competitors (players). The players act reasonably. Every player strives to maximize gains and minimize losses.

Each player has finite number of possible courses of action. The choices are assumed to be made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action. The pay-off is fixed and predetermined. The pay-offs must represent utilities.

Pure Strategy

The simplest type of game is one where the best strategies for both players are pure strategies. This is the case if and only if, the pay-off matrix contains a saddle point. To illustrate, consider the following pay-off matrix concerning zero sum two person game.

	Player B					
		I	**II**	**III**	**IV**	**V**
Player A	**I**	-2	0	0	5	3
	II	4	2	1	3	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

Solution.

We use the maximin (minimax) principle to analyze the game.

	Player B						
		I	**II**	**III**	**IV**	**V**	**Minimum**
Player A	**I**	-2	0	**0**	5	3	-2
	II	**4**	**2**	**1**	**3**	**2**	**1**
	III	-4	-3	**0**	-2	6	-4
	IV	5	3	**-4**	2	-6	-6
Maximum		5	3	**1**	5	6	

Select minimum from the maximum of columns.

Minimax = 1

Player A will choose II strategy, which yields the maximum payoff of 1.

Select maximum from the minimum of rows.

Maximin = 1

Similarly, player B will choose III strategy.

Since the value of maximin coincides with the value of the minimax, therefore, saddle point (equilibrium point) = 1.

The optimal strategies for both players are: Player A must select II strategy and player B must select III strategy. The value of game is 1, which indicates that player A will gain 1 unit and player B will sacrifice 1 unit.

Mixed Strategy

In situations where a saddle point does not exist, the maximin (minimax) principle for solving a game problem breaks down. The concept is illustrated with the help of following example.

Example

Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

	Company B			
		I	II	III
Company A	I	-2	14	-2
	II	-5	-6	-4
	III	-6	20	-8

Determine the optimal strategies for both the companies.

Solution.

First, we apply the maximin (minimax) principle to analyze the game.

	Company B				
		I	II	III	Minimum
Company A	I	-2	14	-2	-2
	II	-5	-6	-4	-6
	III	-6	20	-8	-8
Maximum		-2	20	-2	

Minimax = -2

Maximin = -2

There are two elements whose value is -2. Hence, the solution to such a game is not unique.

In the above problem, there is no saddle point. In such cases, the maximin and minimax principle of solving a game problem can't be applied. Under this situation, both the companies may resort to what is known as mixed strategy.

A mixed strategy game can be solved by following methods:

- Algebraic Method
- Calculus Method
- Linear Programming Method

Example-1

Consider the zero sum two person game given below:

	Player B		
Player A		I	II
	I	a	b
	II	c	d

Formulas:

The solution of the game is:

A plays (p, 1 - p)

where:

$$p = \frac{d - c}{(a + d) - (b + c)}$$

B plays (q, 1 - q)

where:

$$q = \frac{d - b}{(a + d) - (b + c)}$$

$$\text{Value of the game, } V = \frac{ad - bc}{(a + d) - (b + c)}$$

Example-2

Consider the game of matching coins. Two players, A & B, put down a coin. If coins match (i.e., both are heads or both are tails) A gets rewarded, otherwise B. However, matching on heads gives a double premium. Obtain the best strategies for both players and the value of the game.

Solution

	Player B		
Player A		I	II
	I	2	-1
	II	-1	1

Solution.

This game has no saddle point.

$$p = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 - p = 3/5$

$$q = \frac{1 - (-1)}{(2 + 1) - (-1 - 1)} = \frac{2}{5}$$

$1 - q = 3/5$

$$V = \frac{2 X 1 - (-1) X (-1)}{(2 + 1) - (-1 - 1)} = \frac{1}{5}$$

Dominance

The principle of dominance states that if one strategy of a player dominates over the other strategy in all conditions then the later strategy can be ignored. A strategy dominates over the other only if it is preferable over other in all conditions. The concept of dominance is especially useful for the evaluation of two-person zero-sum games where a saddle point does not exist.

Rules

If all the elements of a column (say ith column) are greater than or equal to the corresponding elements of any other column (say jth column), then the ith column is dominated by the jth column and can be deleted from the matrix.

If all the elements of a row (say ith row) are less than or equal to the corresponding elements of any other row (say jth row), then the ith row is dominated by the jth row and can be deleted from the matrix.

Example

	Player B				
		I	II	III	IV
Player A	I	3	5	4	2
	II	5	6	2	4
	III	2	1	4	0
	IV	3	3	5	2

Use the concept of dominance to solve this problem

Solution:

	Player B					
		I	II	III	IV	Minimum
Player A	I	3	5	4	2	2
	II	5	6	2	4	2
	III	2	1	4	0	0
	IV	3	3	5	2	2
Maximum		5	6	5	4	

There is no **saddle point** in this game.

Using dominance property

If a column is greater than another column (compare corresponding elements), then delete that column.

Here, I and II column are greater than the IV column. So, player B has no incentive in using his I and II course of action.

	Player B		
		III	IV
Player A	I	4	2
	II	2	4
	III	4	0
	IV	5	2

If a row is smaller than another row (compare corresponding elements), then delete that row.

Here, I and III row are smaller than IV row. So, player A has no incentive in using his I and III course of action.

Now we can use any one of the following to determine the value of game

- Algebraic Method
- Calculus Method

2 x n Games

Games where one player has only two courses of action while the other has more than two, are called 2 X n or n X 2 games. If these games do not have a saddle point or are reducible by the dominance method, then before solving these games we write all 2 X 2 sub-games and determine the value of each 2 X 2 sub-game. This method is illustrated by the following example.

Determine the solution of game for the pay-off matrix given below:

	Player B			
Player A		I	II	III
	I	-3	-1	7
	II	4	1	-2

Solution.

Obviously, there is no saddle point and also no course of action dominates the other. Therefore, we consider each 2 X 2 sub-game and obtain their values.

(a)

	Player B		
Player A		I	II
	I	-3	-1
	II	4	1

The saddle point is 1. So the value of game, V1 is 1.

(b)

	Player B		
Player A		I	II
	I	-3	-1
	II	4	1

This game has no saddle point, so we use the algebraic method.

$$\text{Value of game, } V2 = \frac{(-3) \text{ X } (-2) - (7 \text{ X } 4)}{(-3 - 2) - (7 + 4)} = \frac{11}{8}$$

(c)

	Player B		
Player A		II	III
	I	-1	7
	II	1	-2

This game has no saddle point, so we use the algebraic method.

$$\text{Value of game, } V3 = \frac{(-1) \times (-2) - (7 \times 1)}{(-1 - 2) - (7 + 1)} = \frac{5}{11}$$

The 2 X 2 sub-game with the lowest value is (c) and hence the solution to this game provides the solution to the larger game.

Using algebraic method:

A plays (3/11, 8/11)

B plays (0, 9/11, 2/11)

Value of game is 5/11.

Graphical Method

The method discussed in the previous section is feasible when the value of n is small, because the larger value of n will yield a larger number of 2 X 2 sub-games. In this section, we discuss another method for solving 2 X n games. This method can only be used in games with no saddle point, and having a pay-off matrix of type n X 2 or 2 X n.

Queuing theory

Objective:

After reading this chapter the student must be able to

d. Formulate a MM1 queuing model

e. Recollect the assumptions of Queuing model

f. Identify the usage of the model in appropriate places

g. Calculate the average waiting time in the system and the queue

h. Calculate the average number of people in system and queue

i. Calculate the probability of the queue length being more than a given number

j. Calculate the probability of having to wait for service for more than a given

k. length of time

Introduction about Queuing theory

Queuing theory deals with problems which involve queuing (or waiting). Typical examples might be:

??banks/supermarkets - waiting for service

??computers - waiting for a response

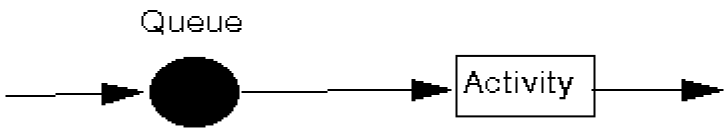
??failure situations - waiting for a failure to occur e.g. in a piece of machinery

??public transport - waiting for a train or a bus

As we know queues are a common every-day experience. Queues form because resources are limited. In fact it makes economic sense to have queues. For example how many supermarket tills you would need to avoid queuing? How many buses or trains would be needed if queues were to be avoided/eliminated?

In designing queueing systems we need to aim for a balance between services to customers (short Queues implying many servers) and economic considerations (not too many servers).

In essence all queuing systems can be broken down into individual sub-systems consisting of entities queuing for some activity (as shown below).



Typically we can talk of this individual sub-system as dealing with customers queuing for service. To analyse this sub-system we need information relating to:

Arrival process:

- how customers arrive e.g. singly or in groups (batch or bulk arrivals)
- how the arrivals are distributed in time (e.g. what is the probability distribution of time between successive arrivals (the interarrival time distribution)
- whether there is a finite population of customers or (effectively) an infinite number The simplest arrival process is one where we have completely regular arrivals (i.e. the same constant time interval between successive arrivals). A Poisson stream of arrivals corresponds to arrivals at random. In a Poisson stream successive customers arrive after intervals which independently are exponentially distributed.
- The Poisson stream is important as it is a convenient mathematical model of many real life queuing systems and is described by a single parameter - the average arrival rate. Other important arrival processes are scheduled arrivals; batch arrivals; and time dependent arrival rates (i.e. the arrival rate varies according to the time of day).

Service mechanism:

- a description of the resources needed for service to begin
- how long the service will take (the service time distribution)
- the number of servers available

❑ whether the servers are in series (each server has a separate queue) or in parallel (one queue for all servers)

❑ whether preemption is allowed (a server can stop processing a customer to deal with another "emergency" customer) Assuming that the service times for customers are independent and do not depend upon the arrival process is common. Another common assumption about service times is that they are exponentially distributed.

Queue characteristics:

❑ how, from the set of customers waiting for service, do we choose the one to be served next (e.g. FIFO (first-in first-out) - also known as FCFS (first-come first served); LIFO (last-in first-out); randomly) (this is often called the queue discipline)

❑ do we have:

_ **balking** (customers deciding not to join the queue if it is too long)

_ **reneging** (customers leave the queue if they have waited too long for service)

_ **jockeying** (customers switch between queues if they think they will get served faster by so doing)

_ **a queue** of finite capacity or (effectively) of infinite capacity

Changing the queue discipline (the rule by which we select the next customer to be served) can often reduce congestion. Often the queue discipline "choose the customer with the lowest service time" results in the smallest value for the time (on average) a customer spends queuing.

Note here that integral to queuing situations is the idea of uncertainty in, for example, interarrival times and service times. This means that probability and statistics are needed to analyse queuing situations.

In terms of the analysis of queuing situations the types of questions in which we are interested are typically concerned with measures of system performance and might include:

??How long does a customer expect to wait in the queue before they are served, and how long will they have to wait before the service is complete?

??What is the probability of a customer having to wait longer than a given time interval before they are served?

1.What is the average length of the queue?

2.?What is the probability that the queue will exceed a certain length?

What is the expected utilization of the server and the expected time period during which he will be fully occupied (remember servers cost us money so we need to keep them busy). In fact if we can assign costs to factors such as customer waiting time and server idle time then we can investigate how to design a system at minimum total cost.

These are questions that need to be answered so that management can evaluate alternatives in an attempt to control/improve the situation. Some of the problems that are often investigated in practice are:

a. Is it worthwhile to invest effort in reducing the service time?

b. How many servers should be employed?

c. Should priorities for certain types of customers be introduced?

d. Is the waiting area for customers adequate?

In order to get answers to the above questions there are two basic approaches:

1 ?analytic methods or queuing theory (formula based); and

2.?simulation (computer based).

The simple queueing systems that can be tackled via queueing theory essentially:

3.?consist of just a single queue; linked systems where customers pass from one queue to another cannot be tackled via queueing theory

4.?have distributions for the arrival and service processes that are well defined (e.g. standard statistical distributions such as Poisson or Normal); systems where these distributions are derived from observed data, or are time dependent, are difficult to analyse via queueing theory The first queueing theory problem was considered by Erlang in 1908 who looked at how large a telephone exchange needed to be in order to keep to a reasonable value the number of telephone calls not connected because the exchange was busy (lost calls). Within ten years he had developed a (complex) formula to solve the problem. Additional queueing theory information can be found.

Queueing notation and a simple example

It is common to use to use the symbols:

lamda to be the mean (or average) number of arrivals per time period, i.e. the mean arrival rate

?µ to be the mean (or average) number of customers served per time period, i.e. the mean service rate

There is a standard notation system to classify queuing systems as A/B/C/D/E, where:

- ?A represents the probability distribution for the arrival process
- ?B represents the probability distribution for the service process

- ?C represents the number of channels (servers)
- ?D represents the maximum number of customers allowed in the queuing system (either being served or waiting for service)
- E represents the maximum number of customers in total

Common options for A and B are:

- ?M for a Poisson arrival distribution (exponential inter arrival distribution) or a exponential service time distribution
- D for a deterministic or constant value
- ?G for a general distribution (but with a known mean and variance)

If D and E are not specified then it is assumed that they are infinite.

For example the M/M/1 queueing system, the simplest queueing system, has a Poisson arrival distribution, an exponential service time distribution and a single channel (one server).

MODEL 1: SINGLE SERVER (M/M/I) : (8/FCFS)

1) Ls = ρ / (1-ρ) **Where ρ = λ/μ**

Ls = Expected number of customers in system

λ = Average rate given customer in the system

μ= Departure rate given customer in the system

ρ = Traffic intensity

2) Lq = Ls - ρ

Lq = Expected number of customers in queue

3) Ws = Ls / λ

Ws = Expected waiting time in system

4) Wq = Lq / λ

Wq = Expected waiting time in queue

5) Expected time of a customer who has to wait is equal to **1 / (μ- λ).**

Example1

1) In a railway marshalling yard goods trains arrived at a rate of 30 trains/day. Assuming that interval arrival time follows in an exponential distribution with service time has an average of 36 min.

Calculate the following:

a) Length of the system

b) Find the average length of the queue

c) Expected waiting time in system

d) Waiting time in the queue

e) Expected waiting time of a train, it has to wait

f) Find the probability that greater than 10

Soln:

Arrival time λ = 30/day

= 30 / (24*60) min

= 1/48 min.

Service time μ= 1/36 min.

Traffic intensity $\rho = \lambda/\mu$

= ((1/48) / (1/36))

=3/4

a) Length of the system: (Ls)

Ls = ρ / (1-ρ)

= (¾) / (1-((¾))

= 3 trains.

b) Average length of the queue:

Lq = Ls - ρ

= 3-(¾)

= 2 trains

c) Expected waiting time in system

Ws = Ls / λ

= 3 / (1/48)

= 144 minutes.

d) Waiting time in the queue:

Wq = Lq / λ

= (9/4) / (1/40)

= 108 minutes.

e) Expected waiting time of a train, it h

= 1 / (μ- λ)

= 1 / ((1/36) - (1/48))

= 144 minutes.

f) Find the probability that greater than

= ρ?

= (3/4) ^10

= 0.056

Example-2 A TV repair men finds that the time spend on his job has an exponential distribution with the mean of 30 min. If he repairs sets in orders in which they came in and if the arrival of sets is poisson with an average rate of 10/8hrs a day.

a) What is his expected idle time/day?

b) How many jobs ahead of the average set just brought in?

Soln:

Arrival time λ = 10 / 8 hrs a day

= 10 / (8*60) min

= 1/48 min.

Service time μ= 1/30 min.

Traffic intensity ρ = λ/μ

= ((1/48) / (1/30))

= 5/8

a) Expected idle time = 1 – busy time

= 1- ρ

= 1 – (5/8)

= 3/8

Therefore out of 8 hrs his idle time is,

= (3/8)*8

= 3 hrs.

b) How many jobs ahead of the average set just brought in?

Ls = ρ / (1-ρ)

= (5/8) / (1-(5/8))

= 1.66

= 2 sets (approx)

Example-3 Cars arrive at a petrol bunk having 1petrol unit in poisson with an average of 10 cars/hr. The service time is distributed an exponentionally with a mean of 3 minutes. Find the following:

a) Average no of cars in system

b) Average waiting time in the queue

c) Average queue length

d) The probability that the no of cars in the system is 2 cars.

Soln:

Arrival time λ = 10/hr

= 10/60 min

= 1/6 min.

Service time μ= 1/3 min.

Traffic intensity $\rho = \lambda/\mu$

= ((1/6) / (1/3))

=1/2

a) Average no of cars in system:

Ls = ρ / (1-ρ)

= (1/2) / (1-(1/2))

= 1 car.

b) Average waiting time in the queue

Wq = Lq / λ

= (1/2) / (1/6)

= 3 minutes.

c) Average queue length:

Lq = Ls - ρ

= 1- (1/2)

= ½ car.

d) The probability that the no of cars in the system is 2 cars.

Pn = ρ? (1- ρ)

= $(1/2)^2$ * (1-(1/2))

= 0.125

MODEL 2: SINGLE SERVER (M/M/I) : (N/FCFS)

The M/M/1 (N/FIFO) system

It is a queuing model where the arrivals follow a Poisson process, service times are exponentially distributed and there is only one server. Capacity of the system is limited to N with first in first out mode.

The first M in the notation stands for Poisson input, second M for Poisson output, 1 for the number of servers and N for capacity of the system.

$? = ? ??$

$$P_o = \frac{1?\,?}{1?\,?^{N+1}}$$

$$L_s = \frac{?}{?-?} - \frac{(N+1)?^{N+1}}{1?\,?^{N+1}}$$

$$L_q = L_s - ? ??$$

$$W_q = \frac{L_q}{?}$$

$$W_s = \frac{L_s}{?}$$

Example1

Students arrive at the head office of www.universalteacher.com according to a Poisson input process with a mean rate of 30 per day. The time required to serve a student has an exponential distribution with a mean of 36 minutes. Assume that the students are served by a single individual, and queue capacity is 9. On the basis of this information, find the following:

- The probability of zero unit in the queue.
- The average line length.

Solution.

$$? = \frac{30}{60 \times 24}$$

= 1/48 students per minute

? = 1/36 students per minute

? = 36/48 = 0.75

N = 9

$$P_o = \frac{1 - 0.75}{1 - (0.75)^{9+1}}$$

= 0.26

$$L_s = \frac{0.75}{1 - 0.75} - \frac{(9+1)(0.75)^{9+1}}{1 - (0.75)^{9+1}}$$

= 2.40 or 2 students.

Exercise

1. What do you understand by a queue? Give some important applications of queuing theory.
2. Explain the basic queuing process.
3. What do you understand by queue discipline and input process
4. Explain the constituents of a queuing model.
5. State some of the important distributions of arrival interval and service times.
6. Give the essential characteristics of the queuing process.
7. Students arrive at the head office of www.universalteacher.com according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a student has an exponential distribution with a mean of 40 per hour. Assume that the students are served by a single individual; find the average waiting time of a student.
8. A barber with a one-man shop takes exactly 30 minutes to complete one haircut. If customers arrive according to a Poisson process at a rate of one every 40 minutes, how long on the average must a customer wait for service?

NOTES

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UNIT - V

NETWORK MODELS

Learning Objectives

After reading this chapter the student must be able to

a. Understand the application of networks

b. Draw a network using activity and event tables

c. Find the expected time of each activity in PERT networks

d. Find the earliest starting and finish time for every activity in the network

e. Find the slack of each activity in a PERT network

f. Distinguish between PERT and CPM

Introduction

Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects. One definition of a project is

A project is a temporary endeavor undertaken to create a "unique" product or Service

This definition serves to highlight some essential features of a project

??it is temporary - it has a beginning and an end

??it is "unique" in some way

With regard to the use of the word unique I personally prefer to use the idea of "non-repetitive" or "non-routine", e.g. building the very first Boeing Jumbo jet was a project - building them now is a repetitive/routine manufacturing process, not a project.

We can think of many projects in real-life, e.g. building the Channel tunnel, building the London Eye, developing a new drug, etc

Typically all projects can be broken down into:

??separate activities (tasks/jobs) - where each activity has an associated duration or completion time (i.e. the time from the start of the activity to its finish)

??precedence relationships - which govern the order in which we may perform the activities, e.g. in a project concerned with building a house the activity "erect all four walls" must be finished before the activity "put roof on" can start Operational project.

Two different techniques for network analysis were developed independently in the late 1950's - these were:

??PERT (for Program Evaluation and Review Technique); and

??CPM (for Critical Path Management)

PERT was developed to aid the US Navy in the planning and control of its Polaris missile program . This was a project to build a strategic weapons system, namely the first submarine launched intercontinental ballistic missile, at the time of the Cold War between the USA and Russia. Military doctrine at that time emphasised 'MAD - mutually assured destruction', namely if the other side struck first then sufficient nuclear weapons would remain to obliterate their homeland. That way peace was preserved. By the late 1950s the USA believed (or more importantly believed that the Russians believed) that American land based missiles and nuclear bombers were vulnerable to a first strike. Hence there was a strategic emphasis on completing the Polaris project as quickly as possible, cost was not an issue. However no one had ever build a submarine launched intercontinental ballistic missile before, so dealing with uncertainty was a key issue. PERT has the ability to cope with uncertain activity completion times (e.g. for a particular activity the most likely completion time is 4 weeks but it could be any time between 3 weeks and 8 weeks).

CPM was developed in the 1950's as a result of a joint effort by the DuPont Company and Remington Rand Univac. As these were commercial companies cost was an issue, unlike the Polaris project mentioned above. In CPM the emphasis is on the trade-off between the cost of the project and its overall completion time (e.g. for certain activities it may be possible to decrease their completion times by spending more money - how does this affect the overall completion time of the project?)

Modern commercial software packages tend to blur the distinction between PERT and CPM and include options for uncertain activity completion times and project completion time/project cost trade-off analysis. Note here that many such packages exist for doing network analysis.

There is no clear terminology in the literature and you will see this area referred to by the phrases: network analysis, PERT, CPM, PERT/CPM, critical path analysis and project planning.

Network analysis is a vital technique in **PROJECT MANAGEMENT.** It enables us to take a **systematic quantitative structured approach** to the problem of managing a project through to successful completion. Moreover, as will become clear below, it has a graphical representation which means it can be understood and used by those with a less technical background.

Example

We will illustrate network analysis with reference to the following example: suppose that we are going to carry out a minor redesign of a product and its associated packaging. We intend to test market this redesigned product and then revise it in the light of the test market results, finally presenting the results to the Board of the company.

The key question is: How long will it take to complete this project?

After much thought we have identified the following list of separate activities together with their associated completion times (assumed known with certainty).

Activity Completion number time (weeks)

1 Redesign product 6

2 Redesign packaging 2

3 Order and receive components for redesigned product 3

4 Order and receive material for redesigned packaging 2

5 Assemble products 4

6 Make up packaging 1

7 Package redesigned product 1

8 Test market redesigned product 6

9 Revise redesigned product 3

10 Revise redesigned packaging 1

11 Present results to the Board 1

It is clear that in constructing this list of activities we must make judgements as to the level of detail (timescale) to adopt. At one extreme we could have just a single activity "do the project" and at the other extreme we could try to break the project down into hourly activities. The appropriate timescale to adopt (which can be different for different activities) grows out of our knowledge of the situation and experience.

Aside from this list of activities we must also prepare a list of precedence relationships indicating activities which, because of the logic of the situation, must be finished before other activities can start e.g. in the above list activity number 1 must be finished before activity number 3 can start.

It is important to note that, for clarity, we try to keep this list to a minimum by specifying only immediate relationships, that is relationships involving activities that "occur near to each other in time".

For example it is plain that activity 1 must be finished before activity 9 can start but these two activities can hardly be said to have an immediate relationship (since many other activities after activity 1 need to be finished before we can start activity 9).

Activities 8 and 9 would be examples of activities that have an immediate relationship (activity 8 must be finished before activity 9 can start).

Note here that specifying non-immediate relationships merely complicates the calculations that need to be done - it does not affect the final result. Note too that, in the real-world, the consequences of missing out precedence relationships are much more serious than the consequences of including unnecessary (non-immediate) relationships.

Again after much thought (and aided by the fact that we listed the activities in a logical/chronological order) we come up with the following list of immediate precedence relationships.

Activity Activity

number number

1 must be finished before 3 can start

2 4

3 5

4 6

5,6 7

7 8

8 9

8 10

9,10 11

The key to constructing this table is, for each activity in turn, to ask the question: **"What activities must be finished before this activity can start"** Note here that:

??activities 1 and 2 do not appear in the right hand column of the above table, this is because there are no activities which must finish before they can start, i.e. both activities 1 and 2 can start immediately

??two activities (5 and 6) must be finished before activity 7 can start

??it is plain from this table that non-immediate precedence relationships (e.g. "activity 1 must be finished before activity 9 can start") need not be included in the list since they can be deduced from the relationships already in the list.

Once we have completed our list of activities and our list of precedence relationships we combine them into a diagram/picture (called a network - which is where the name network analysis comes from). We do this below.

Note first however that we asked the key question above:

How long will it take to complete this project?

(i.e. complete all the activities whilst respecting the precedence relationships).

What would you say, e.g.

??could we complete this project in 3 years?

??could we complete this project in 2 weeks?

One answer could be if we first do activity 1, then activity 2, then activity 3, ..., then activity 10, then activity 11. Such an arrangement would be possible here, (check the precedence relationships above), and the project would then take the sum of the activity completion times, 30 weeks.

However could we complete the project in less time? It is clear that logically we need to amend our key question to be:

What is the minimum possible time in which we can complete this project?

We shall see below how the network analysis diagram/picture we construct helps us to answer this question.

Note too here that, at this stage, we are merely planning the project - we have not started any of the various activities mentioned above.

Network diagram construction

The network diagram we construct is different depending upon whether we are going to use an activity on node or an activity on arc network.

As will become apparent below this network diagram **represents clearly and concisely the entire project -** enabling us to get a pictorial overview of what is to happen, the various activities and their relationship one to another.

PROBLEM : 1

In a boiler overhauling project following activities are to be performed:

A Inspection of boiler by boiler engineer and preparation of list of parts to be replaced/repaired.

B Collecting quotations for the parts to be purchased.

C Placing the orders and purchasing.

D Dismantling of the defective parts from the boiler.

E Preparation of necessary instruction for repairs.

F Repair of parts in the workshop.

G Cleaning of the various mountings and fittings.

H Installation of the repaired parts.

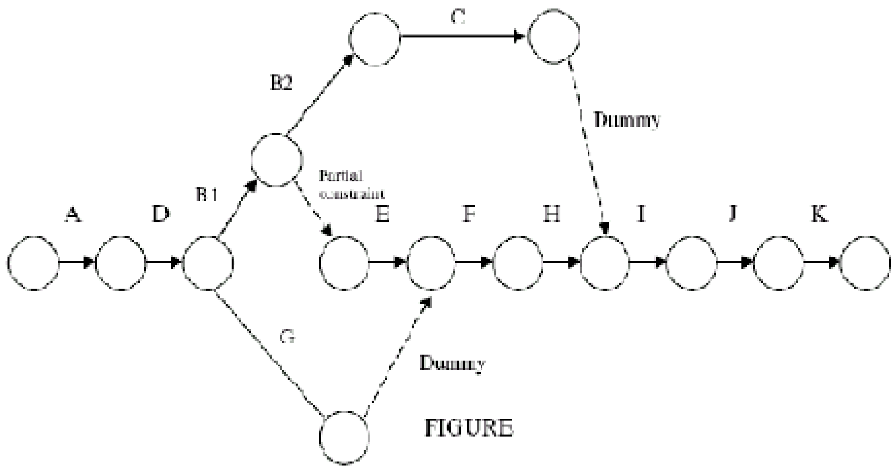
J Installation of the purchased parts.

K Trial run.

Assuming that the work is assigned to the boiler engineer who has one boiler mechanic and one boiler attendant at his disposal, draw a network showing the precedence relationships.

Solution : When we look at the list of activities, we note that activity A (inspection of boiler) is to be followed by dismantling (D) and only after that it can be decided which parts can be repaired and which will have to be replaced. Now the repairing and purchasing can go side by side. But the instructions for repairs may be prepared after sending the letters for quotations.

Note that it becomes a partial constraint. Also the cleaning of the boiler which is to be done by the attendant can be started after activity D. Now we assume that repairing will take less time than purchasing. But the installation of repaired parts can be started only when the cleaning is complete. This results in the use of a dummy activity. After the installation of repaired parts, installation of purchased parts can be taken up. This will be followed by inspection and trial run.



FIGURE

PROBLEM : 2

Following are the activities which are to be performed for a building site preparation.

Determine the precedence relationship and draw the network.

A Clear the site.

B Survey and layout.

C Rough grade.

D Excavate for sewer.

E Excavate for electrical manholes.

F Install sewer and backfill.

G Install electrical manholes.

H Construct the boundary wall.

Solution : Looking at the list of activities we can fix the following precedence order :

B succeeds A and C succeeds B i.e., B > A; C > B.

D and E can start together after the completion of C, i.e., D, E > C.

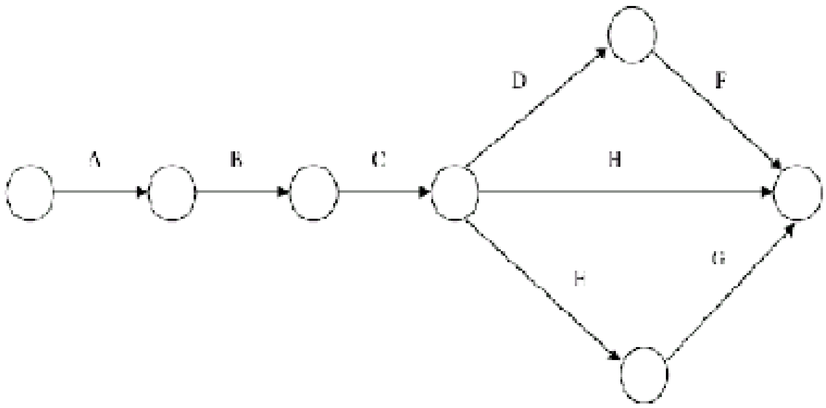
F will follow D and G will follow E, i.e., F > D; G>E.

H can start after C i.e., H > C.

Thus the precedence relationships are :

B > A; C > B; D, E, H > C; F > D and G > E.

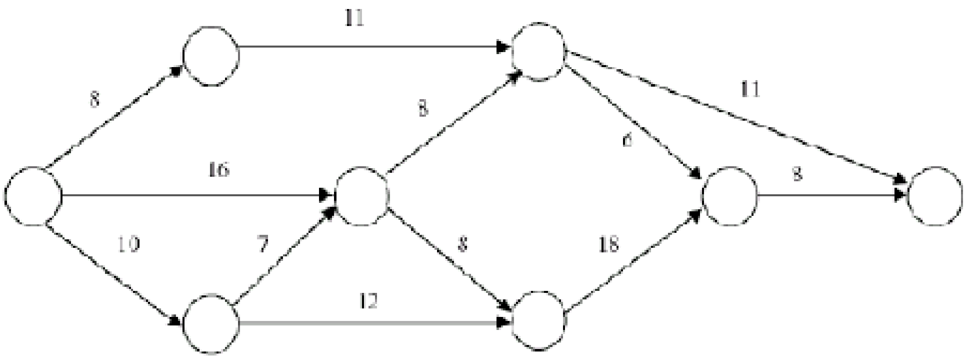
The project can be represented in the form of a network as shown in figure.



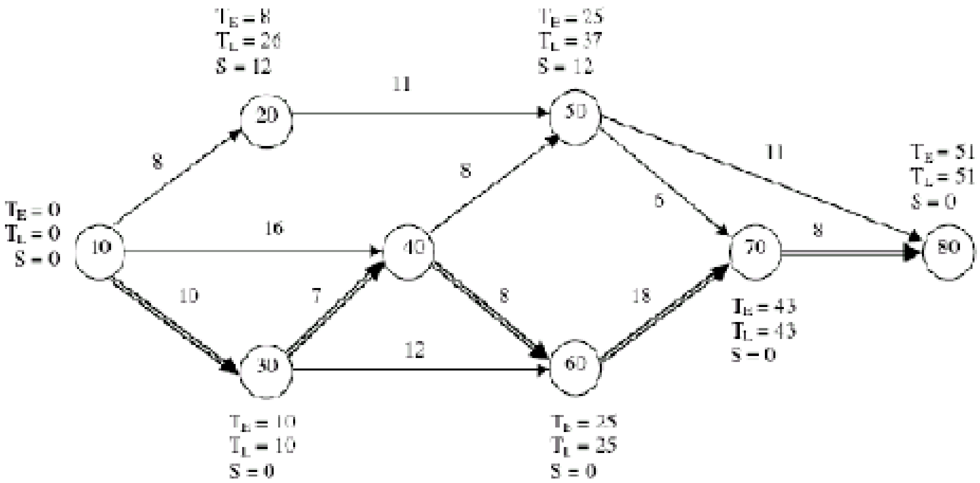
PROBLEM : 3

Consider the network shown in figure 15.5. The activitiy times in days are given along the arrows. Calculate the slacks for the events and determine the critical path. Put the calculations in tabular form.

Solution : The events of the network are first numbered and then TE and TL values are calculated. Figure 15.6 shows the network with the numbers in the node circles and TE, TL and S values along the nodes.



FIGURE



Activity ij		Activity	Earliest		Latest		
Predecessor event (i)	Successor event (j)	duration (T_E^{ij})	Start time (T_E^i)	Finish time (T_E^j)	Start time (T_L^i)	Finish time (T_L^j)	Slack $S_{ij}=T_L^j - T_E^j$ ($=T_L^i - T_E^i$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10	20	8	0	8	18	26	18
10	30	10	0	10	0	10	0
10	40	16	0	16	1	17	1
20	50	11	8	19	26	37	18
30	40	7	10	17	10	17	0
30	60	12	10	22	13	25	3
40	50	8	17	25	29	37	12
40	60	8	17	25	17	25	0
50	70	6	25	34	37	43	12
50	80	11	25	36	40	51	15
60	70	18	25	43	25	43	0
70	80	8	43	51	43	51	0

From the computations carried on the network as well as in table 15.3, we find that the slack is zero for activities 10-30, 30-40, 40-60, 60-70, 70-80 and non-zero for all others.

Now, to determine the critical path we apply the conditions of section 15.5. Note activity, thought the slack at both the end events is zero. This does not satisfy the condition TE

j -

TE

i = TL

j - TL

i = TE

ij. This activity can be expanded by three days. Thus the critical path is 10-30-40-60-70-80, and the same is shown with heavy lines on the network.

Example-4

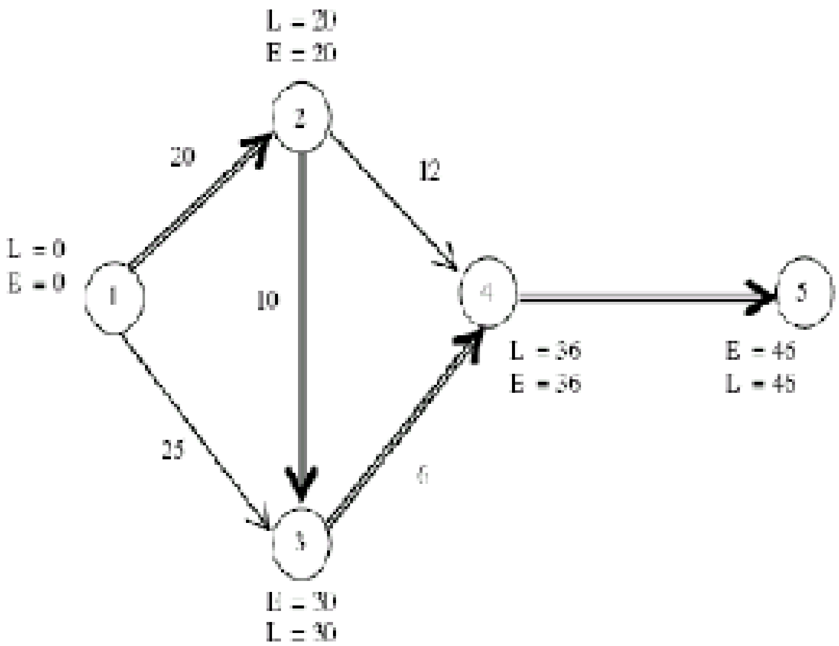
The following table gives the activities in a construction project and other relevant information.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration (days) :	20	25	10	12	6	10

(i) Draw the network for the project.

(ii) Find the critical path the project duration.

(iii) Find the total float for each activity.



FIGURE

Activity	Duration	EST	EFT	LST	LFT	TF
1-2	20	0	20	0	20	0
1-3	25	0	25	5	30	5
2-3	10	20	30	20	30	0
2-4	12	20	32	24	36	4
3-4	6	30	36	30	36	0
4-5	10	36	46	36	46	0

The critical path is 1-2-3-4-5

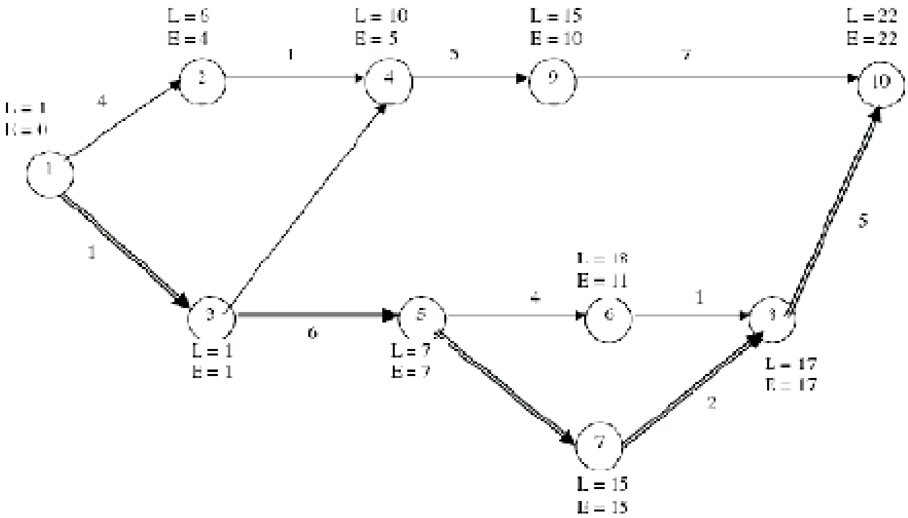
The project duration is 46 days.

Example-5

A project scheduling has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-0
Time	4	1	1	1	6	5	4	8	1	2	5	7

Summarise the CPM calculations in a tabular form and determine the critical path.



FIGURE

Activity	Duration	EST	EFT	LST	LFT	TF
1 – 2	4	0	4	2	9	5
1 – 3	1	0	1	0	1	0
2 – 4	1	4	5	9	10	5
3 – 4	1	1	2	9	10	8
3 – 5	6	1	7	1	7	0
4 – 9	5	5	10	10	15	5
5 – 6	4	7	11	12	16	5
5 – 7	8	7	15	7	15	0
6 – 8	1	11	12	16	17	5
7 – 8	2	15	17	15	17	0
8 – 10	5	17	22	17	22	0
9 – 10	7	10	17	15	22	5

Example 8

The following table lists the jobs of a network with their time estimates

Job i-j	Duration (days)		
	Optimistic	Most likely	Pessimistic
1 2	3	6	15
1 6	2	5	14
2 3	6	12	30
2 4	2	5	8
3 5	5	11	17
4 5	3	6	15
6 7	3	9	27
5 8	1	4	7
7 8	4	19	28

(a) Draw the project network

(b) Calculate the length and variance of the critical path.

(c) What the is approximate probability that the jobs on the critical path will be completed path will be completed by the due date of 42 days?

(d) What due date has about 90% chance of being met?

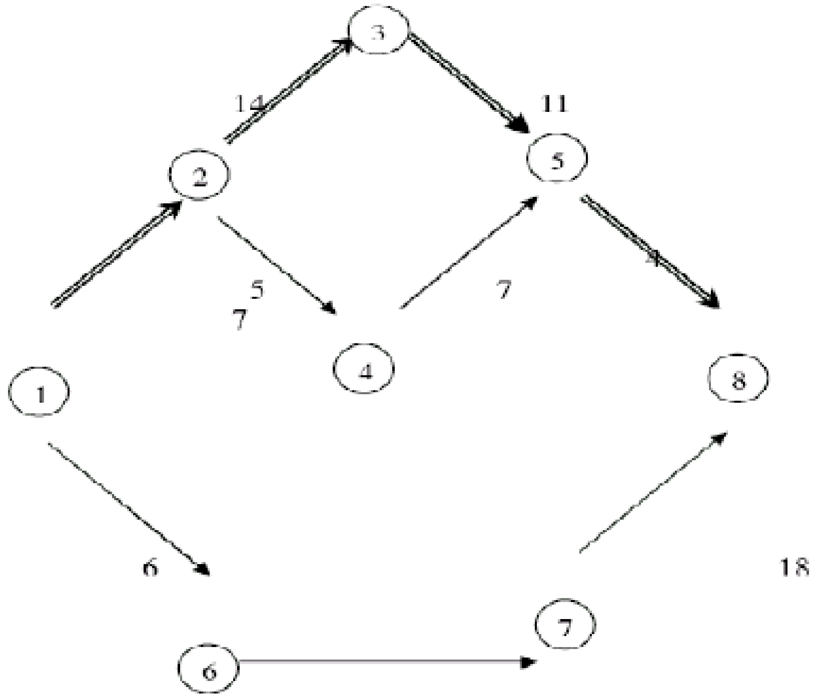
Solution :

Before proceeding to draw the project network, let us calculate the expected time of activity t e, standard deviation and variance of the excepted time of activity using.

$$t_e = \frac{(t_o + 4t_m + t_p)}{6}$$

$$S.D = (t_p - t_o) / 6; \text{ Variance } = (S.D)^2$$

Activity	t_e (Days)	S.D(Days)	Variance
1-2	7	2	4
1-6	6	2	4
2-3	14	4	16
2-4	5	1	1
3-5	11	2	4
4-5	7	2	4
6-7	11	4	16
5-8	4	1	1
7-8	18	4	16



(b) There are three paths:

1-2-3-5-8 = 36 days

1-2-4-5-8 = 23 days

1-6-7-8= 35 days

1-2-3-5-8 is the longest path and hence the critical path.

Expected length of the critical path is 36 days. The variance for 1-2, 2-3, 3-5 and 5-8 are

4, 16, 4 and 1 respectively and variance of the projection duration is 25. and hence.

Standard deviation of the project duration = 25 = 5 days.

(c) Due date = 42 days (T)

Expected duration = 36 days (Te) and S.D = 5 days (0)

Exercise

1. From the following find

Activity	Optimistic	Most Likely	Pessimistic
0-1	2 days	3.5 days	8 days
0-2	3	3.75	6
0-3	1	2.5	7
1-2	3	7.5	9
1-5	4	5.5	10
2-4	2	5	8
3-4	2	2.75	5
3-5	3	6	9
4-5	2	5	8

(i) Construct a network for the data and find the expected completion time of the project.

(ii) Find the probability of completing the project 3 days ahead of the expected schedule.

2. consider the project network given below. The project executive has made estimates of the optimistic most likely and pessimistic times (indays) for completion of the various activities as given below.

(i) Find the critical path.

(ii) Determine the expected project completion time and its variance.

(iii) What is the probability that the project will be completed in 30 days?

Activity	Optimistic	Most Likely	Pessimistic
A (1-2)	2	5	14
B (1-3)	9	12	15
C (2-4)	5	14	17
D (3-4)	2	5	8
E (4-5)	6	9	12
F (3-5)	8	17	20

3. from the following find

(i) Find the critical path.

(ii) Determine the expected project completion time and its variance.

(iii) What is the probability that the project will be completed in 20 day

Activity	Preceding Activity	Succeeding Activity	Time Optimistic	Estimates in most likely	Weeks Pessimistic
A	-	C	5	7	15
B	-	D	2	3	4
C	A	E	7	9	17
D	B	F	5	6	13
E	C	-	4	5	12
F	D	-	2	4	6

NVENTORY MODELS

Objective

- To minimize the possibility of delays in production through regular supply of raw materials.
- To keep inactive, waste, surplus, scrap and obsolete items at the minimum level.
- To maintain the overall investment in inventory at the lowest level, consistent with operating requirements.
- To exercise economies in ordering, obtaining, and storing of the materials.

Introduction

The word 'inventory' means simply a stock of idle resources of any kind having an economic value. In other words, inventory means a physical stock of goods, which is kept in hand for smooth and efficient running of future affairs of an organization. It may consist of raw materials, work-in-progress, spare parts/consumables, finished goods, human resources such as unutilized labour, financial resources such as working capital, etc. It is not necessary that an organization has all these inventory classes. But whatever may be the inventory items, they need efficient management as generally, a substantial amount of money is invested in them. The basic inventory decisions include:

- How much to order?
- When to order?
- How much safety stock should be kept?

The problems faced by different organizations have necessitated the use of scientific techniques in the management of inventories known as inventory control.

Inventory models can be classified according to the following factors:

1. Inventory related costs

Inventory related costs are classified as

- **Purchase (or production) cost**. It is the cost at which an item is purchased, or if an item is produced, it is the direct manufacturing cost. In many practical situations, the unit purchase price depends on the quantity purchased so the purchase price is of special interest when large quantities are bought or when large production runs may result in a decrease in the production cost.

- **Ordering (or replenishment or set up) cost.** The cost incurred in replenishing the inventory is known as ordering cost. It includes all the costs relating to administration (such as salaries of the persons working for purchasing, telephone calls, computer costs, postage, etc.), transportation, receiving and inspection of goods, processing payments, etc. If a firm produces its own goods instead of purchasing the same from an outside source, then it is the cost of resetting the equipment for production. This cost is expressed as the cost per order or per set up. It is denoted by Co.

- **Carrying (or holding) cost.** The cost associated with maintaining the inventory level is known as holding cost. It is directly proportional to the quantity to be kept in stock and the time for which an item is held in stock. It includes handling cost, maintenance cost, depreciation, insurance, warehouse rent, taxes, etc.

This cost may be expressed either as per unit of item held per unit of time or as a percentage of average rupee value of inventory held. It is denoted by Ch.

- **Shortage (or stock out) cost.** It is the cost, which arises due to running out of stock (i.e., when an item can not be supplied on the customer's demand). It includes the cost of production stoppage, loss of goodwill, loss of profitability, special orders at higher price, overtime/idle time payments, expediting, loss of opportunity to sell, etc. It is denoted by Cs.

2. Demand

It is an effective desire which is related with a particular time, price, and quantity. The demand pattern of a commodity may be either deterministic or probabilistic. In case of deterministic, it is assumed that the quantities needed in future are known with certainty. This can be fixed (static) or can vary (dynamic) from time to time. To the contrary, in case of probabilistic, the demand over a certain period of time is uncertain, but its pattern can be described by a known probability distribution.

3. Ordering cycle

An ordering cycle is defined as the time period between two successive placement of orders. The order may be placed on the basis of following two types of inventory review systems:

- **Continuous review:** In this case, record of the inventory level is updated continuously until a specified point (known as reorder point) is reached, at this point a new order is placed. Sometimes, this is referred to as the two-bin system. The inventory is divided into two parts (two bins). Initially, items are used only from one bin, and when it becomes empty, a new order is placed. Demand is then satisfied from the second bin until the order is received. After receiving the order, the second bin is filled to make up the earlier total. The remaining items are placed in the first bin.
- **Periodic review:** In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.

4. Time horizon

This is also known as planning period over which the inventory level is to be controlled. This can be finite or infinite depending on the nature of demand.

5. Lead time or delivery lag

The time gap between the moment of placing an order and actually receiving the order is referred to as lead time. The lead time can be deterministic, constant or variable, or probabilistic. If there is no such gap, then we say that lead time is zero. If the lead time exists (i.e., it is not zero), then it is required to place an order in advance by an amount of time equal to the lead time.

6. Buffer (or safety) stock

Normally, demand and lead time are uncertain and cannot be predetermined completely. So to absorb the variation in demand and supply, some extra stock is kept. This extra stock is known as buffer stock.

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12. Number of items

Generally, an inventory system involves more than one commodity. The number of items held in inventory affect the situation when these items compete for limited floor space or limited total capital.

13. Government's policy

For items to be imported as well as for other items like explosive, highly inflammable, and other essential items, the Government has laid down some policy norms. All these affect the level of inventories in an organization.

Advantages

- It enables the material to be procured in economic quantities.
- It eliminates delays in production caused by the non-availability of required materials.
- It works as a check on the over accumulation of inventories and thereby results in minimum investment consistent with production requirements.
- It reduces inventory losses caused by inadequate inspection of incoming materials and losses due to obsolescence, deterioration, waste and theft while in storage.
- It ensures proper execution of policies covering procurement and use of materials. It also facilitates timely adjustment with changing conditions in the market.

Techniques of Inventory Control

- ABC analysis
- VED analysis
- ABC analysis

Under this technique, the inventory items are divided into three groups, viz., A, B and C on the basis of the investment involved.

Category (or group)	Percentage of the items	Percentages of the total annual value of the inventories (Rs.)
A	10-20	70-85
B	20-30	10-25
C	60-70	5-15

In fact, ABC analysis indicates the items of raw materials to be controlled by managers at different levels. The managers are responsible for ensuring optimal investment in raw materials.

VED analysis

This analysis consists of separating the inventory items into three groups according to their critically as under:

1. Vital items (or V items) - These items are considered vital for smooth running of the system and without these items the whole system becomes inoperative. Thus, close attention is paid to V items.

2. Essential items (or E items) - These items are considered essential for efficient running of the system.

3. Desirable items (or D items) - The availability of these items help in increasing the efficiency.

The criticality of the item may either be on technical grounds or on environmental grounds or on both.

ABC analysis coupled with VED analysis enhances the efficiency of control on inventories.

The Basic Deterministic Inventory Models

Before examining the solution of specific inventory models, we provide the notations used in the development of these models.

Q = Number of units ordered per order.

D = Rate of demand.

N = Number of orders placed per year.

TC = Total inventory cost

Co = Cost of ordering per order

C = Purchase or manufacturing price per unit

Ch = Cost of holding stock per unit per period of time.

Cs = Shortage cost per unit.

R = Reorder point

L = Lead time (weeks or months)

t = The elapsed time between placement of two successive orders.

Model I - Economic Order Quantity Model With Uniform Demand

The main problem while purchasing material is how much to buy at a time. If large quantities are bought, the cost of carrying the inventory would be high. To the contrary, if frequent purchases are made in small quantities, costs relating to ordering will be high. So the problem is of indecision. How this problem can be resolved ?

EOQ is that size of the order for which the cost of maintaining inventories is minimum. Therefore, the quantity to be ordered at a given time should be determined by taking into account two factors, i.e., the acquisition cost and the cost of possessing materials. We illustrate this model after making the following assumptions:

Assumptions

· Demand rate is uniform over time and is known with certainty.

- The inventory is replenished as soon as the level of the inventory reaches to zero. Thus shortages are not allowed.
- Lead time is zero.
- The rate of inventory replenishment is instantaneous.
- Quantity discounts are not allowed.

The inventory costs are determined as follows:

1. Ordering cost = Number of orders per year X Ordering cost per order = N X Co = Total annual demand/Number of units ordered X Co

$$= \frac{D}{Q} \times C_o$$

2. Carrying cost = Average inventory X carrying cost per unit

$$= \frac{Q}{2} \times C_h$$

The total inventory cost is the sum of ordering cost and carrying cost.

$$TC = \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

The total cost is minimum at a point where ordering cost equals carrying cost. Thus, economic order quantity occurs at a point where

Ordering cost = Carrying cost

$$\frac{D}{Q} C_o = \frac{Q}{2} C_h$$

Thus, optimal Q* (EOQ) is derived to be

$$EOQ = Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

The period t is given by $t^* = \frac{Q^*}{D} = \frac{2C_o}{C_h D}$

Optimal number of orders per year is given by $N^* = \frac{D}{Q^*} = \frac{1}{t^*}$

Minimum total yearly inventory cost $TC^* = 2DC_oC_h$

Example -1

Annual usage	500 pieces
Cost per piece	Rs. 100
Ordering cost	Rs. 10 per order
Inventory holding cost	20% of Average Inventory

Find EOQ

Solution.

D = 500 pieces

Co = 10

Ch = 100 X 20% = Rs. 20

$$EOQ = \frac{(2 \times 10 \times 500)}{20}$$

EOQ = 22 pieces (rounded)

Example 2

The Newtech Hardware Company sells hardware items. Consider the following information.

Annual sales = Rs. 10000

Ordering cost = Rs. 25 per order

Carrying cost = 12.5% of average inventory value.

Find the optimal order size, number of orders per year, and cycle period.

Solution.

D = Rs. 10000, Co = Rs. 25, Ch = 12.5% of average inventory value/unit

$$EOQ = Q^* = \frac{(2 \times 25 \times 10000)}{(0.125)} = \text{Rs. } 2000$$

$$t^* = \frac{Q^*}{D} = \frac{2000}{10000} = 1/5 \text{ yrs} = 73 \text{ days}$$

$$N^* = \frac{1}{t^*} = \frac{1}{1/5} = 5$$

Model II - EOQ When Shortages Are Allowed

In this case, shortages are permitted which implies that shortage cost is finite or it is not large. The cost of a shortage is assumed to be directly proportional to the mean number of units short. Further, all the assumptions of model I hold good here also. The model is graphed in the following figure.

where

S = Back order quantity.

M = Maximum inventory level.

t1 = Time during which stock is available.

t2 = Time during which there is a shortage.

t = Time between receipt of orders.

$$Q^{*} = \sqrt{\frac{2DC_o}{C_h} \times \frac{(C_h + C_o)}{C_s}}$$

$$M^{*} = \sqrt{\frac{(2DC_o)}{(C_h)} \times \frac{C_s}{(C_h + C_s)}}$$

$$t^{*} = \sqrt{\frac{2C_o}{DC_h} \times \frac{(C_s + C_h)}{C_s}}$$

$$T^{C*} = \sqrt{2DC_o C_h \times \frac{C_s}{(C_s + C_h)}}$$

The Wartsila Diesel Company has to supply diesel engines to a truck manufacturer at a rate of 10 engines per day. The ordering cost is Rs. 150 per order. The penalty in the contract is Rs. 90 per engine per day late for missing the scheduled delivery date. The cost of holding an engine in stock for one month is Rs. 140. His production process is such that each month (30 days) he starts procuring a batch of engines through the agencies and all are available for supply after the end of the month. Determine the maximum inventory level at the beginning of each month.

Solution.

Given

Demand (D) = 10 engines per day

Shortage cost (Cs) = Rs. 90 per day per engine

Carrying cost (Ch) = 140/30 = 14/3 per engine per day

Ordering cost (Co) = Rs. 150 per order

$$M^{*} = \sqrt{\frac{2 \times 10 \times 150}{14/3} \times \frac{90}{14/3 + 90}} \times 30$$

= 741.65 = 742 engines (approx.)

Exercise

1.A particular Company sells hardware items. Consider the following information.

Annual sales = Rs. 20000

Ordering cost = Rs. 50 per order

Carrying cost = 13.5% of average inventory value.

Find the optimal order size, number of orders per year, and cycle period.

2.

Annual usage	200 pieces
Cost per piece	Rs. 10
Ordering cost	Rs. 10per order
Inventory holding cost	10% of Average Inventory

Find EOQ

NOTES